Critical Point

Happy Birthday to Marek Gazdzicki

60

Marek Gazdzicki

Short Scientific Summary

Dubna SKM-200	CERN NA35	NA49	NA61
(1982-86)	(1987-95)	(1996-2010)	(2011-...)

STAR BNL, ALLICE CERN

Total number of publications 336
Total number of citations 14811
H-index 64

Short history of my collaboration with Marek Gazdzicki

1983 – we met in Dubna (Marek Gazdzicki and Stanislaw Mrowczynski were PhD students)

1988 – we met in Frankfurt

1998 – we published our first joint paper

36 joint theoretical papers are published

First Paper of Marek Gazdzicki

объединенный
институт
ядерных
исследований
дубна

E1-80-651

M.Kh.Anikina, L.V.Chkhaidze,[1] M.Gazdzicki,[2]
A.I.Golokhvastov, S.A.Khorozov, E.S.Kuznetsova,
J.Lukstins, E.O.Okonov, T.G.Ostanevich,
E.Skrzypczak,[2] R.Szwed,[2] G.L.Vardenga,
M.S.Zhuravleva

CHARACTERISTICS OF π^- -MESON MULTIPLICITY DISTRIBUTIONS IN CENTRAL COLLISIONS OF ^{12}C AND ^{16}O WITH NUCLEI AT P = 4.5 GeV/c PER INCIDENT NUCLEON

Submitted to "Nuclear Physics"

[1] Tbilisi State University, USSR.
[2] Institute of Experimental Physics, University of Warsaw, Poland.

1980

INSTITUTE OF EXPERIMENTAL PHYSICS
WARSAW UNIVERSITY
Hoza 69, 00-681 Warsaw Polland
IFD/4/81, Preprint

Thermodynamic models approach to the π-meson production in relativistic ion collisions

M. Gazdzicki, P. Danielewicz
University of Warsaw, Warsaw, Poland

K. Lang
Institute of Nuclear Research, Warsaw, Poland

Warsaw, 1981

First (theoretical) paper of Marek Gazdzicki

PHYSICAL REVIEW C VOLUME 30, NUMBER 1 JULY 1984

Difficulties of the thermodynamical model approach to pion production in relativistic ion collisions

M. Gaździcki
Institute of Experimental Physics, University of Warsaw, Warsaw, Poland and High Energy Laboratory, Joint Institute for Nuclear Research, Dubna, USSR

St. Mrówczyński
Institute for Nuclear Studies, Warsaw, Poland and High Energy Laboratory, Joint Institute for Nuclear Research, Dubna, USSR

(Received 17 November 1983)

Thermodynamical models with various forms of partial transparency of nuclear matter are considered. It is shown that the introduction of transparency, however, significantly improves agreement with pion data concerning multiplicities and transverse momenta leads to a serious discrepancy with average rapidities of pions. Qualitative arguments are given that difficulties of the thermodynamical approach can be overcome if one assumes hydrodynamical expansion in the first stage of nuclear interactions.

Another theoretical paper of Marek Gazdzicki

Volume 252, number 3 PHYSICS LETTERS B

A new variable to study intermittency

A. Bialas [1] and M. Gazdzicki [2]
CERN, CH-1211 Geneva 23, Switzerland

Received 17 September 1990

It is proposed to study intermittency properties of particle spectra using the variables for which the single-particle distribution is constant. The construction of such a set of variables is described. It is shown that this method drastically reduces distortions of intermittency due to a non-uniform single-particle density. A method of systematic analysis of intermittency in three dimensions based on the new variable is suggested.

One more theoretical paper of Marek Gazdzicki

Our first joint papers, Frankfurt, 1998

General Scheme:

1. **Marek suggested new physical idea.**

2. **Marek published short paper on this subject.**

3. **Marek published joint paper with Stanislaw Mrowczynski on this subject .**

4. **Marek proposed me: "let us do now the rigorous statistical formulation of this".**

Z. Phys. C 66 (1995) 659–662

ZEITSCHRIFT
FÜR PHYSIK C

Entropy in nuclear collisions

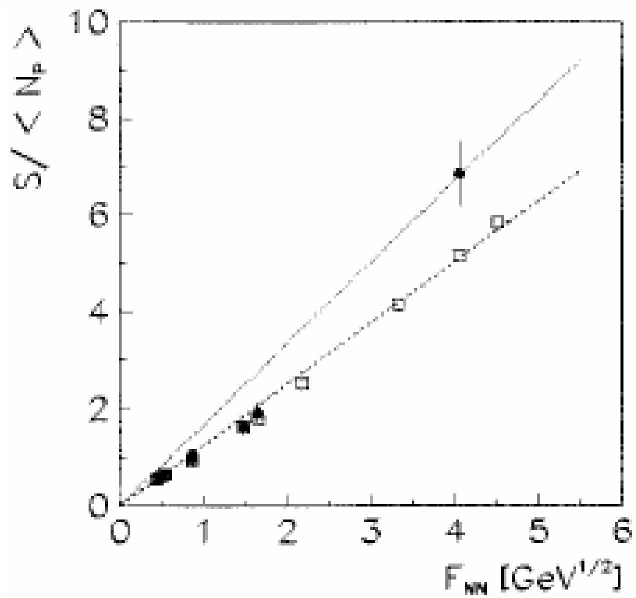
Marek Gaździcki

Institute für Kernphysik, Universität Frankfurt, August-Euler-Strasse 6, D-60486 Frankfurt, Germany
(e-mail: marek@hvax.ikf.physik.uni-frankfurt.de)

Received: 25 October 1994

$$S \sim g^{1/4} \langle N_P \rangle F$$

$$\frac{g_P}{g_H} = 1.33^4 \approx 3 .$$



Eur. Phys. J. C 5, 129–133 (1998)
DOI 10.1007/s100529800830

THE EUROPEAN PHYSICAL JOURNAL C

Pion suppression in nuclear collisions

Marek Gaździcki[1], Mark I. Gorenstein[2], Stanisław Mrówczyński[3,4]

[1] Institut für Kernphysik, Universität Frankfurt, August-Euler-Strasse 6, D-60486 Frankfurt, Germany (e-mail: marek@ikf.physik.uni-frankfurt.de)
[2] Bogolyubov Institute for Theoretical Physics, UK-252143 Kiev, Ukraine (e-mail: goren@ap3.gluk.apc.org)
[3] Sołtan Institute for Nuclear Studies, ul. Hoża 69, PL-00-681 Warsaw, Poland (e-mail: mrow@fuw.edu.pl)
[4] Institute of Physics, Pedagogical University, ul. Leśna 16, PL-25-509 Kielce, Poland

Received: 21 July 1997

Abstract. The pion multiplicity per participating nucleon in central nucleus–nucleus collisions at the energies 2–15 A·GeV is significantly smaller than in nucleon–nucleon interactions at the same collision energy. This effect of pion *suppression* is argued to appear due to the evolution of the system produced at the early stage of heavy–ion collisions towards a local thermodynamic equilibrium and further isentropic expansion.

Our first joint paper with Marek


Mehr als Durchschnitt...
Renner
CHOPIN

INSTITUT FÜR KERNPHYSIK, UNIVERSITÄT FRANKFURT
D - 60486 Frankfurt, August–Euler–Strasse 6, Germany

IKF–HENPG/2–98

On the Early Stage of Nucleus–Nucleus Collisions

Marek Gaździcki[1]
Institut für Kernphysik, Universität Frankfurt, Germany

Mark I. Gorenstein[2,3]
Institute for Theoretical Physics, University of Frankfurt, Germany
and
School of Physics and Astronomy, Tel Aviv University, Israel

A statistical model of the early stage of central nucleus–nucleus (A+A) collisions is developed. We suggest a description of the confined state with several free parameters fitted to a compilation of A+A data at the AGS. For the deconfined state a simple Bag model equation of state is assumed. The model leads to the conclusion that a Quark Gluon Plasma is created in central nucleus–nucleus collisions at the SPS. This result is in quantitative agreement with existing SPS data on pion and strangeness production and gives a natural explanation for their scaling behaviour. The localization and the properties of the transition region are discussed. It is shown that the deconfinement transition can be detected by observation of the characteristic energy dependence of pion and strangeness multiplicities, and by an increase of the event–by–event fluctuations. An attempt to understand the data on J/ψ production in Pb+Pb collisions at the SPS within the same approach is presented.

February 7, 2008

[1]E–mail: marek@ikf.physik.uni–frankfurt.de
[2]Permanent address: Bogolyubov Institute for Theoretical Physics, Kiev, Ukraine
[3]E–mail: goren@th.physik.uni-frankfurt.de

Statistical Model of the Early Stage In A+A Collisions

1. Different EoS for confined and deconfined matter.

2. Different masses and degeneracy factors for strange and non-strange particles.

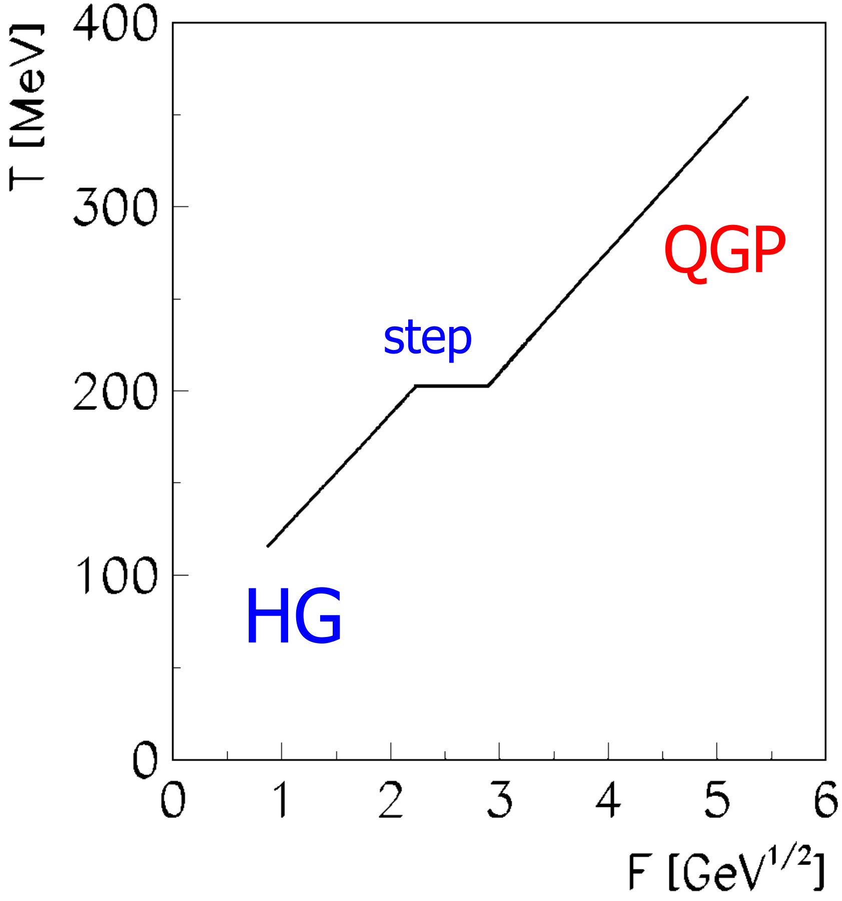
3. Bag model EoS for the deconfined phase.

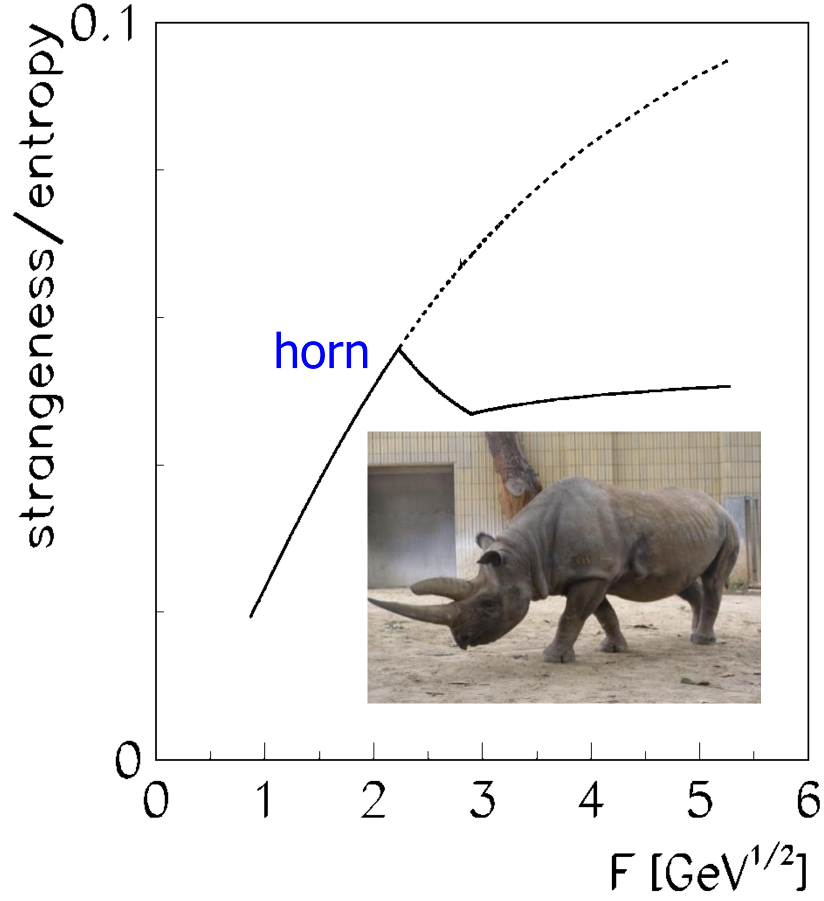
4. First order phase transition (Gibbs criteria).

Let us do now the rigorous statistical formulation of this!

For example: Gibbs criteria = Maximum of Entropy (Appendix)

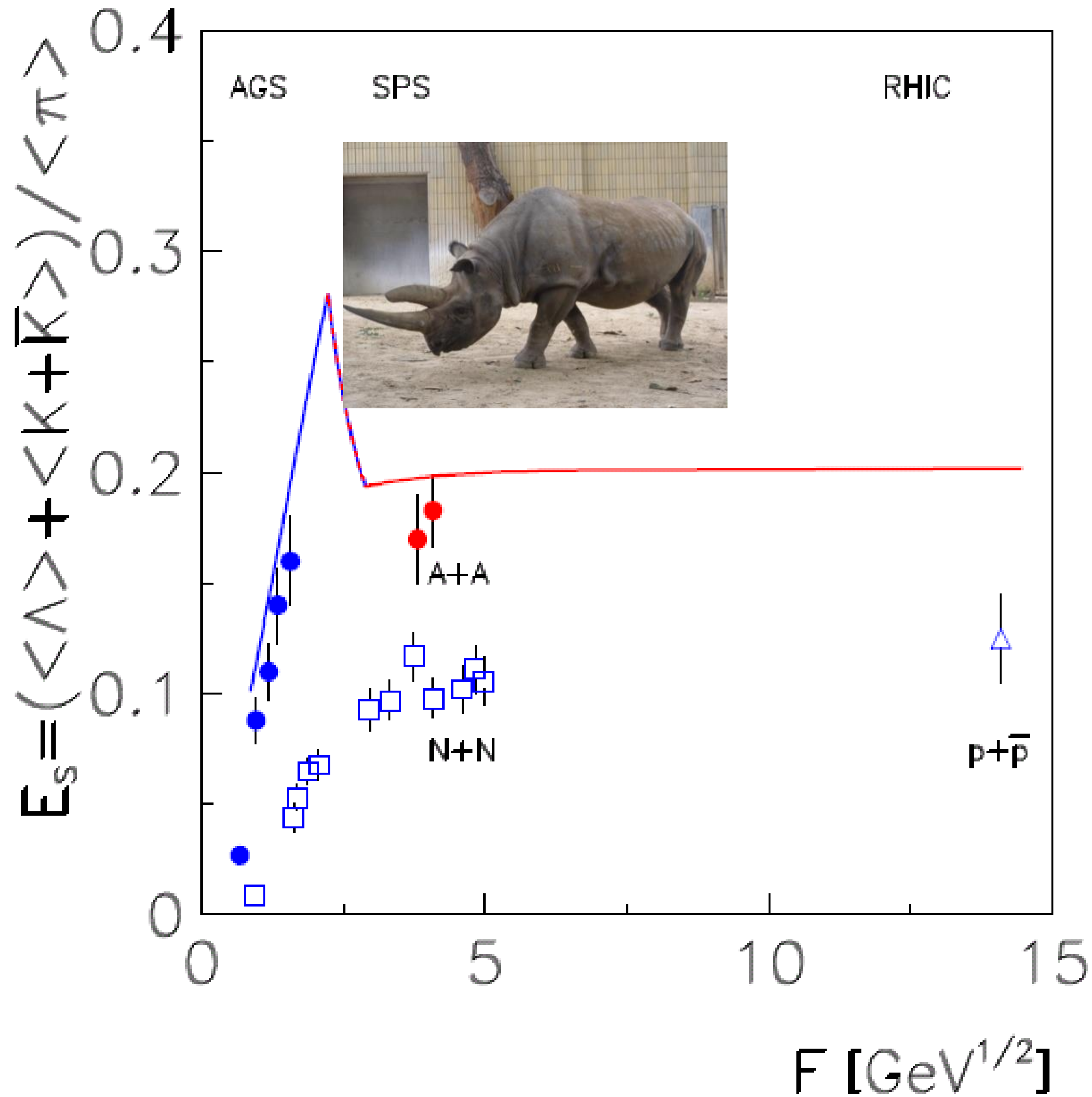
Statistical Model of Early Stage



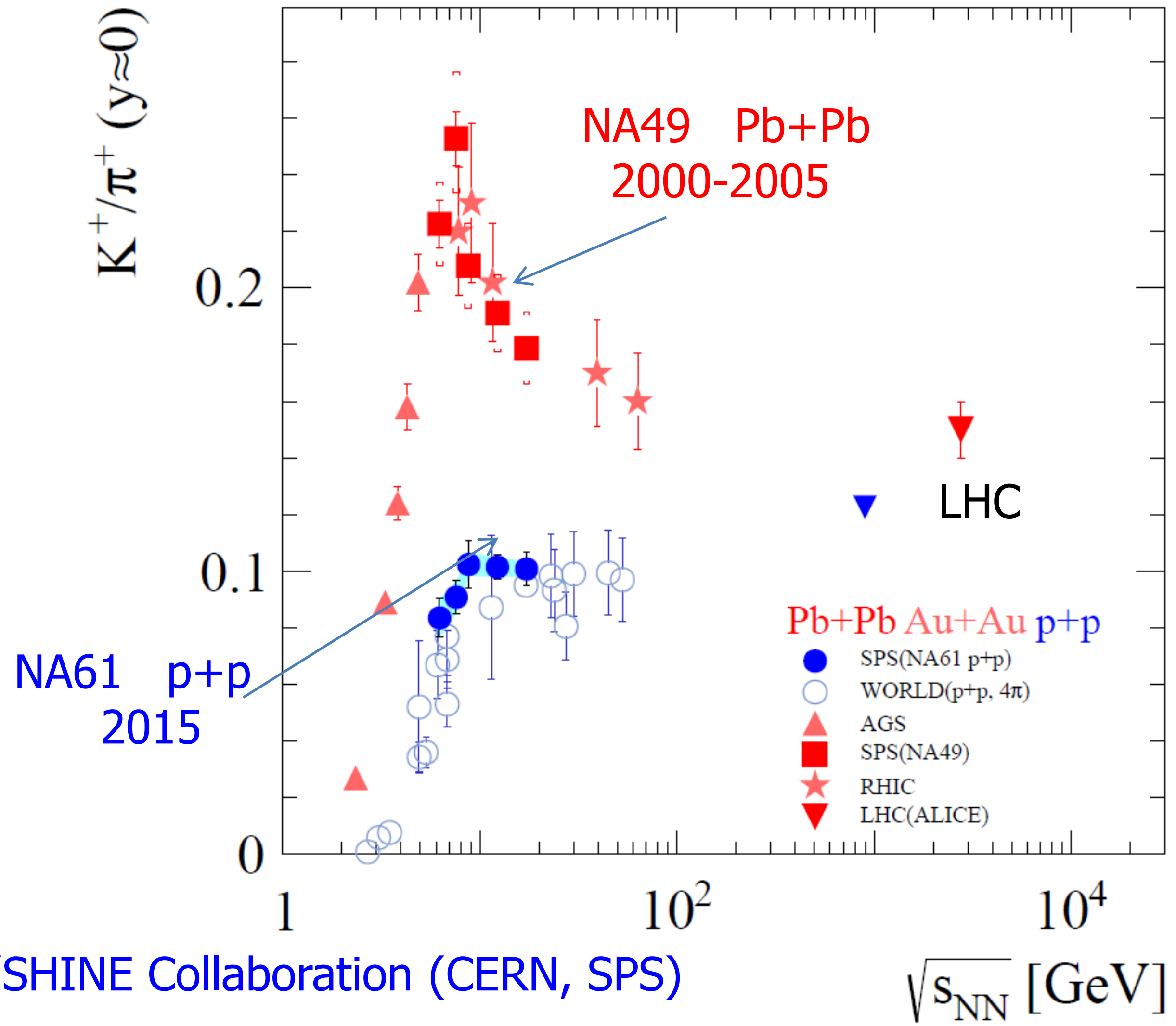


Gazdzicki and M.I.G., Acta Phys. Pol. (1999)

Experimental Data in 1998



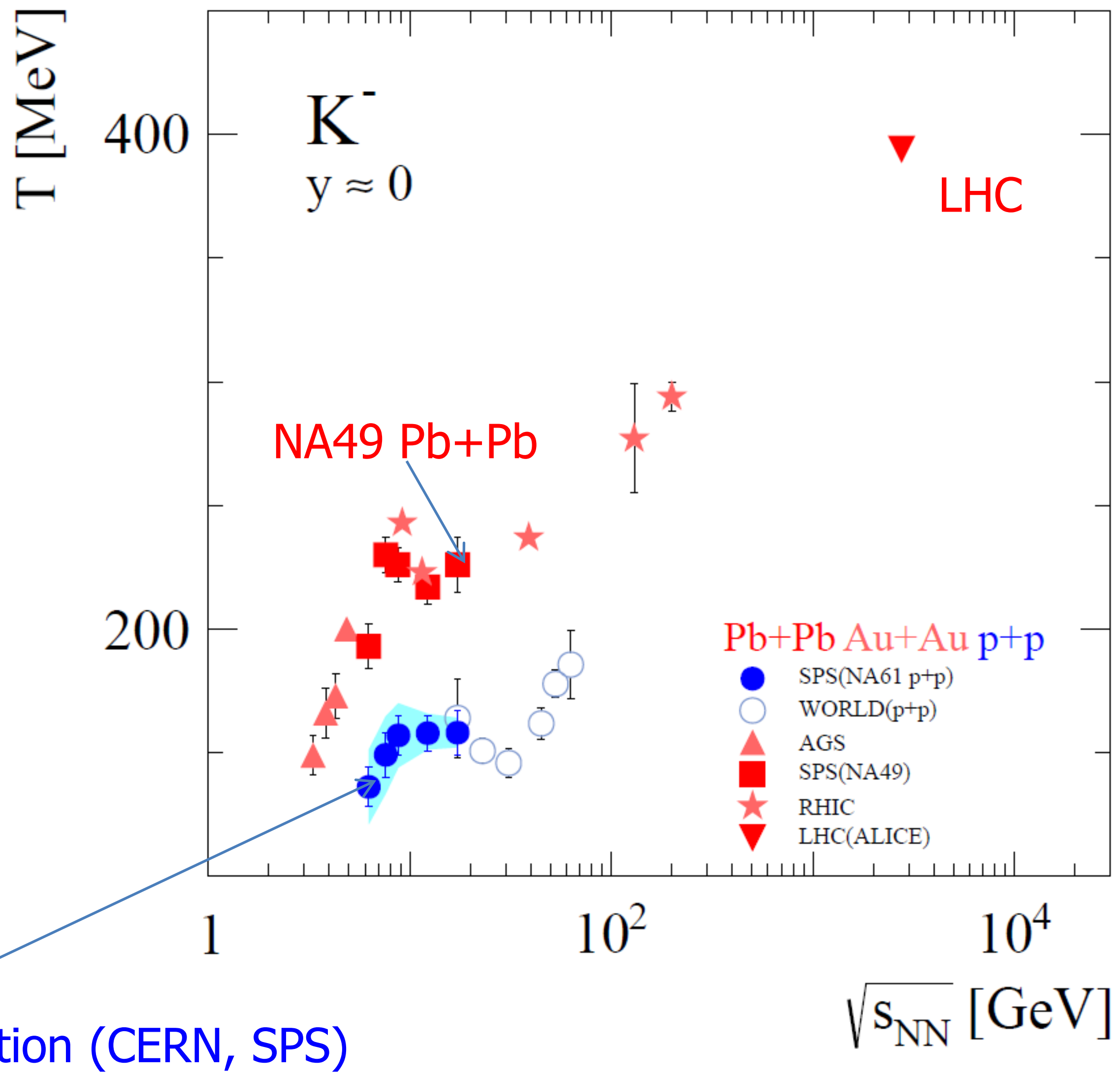
The Horn: Pb+Pb vs p+p



NA61/SHINE Collaboration (CERN, SPS)

arXiv:1502.07916 [nucl-ex]

The Step: Pb+Pb vs p+p



NA61/SHINE Collaboration (CERN, SPS)

arXiv:1502.07916 [nucl-ex]

Available online at www.sciencedirect.com
SCIENCE DIRECT®
PHYSICS LETTERS B
Physics Letters B 585 (2004) 115–121
www.elsevier.com/locate/physletb

Event-by-Event Fluctuations

Fluctuations and deconfinement phase transition in nucleus–nucleus collisions

M. Gaździcki [a,b], M.I. Gorenstein [c,d], St. Mrówczyński [e,b]

[a] *Institut für Kernphysik, Universität Frankfurt, Germany*
[b] *Institute of Physics, Świętokrzyska Academy, Kielce, Poland*
[c] *Bogolyubov Institute for Theoretical Physics, Kiev, Ukraine*
[d] *Institut für Theoretische Physik, Universität Frankfurt, Germany*
[e] *Sołtan Institute for Nuclear Studies, Warsaw, Poland*

Received 12 October 2003; received in revised form 23 January 2004; accepted 28 January 2004

Editor: P.V. Landshoff

Abstract

We propose a method to experimentally study the equation of state of strongly interacting matter created at the early stage of nucleus–nucleus collisions. The method exploits the relation between relative entropy and energy fluctuations and equation of state. As a measurable quantity, the ratio of properly filtered multiplicity to energy fluctuations is proposed. Within a statistical approach to the early stage of nucleus–nucleus collisions, the fluctuation ratio manifests a non-monotonic collision energy dependence with a maximum in the domain where the onset of deconfinement occurs.

Fluctuations in different statistical ensembles

PHYSICAL REVIEW C **70**, 034901 (2004)

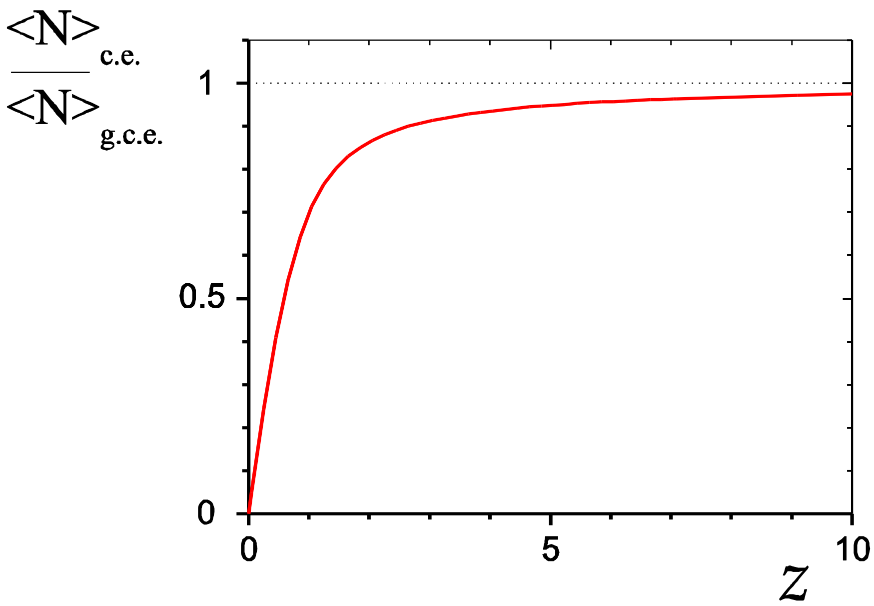
Particle number fluctuations in a canonical ensemble

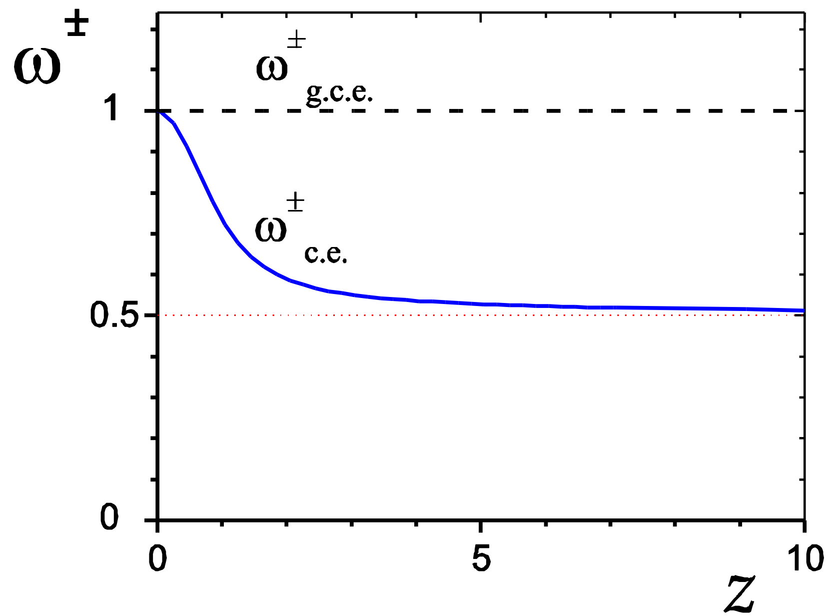
V. V. Begun,[1] M. Gaździcki,[2,3] M. I. Gorenstein,[1,4] and O. S. Zozulya[4,5]
[1]*Bogolyubov Institute for Theoretical Physics, Kiev, Ukraine*
[2]*Institut für Kernphysik, Universität Frankfurt, Frankfurt, Germany*
[3]*Świętokrzyska Academy, Kielce, Poland*
[4]*Institut für Theoretische Physik, Universität Frankfurt, Frankfurt, Germany*
[5]*Taras Shevchenko Kiev National University, Kiev, Ukraine*
(Received 21 April 2004; published 2 September 2004)

Fluctuations of charged particle number are studied in the canonical ensemble. In the infinite volume limit the fluctuations in the canonical ensemble are different from the fluctuations in the grand canonical one. Thus, the well-known equivalence of both ensembles for the average quantities does not extend for the fluctuations. In view of the possible relevance of the results for the analysis of fluctuations in nuclear collisions at high energies, a role of the limited kinematical acceptance is studied.

DOI: 10.1103/PhysRevC.70.034901 PACS number(s): 25.75.−q

GCE and CE with Q=0





Rafelski, Danos, Phys. Lett. B (1980)

Begun, Gazdzicki, M.I.G., Zozulya, Phys. Rev. C (2004)

$$\langle N_- \rangle_{ce} \;=\; z\,\frac{I_1(2z)}{I_0(2z)}\,, \qquad \omega^-_{ce} \;=\; 1 \;-\; z\left[\frac{I_1(2z)}{I_0(2z)} \;-\; \frac{I_2(2z)}{I_1(2z)}\right]$$

Z. Phys. C – Particles and Fields 54, 127–132 (1992)

Zeitschrift für Physik C Particles and Fields

Strongly Intensive Measures

A method to study "equilibration" in nucleus–nucleus collisions

Marek Gaździcki[1],* and Stanisław Mrówczyński[2],**

[1] Institute of Experimental Physics, Warsaw University, ul. Hoża 69, 00-681 Warsaw, Poland
[2] High Energy Department, Sołtan Institute for Nuclear Studies, ul. Hoża 69, 00-681 Warsaw, Poland

Received 14 October 1991

Eur. Phys. J. C 8, 131–133 (1999)
DOI 10.1007/s100529901070

THE EUROPEAN PHYSICAL JOURNAL C

ϕ Measure

A method to study "chemical" fluctuations in nucleus–nucleus collisions

M. Gaździcki[a]

Institut für Kernphysik, Universität Frankfurt, August–Euler–Strasse 6, D-60486 Frankfurt, Germany

Received: 6 May 1998 / Revised version: 23 July 1998 / Published online: 11 March 1999

Abstract. A method to study event–by–event fluctuations of the "chemical" (particle type) composition of the final state of high energy collisions is proposed.

PHYSICAL REVIEW C **84**, 014904 (2011)

Strongly intensive quantities

M. I. Gorenstein[1,2] and M. Gaździcki[3,4]
[1]*Bogolyubov Institute for Theoretical Physics, Kiev, Ukraine*
[2]*Frankfurt Institute for Advanced Studies, Frankfurt, Germany*
[3]*Institut für Kernphysik, University of Frankfurt, Frankfurt, Germany*
[4]*Jan Kochanowski University, Kielce, Poland*
(Received 8 February 2011; published 21 July 2011)

Analysis of fluctuations of hadron production properties in collisions of relativistic particles profits from use of measurable intensive quantities which are independent of system size variations. The first family of such quantities was proposed in 1992; another is introduced in this paper. Furthermore we present a proof of independence of volume fluctuations for quantities from both families within the framework of the grand canonical ensemble. These quantities are referred to as strongly intensive ones. Influence of conservation laws and resonance decays is also discussed.

$$\Delta[A, B] = \frac{1}{C_\Delta}[\langle B\rangle\,\omega[A] - \langle A\rangle\,\omega[B]], \qquad (2)$$

$$\Sigma[A, B] = \frac{1}{C_\Sigma}[\langle B\rangle\,\omega[A] + \langle A\rangle\,\omega[B] - 2\,(\langle AB\rangle - \langle A\rangle\langle B\rangle)], \qquad (3)$$

Identity Methods

PHYSICAL REVIEW C **83**, 054907 (2011)

Identity method to study chemical fluctuations in relativistic heavy-ion collisions

Marek Gaździcki
Goethe-Universität Frankfurt, Max-von-Laue Strasse 1, D-60438 Frankfurt am Main, Germany, and
Institute of Physics, Jan Kochanowski University, ul. Świętokrzyska 15, PL-25-406 Kielce, Poland

Katarzyna Grebieszkow and Maja Maćkowiak
Faculty of Physics, Warsaw University of Technology, ul. Koszykowa 75, PL-00-662 Warszawa, Poland

Stanisław Mrówczyński
Institute of Physics, Jan Kochanowski University, ul. Świętokrzyska 15, PL-25-406 Kielce, Poland and
Sołtan Institute for Nuclear Studies, ul. Hoża 69, PL-00-681 Warszawa, Poland
(Received 24 March 2011; published 17 May 2011)

PHYSICAL REVIEW C **84**, 024902 (2011)

Identity method for particle number fluctuations and correlations

M. I. Gorenstein
Bogolyubov Institute for Theoretical Physics, Kiev, Ukraine and
Frankfurt Institute for Advanced Studies, Frankfurt, Germany
(Received 22 June 2011; published 8 August 2011)

An incomplete particle identification distorts the observed event-by-event fluctuations of the hadron chemical composition in nucleus-nucleus collisions. A new experimental technique called the *identity method* was recently proposed. It eliminated the misidentification problem for one specific combination of the second moments in a system of two hadron species. In the present paper, this method is extended to calculate all the second moments in a system with an arbitrary number of hadron species. Special linear combinations of the second moments are introduced. These combinations are presented in terms of single-particle variables and can be found experimentally from the event-by-event averaging. The mathematical problem is then reduced to solving a system of linear equations. The effect of incomplete particle identification is fully eliminated from the final results.

PHYSICAL REVIEW C **86**, 044906 (2012)

Identity method for the determination of the moments of multiplicity distributions

A. Rustamov[1] and M. I. Gorenstein[2,3]
[1]*Institut für Kernphysik, Johann Wolfgang Goethe Universität Frankfurt, Frankfurt am Main, Germany*
[2]*Bogolyubov Institute for Theoretical Physics, Kiev, Ukraine*
[3]*Frankfurt Institute for Advanced Studies, Frankfurt, Germany*
(Received 17 July 2012; revised manuscript received 26 September 2012; published 26 October 2012)

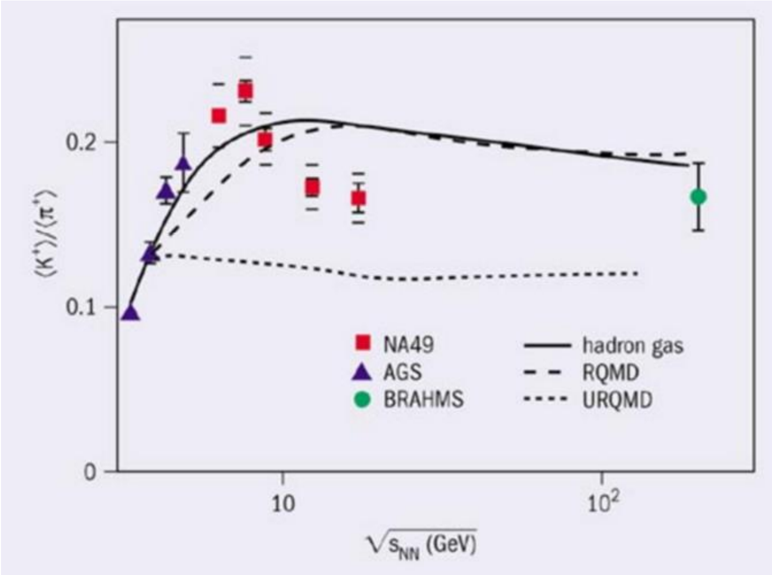
Recently the identity method was proposed to calculate second moments of the multiplicity distributions from event-by-event measurements in the presence of the effects of incomplete particle identification. In this paper the method is extended for higher moments. The moments of smeared multiplicity distributions are calculated using single-particle identity variables. The problem of finding the moments of the multiplicity distributions is reduced to solving a system of linear equations.

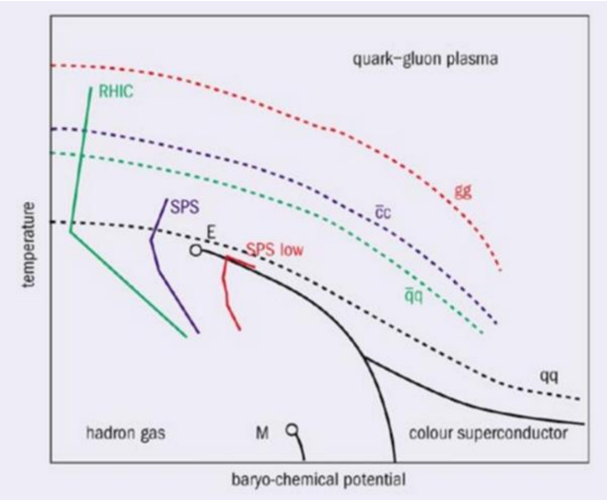
CPOD Workshops
April 24-29, 2004, Trento
Tracing the Onset of Deconfinement in Nucleus-Nucleus Collisions

CERN COURIER

Sep 6, 2004

When quarks and gluons become free





About the author

Marek Gazdzicki, Frankfurt and Kielce, Peter Seyboth, Munich, and Edward Shuryak, Stony Brook.

Critical Point and Onset of Deconfinement

CPOD-1, Trento, 2004

CPOD-2, Bergen, 2005

CPOD-3, Florence, 2006

CPOD-4, Darmstadt, 2007

CPOD-5, Brookhaven, 2009

CPOD-6, Dubna, 2010

CPOD-7, Wuhan, 2011

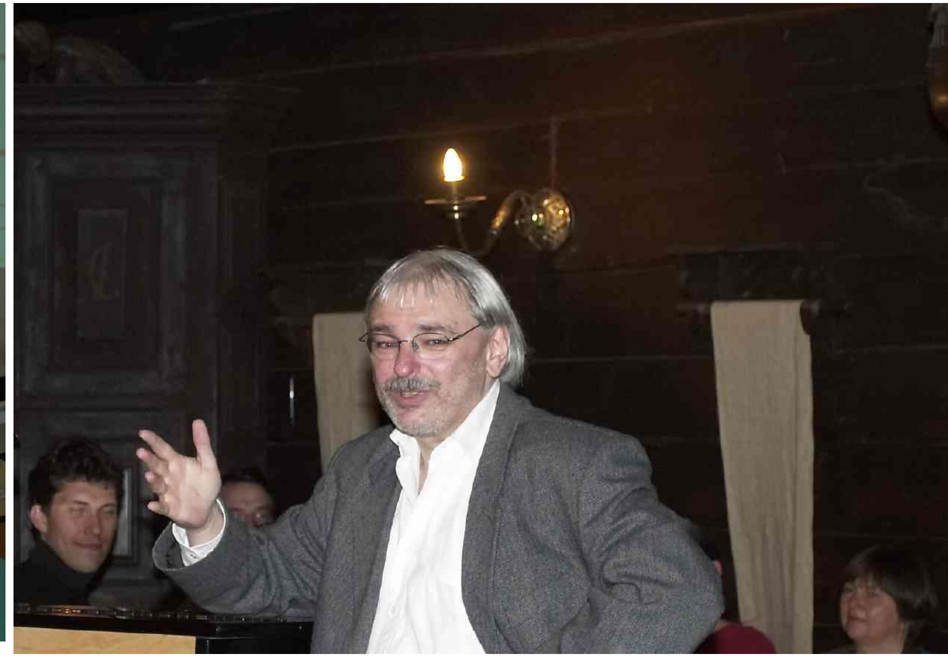
CPOD-8, Napa, 2013

CPOD-9, Bielefeld, 2014

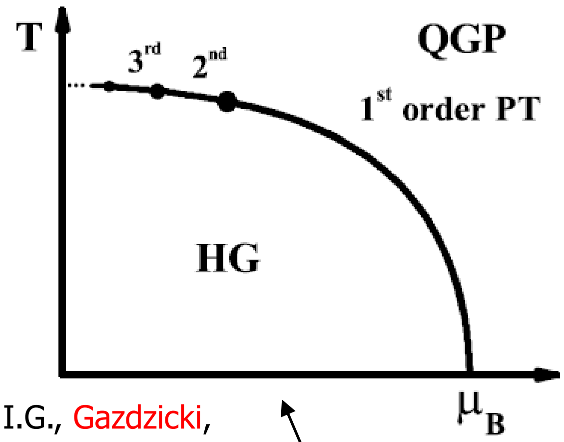
CPOD-10, Wroclaw, 2016

CPOD – 2, 2005, Bergen

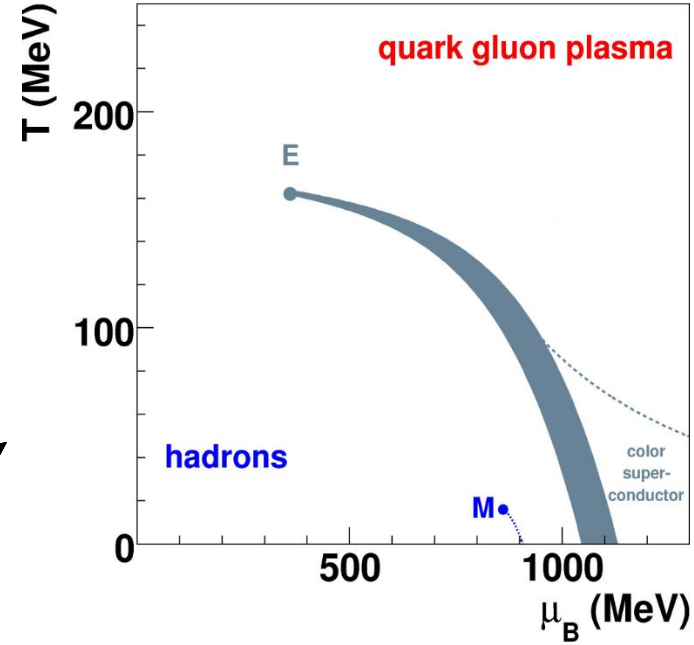

Importance of non-equilibrium effects ?
Tendency towards soft EOS
Medium effects due to finite baryon density are able to explain:
K^+ yield
K^+ flow
K^- / K^+ differential phase space distributions
needs more data
New probes could become accessible by invariant mass spectroscopy:
Excited strange baryons
K^- - clusters



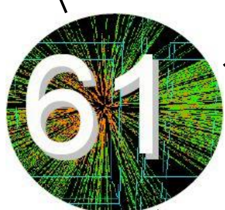
CPOD – 3, 2006, Florence



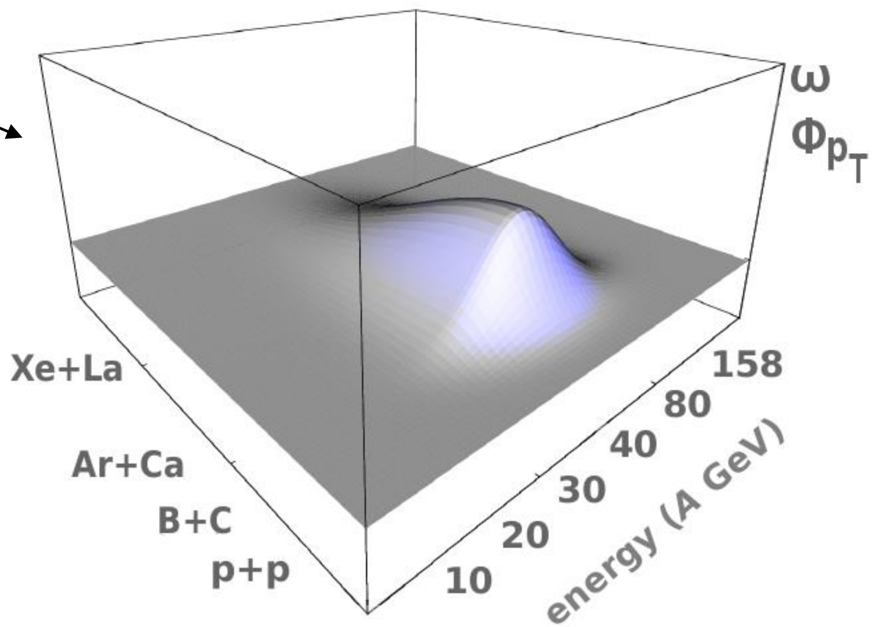
M.I.G., Gazdzicki, and W. Greiner, Phy. Rev. C (2005)



60...



Critical Point



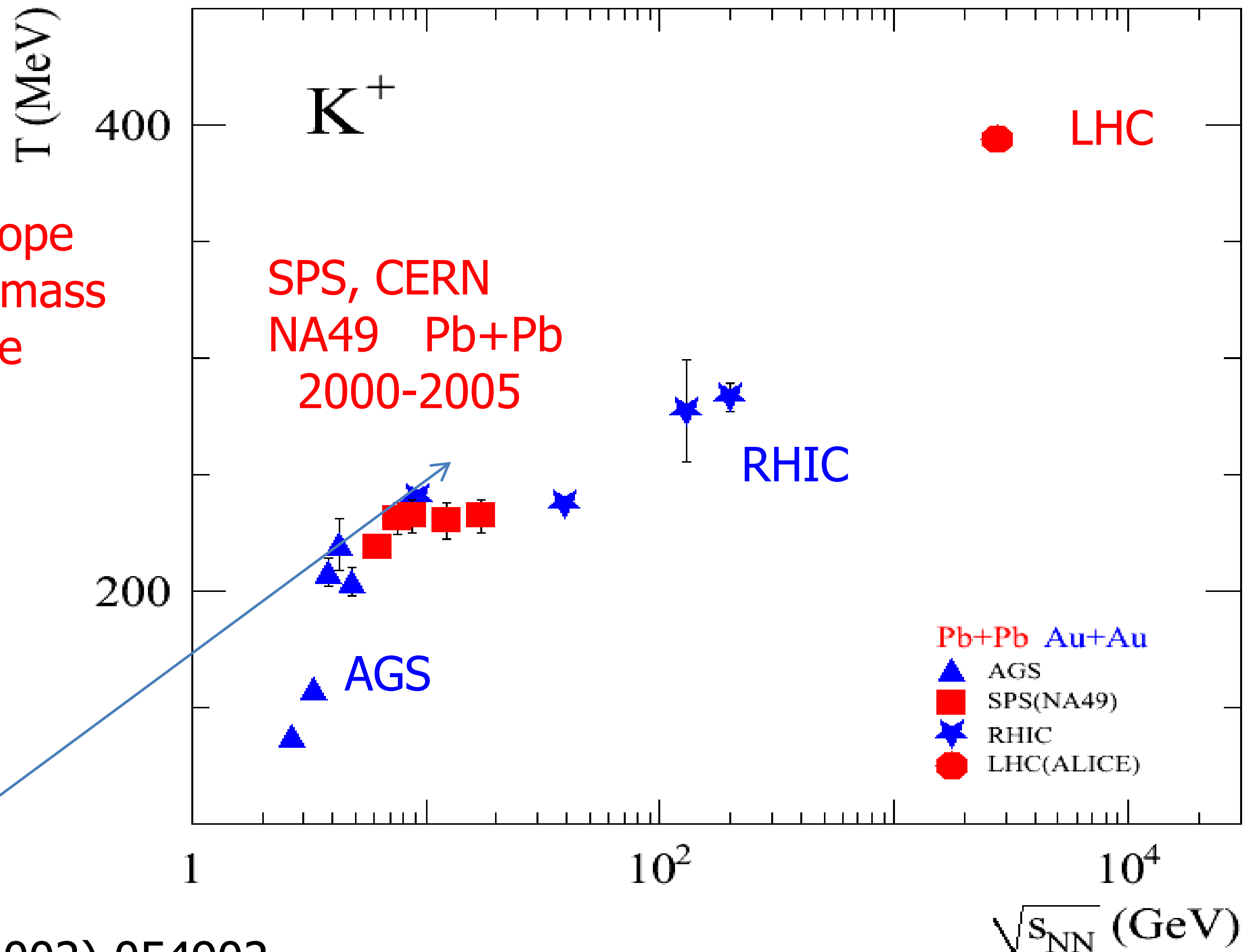
?

Thank you

Thank You

The Step: Pb+Pb

T is the inverse slope of the transverse mass spectra = effective temperature.



NA49

Phys.Rev. C66 (2002) 054902

Phys.Rev. C77 (2008) 024903

PHYSICAL REVIEW C **71**, 054904 (2005)

Particle number fluctuations in the microcanonical ensemble

V. V. Begun,[1] M. I. Gorenstein,[1,2,3] A. P. Kostyuk,[1,2,3] and O. S. Zozulya[1,4]
[1]*Bogolyubov Institute for Theoretical Physics, Kiev, Ukraine*
[2]*Institut für Theoretische Physik, Universität Frankfurt, Germany*
[3]*Frankfurt Institute for Advanced Studies, Frankfurt, Germany*
[4]*Utrecht University, Utrecht, The Netherlands*
(Received 14 October 2004; published 26 May 2005)

PHYSICAL REVIEW C **73**, 054904 (2006)

Particle number fluctuations in relativistic Bose and Fermi gases

V. V. Begun[1] and M. I. Gorenstein[1,2]
[1]*Bogolyubov Institute for Theoretical Physics, Kiev, Ukraine*
[2]*Frankfurt Institute for Advanced Studies, Frankfurt, Germany*
(Received 7 October 2005; published 23 May 2006)

PHYSICAL REVIEW C **74**, 044903 (2006)

Multiplicity fluctuations in hadron-resonance gas

V. V. Begun,[1,2] M. I. Gorenstein,[2,3] M. Hauer,[4] V. P. Konchakovski,[2] and O. S. Zozulya[2,5]
[1]*Museo Storico della Fisica e Centro Studi e Ricerche Enrico Fermi, Rome, Italy*
[2]*Bogolyubov Institute for Theoretical Physics, Kiev, Ukraine*
[3]*Frankfurt Institute for Advanced Studies, Frankfurt, Germany*
[4]*University of Cape Town, Cape Town, South Africa*
[5]*Utrecht University, Utrecht, The Netherlands*
(Received 23 June 2006; published 5 October 2006)

PHYSICAL REVIEW C **76**, 024902 (2007)

Multiplicity fluctuations in relativistic nuclear collisions: Statistical model versus experimental data

V. V. Begun,[1,2] M. Gaździcki,[3,4] M. I. Gorenstein,[2,5] M. Hauer,[6,7] V. P. Konchakovski,[2,6] and B. Lungwitz[3]
[1]*Museo Storico della Fisica e Centro Studi e Ricerche Enrico Fermi, Rome, Italy*
[2]*Bogolyubov Institute for Theoretical Physics, Kiev, Ukraine*
[3]*Institut für Kernphysik, University of Frankfurt, Frankfurt, Germany*
[4]*Świętokrzyska Academy, Kielce, Poland*
[5]*Frankfurt Institute for Advanced Studies, Frankfurt, Germany*
[6]*Helmholtz Research School, University of Frankfurt, Frankfurt, Germany*
[7]*University of Cape Town, Cape Town, South Africa*
(Received 11 December 2006; published 6 August 2007)

$$R_e \equiv \frac{(\delta S)^2/S^2}{(\delta E)^2/E^2} = \left(1 + \frac{p}{\varepsilon}\right)^{-2}$$

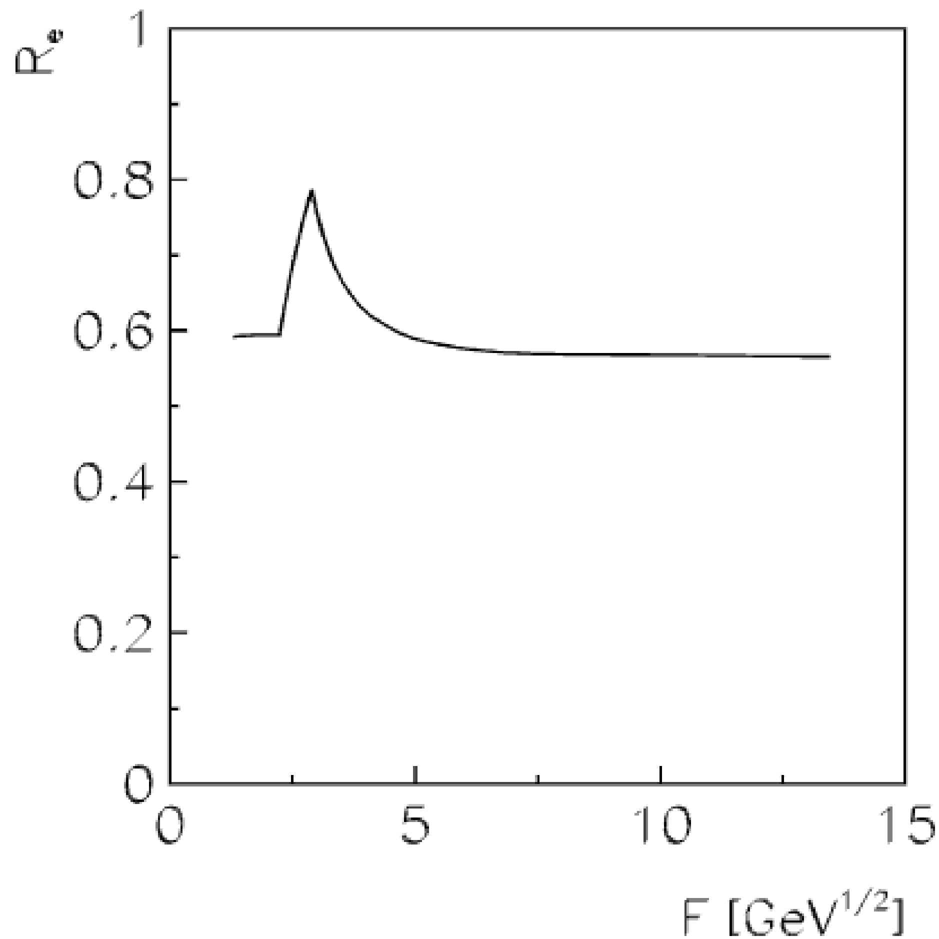


Fig. 1. The dependence of R_e calculated within SMES [8] on the Fermi's collision energy measure $F \equiv (\sqrt{s} - 2m)^{3/4}/s^{1/8}$ where $\sqrt{s}$ is the c.m.s. energy per nucleon–nucleon pair and m is the nucleon mass. The 'shark fin' structure is caused by the large fluctuations in the mixed phase region.

PHYSICAL REVIEW C **86**, 044906 (2012)

Identity method for the determination of the moments of multiplicity distributions

A. Rustamov[1] and M. I. Gorenstein[2,3]

[1]*Institut für Kernphysik, Johann Wolfgang Goethe Universität Frankfurt, Frankfurt am Main, Germany*

[2]*Bogolyubov Institute for Theoretical Physics, Kiev, Ukraine*

[3]*Frankfurt Institute for Advanced Studies, Frankfurt, Germany*

(Received 17 July 2012; revised manuscript received 26 September 2012; published 26 October 2012)

Recently the identity method was proposed to calculate second moments of the multiplicity distributions from event-by-event measurements in the presence of the effects of incomplete particle identification. In this paper the method is extended for higher moments. The moments of smeared multiplicity distributions are calculated using single-particle identity variables. The problem of finding the moments of the multiplicity distributions is reduced to solving a system of linear equations.

$$
\begin{aligned}
\langle W_p^3\rangle = & \sum_{N_1=0}^{N_1=\infty}\sum_{N_2=0}^{N_2=\infty}\cdots\sum_{N_k=0}^{N_k=\infty}\mathcal{P}(N_1,N_2,\ldots,N_k)\int dx_1^1 P_1(x_1^1)\cdots\int dx_{N_1}^1 P_1(x_{N_1}^1)\\
& \times\int dx_1^2 P_2(x_1^2)\cdots\int dx_{N_2}^2 P_2(x_{N_2}^2)\times\cdots\times\int dx_1^k P_k(x_1^k)\cdots\times\int dx_{N_k}^k P_k(x_{N_k}^k)\\
& \times\left[w_p(x_1^1)+\cdots+w_p(x_{N_1}^1)+w_p(x_1^2)+\cdots+w_p(x_{N_2}^2)+w_p(x_1^k)+\cdots+w_p(x_{N_k}^k)\right]^3\\
= & \sum_{i=1}^{k}\langle N_i^3\rangle\overline{w_{p,i}}^3+3\sum_{1\leqslant i<l\leqslant k}\langle N_i^2N_l\rangle\left(\overline{w_{p,i}}^2\overline{w_{p,l}}\right)+3\sum_{1\leqslant i<l\leqslant k}\langle N_iN_l^2\rangle\left(\overline{w_{p,l}}^2\overline{w_{p,i}}\right)\\
& +6\sum_{1\leqslant i<l<m\leqslant k}\langle N_iN_lN_m\rangle W_{ilm}^{ppp}+3\sum_{i=1}^{k}\langle N_i^2\rangle\left(\overline{w_{p,i}^2}\,\overline{w_{p,i}}-\overline{w_{p,i}}^3\right)\\
& +3\sum_{1\leqslant i<l\leqslant k}\langle N_iN_l\rangle\left(\overline{w_{p,i}^2}\,\overline{w_{p,l}}+\overline{w_{p,l}^2}\,\overline{w_{p,i}}-\overline{w_{p,i}}^2\overline{w_{p,l}}-\overline{w_{p,l}}^2\overline{w_{p,i}}\right)\\
& +\sum_{i=1}^{k}\langle N_i\rangle\left(2\overline{w_{p,i}}^3+\overline{w_{p,i}^3}-3\overline{w_{p,i}^2}\,\overline{w_{p,i}}\right),
\end{aligned}
$$