Off-equilibrium Non-Gaussian Cumulants: criticality, complexity, and universality

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SM, R. Venugopalan, Y. Yin:
arXiv:1605.09341 & arXiv:1506.00645

BEST COLLABORATION

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hope: observe something universal

static universality

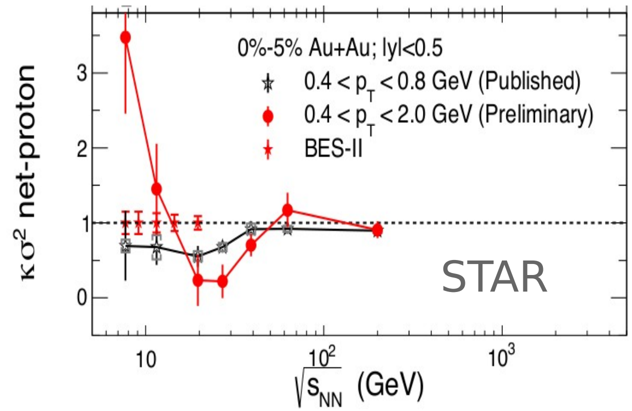
QCD critical point ⟷ 3-d Ising

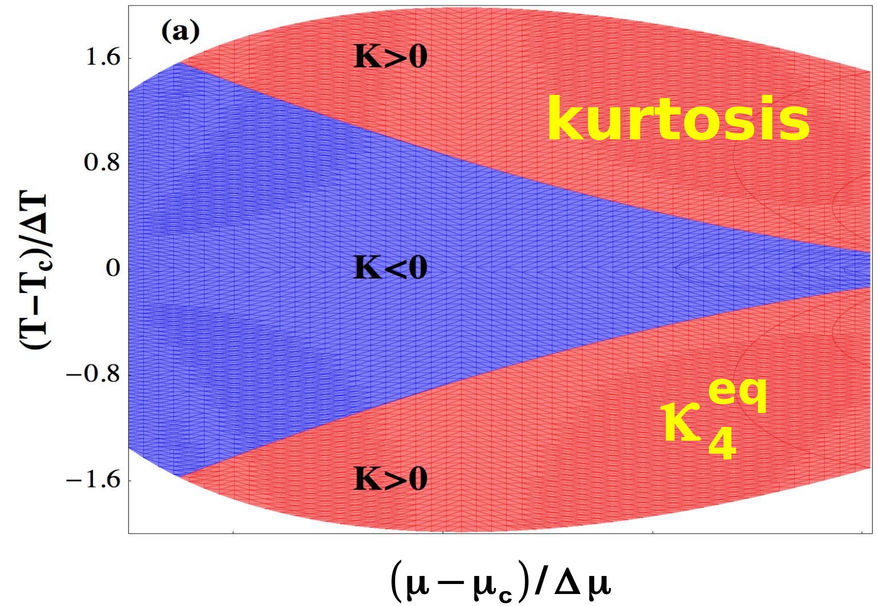
$$\kappa_n^{eq} \sim \xi_{eq}^{-\frac{1}{2}+\frac{5}{2}(n-1)} \, f_n^{eq}(\theta)$$

$$\theta \sim r^{-5/3} h$$

r ← reduced temperature

h ← reduced magnetic field

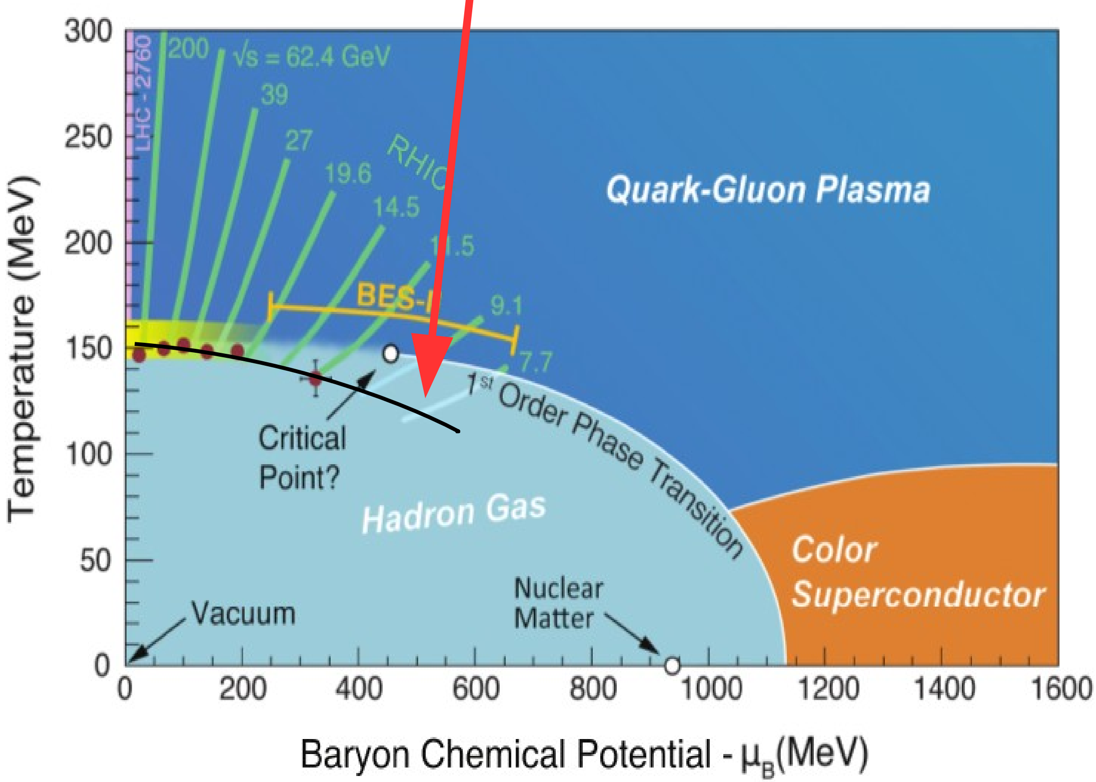




Stephanov: arXiv:1104.1627

why non-equilibrium ? its necessary

where is freeze-out
w.r.t. critical point?



memory effects needed
to preserve remnant
critical signatures

unless accidental freeze-out
very close to critical point

why non-equilibrium ? its unavoidable

dynamical universality

QCD critical point ⟺ model-H

$$\tau_{eff} \sim \xi_{eq}^{z}$$

$$z = 3$$

slow relaxation of critical mode
→ critical mode out of equilibrium

Son, Stephanov: arXiv:hep-ph/0401052

τ_{eff} : relaxation time for critical mode

critical mode →
linear combination of chiral condensate & baryon current

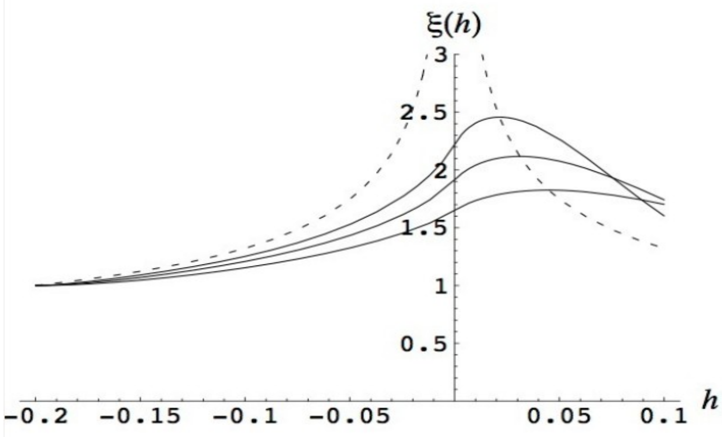
simple extension to non-equilibrium

Berdnikov, Rajagopal:
arXiv:hep-ph/9912274

Ansatz for evolution of correlation length:

$$\partial_\tau \xi^{-1} = -\tau_{eff}^{-1}\left[\xi^{-1} - \xi_{eq}^{-1}\right]$$

with dynamical universality: $\tau_{eff} \sim \xi_{eq}^{z}$



non-Gaussian cumulants
just as in equilibrium

$$\kappa_n(\tau) \sim [\xi(\tau)]^{-\frac{1}{2}+\frac{5}{2}(n-1)}$$

- scaling holds off-equilibrium ?
- signs of non-Gaussian cumulants ?

real-time evolution of cumulants

SM, R. Venugopalan, Y. Yin:
arXiv:1506.00645

effective action
3-d Ising

$$\Omega_0(\sigma) = \frac{1}{2} m_\sigma^2 (\sigma - \sigma_0)^2 + \frac{\lambda_3}{3}(\sigma - \sigma_0)^3 + \frac{\lambda_4}{4}(\sigma - \sigma_0)^4$$

σ : critical mode

$\xi_{eq} \equiv m_\sigma^{-1}$

remain within scaling regime,
but not at the critical point

$\epsilon = \sqrt{\xi^3/V} < 1$

$L_{micr} < \xi < L$

mass term $\sim \sigma^2/\xi_{eq}^2$

momentum dependence
kinetic term $\sim \sigma^2/L^2$ ← neglected

Langevin dynamics

only soft critical mode out of equilibrium,
hard modes are in equilibrium,
soft mode receive random kicks from
thermal bath of hard modes

Fokker-Planck evolution

$$\partial_\tau P(\sigma;\tau) = \frac{1}{m_\sigma^2 \tau_{\rm eff}} \left[\partial_\sigma \left[\partial_\sigma \Omega_0(\sigma) + V_4^{-1} \partial_\sigma \right] P(\sigma;\tau) \right]$$

$$\tau_{\rm eff} \sim \xi_{\rm eq}^3$$

evolution of cumulants

$$\partial_\tau \langle f(\sigma) \rangle = -\frac{1}{m_\sigma^2 \tau_{\rm eff}} \left[\langle f'(\sigma) \Omega_0'(\sigma) \rangle - V_4^{-1} \langle f''(\sigma) \rangle \right]$$

systematic expansion in $\epsilon = \sqrt{\xi^3/V} < 1$

closed set of coupled time evolution equations

$$\partial_\tau \kappa_n = -n\tau_{eff}^{-1} F_n(\kappa_1, \cdots, \kappa_n) + O(\epsilon)$$

lower cumulatns relax back to equilibrium first

$\epsilon < 1$ ⇒ evolution of the higher cumulants couples only to lower ones

Gaussian limit

$$\Omega_0(\sigma) = \frac{1}{2} m_\sigma^2 (\sigma - \sigma_0)^2 \qquad \kappa_3^{eq} = \kappa_4^{eq} = 0$$

$$\partial_\tau \kappa_n = -n\tau_{eff}^{-1} \left[\kappa_n - \kappa_n^{eq}\right]$$

for n=2 reduces to the old Berdnikov-Rajagopal Ansatz

$$\partial_\tau \xi^{-1} = -\tau_{eff}^{-1} \left[\xi^{-1} - \xi_{eq}^{-1}\right]$$

modeling heavy-ion collisions …

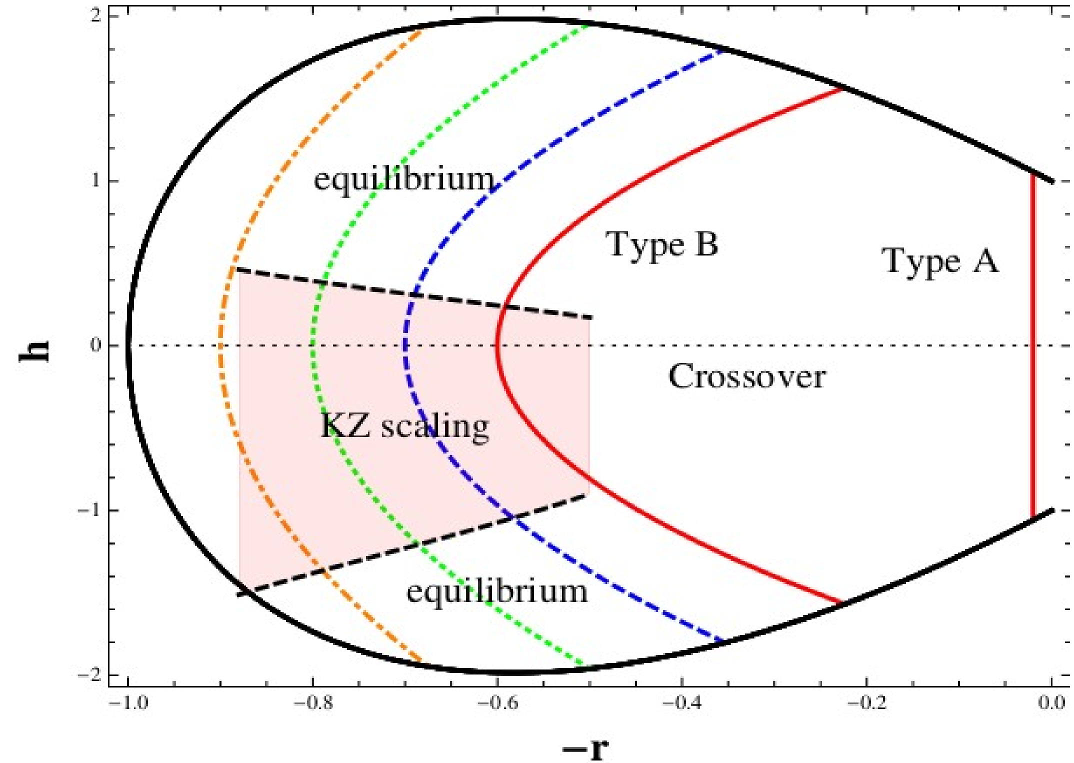
… or introducing non-universality

- Ising → thermodynamic variables: $(r, h) \rightarrow (\mu_B, T)$
- details of trajectory in (μ_B, T)-plane
- relaxation time of the critical mode: τ_{eff}
- location of freeze-out in (μ_B, T)-plane

example:

trajectory Type-A, vary τ_{eff}

$$h = \frac{T - T_c}{\Delta T} \qquad r = -\frac{\mu - \mu_c}{\Delta \mu}$$



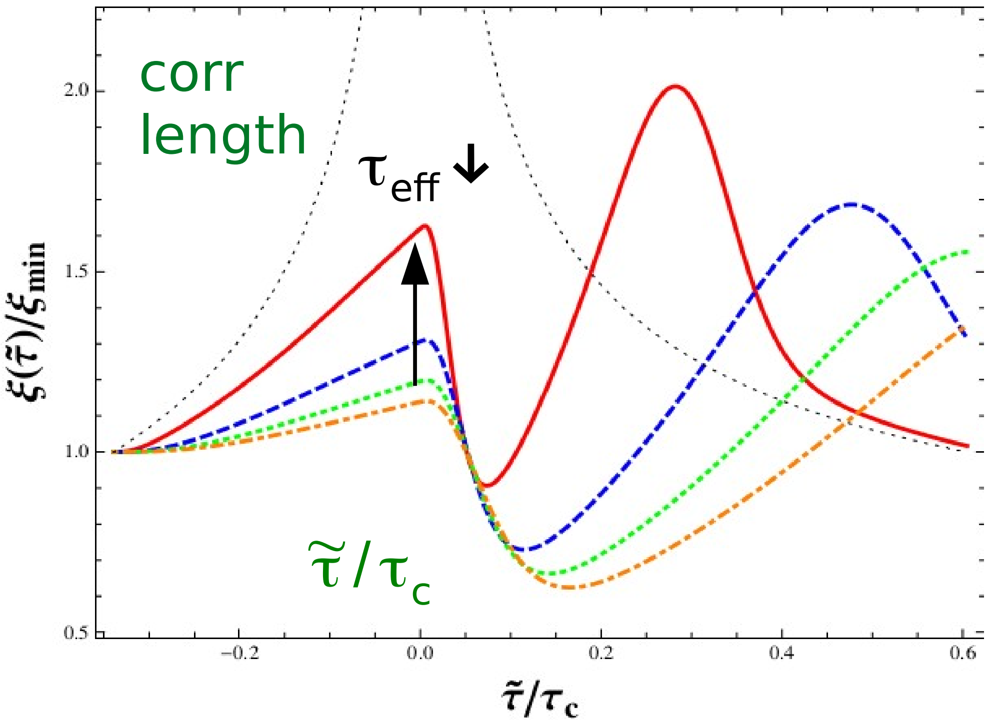
3-d Hubble-like expansion

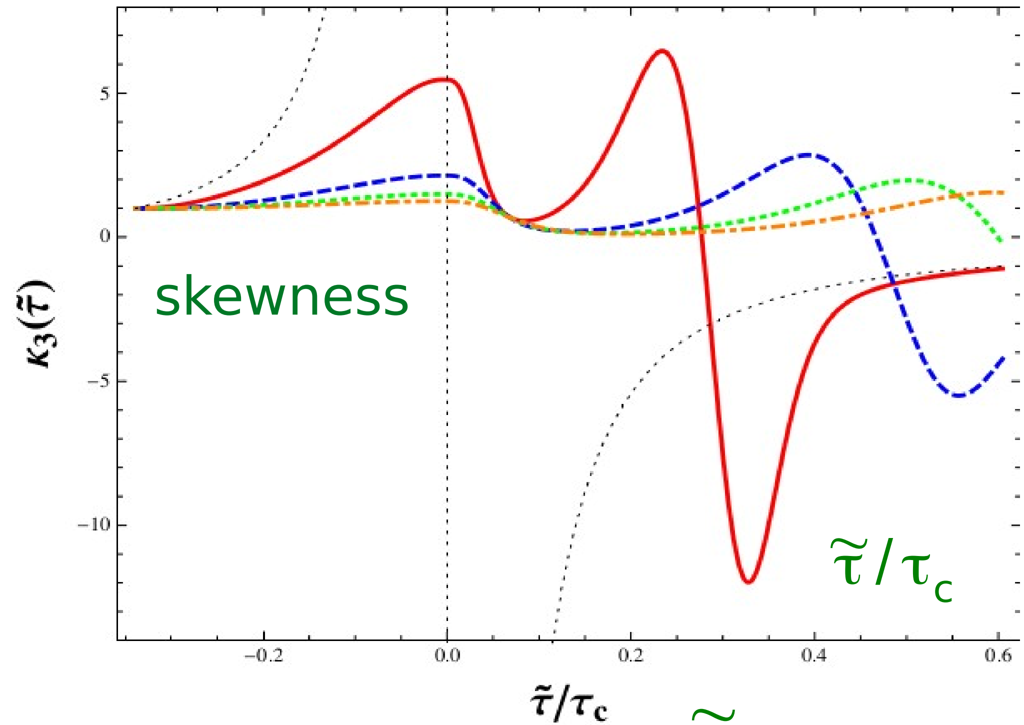
$$T(\tau) = T_c \left(\tau/\tau_c\right)^{-3c_s^2}$$

τ_c ← time when trajectory crosses h=0, crossover, line

$c_s^2 = 0.1$ ← speed of sound

universality lost …





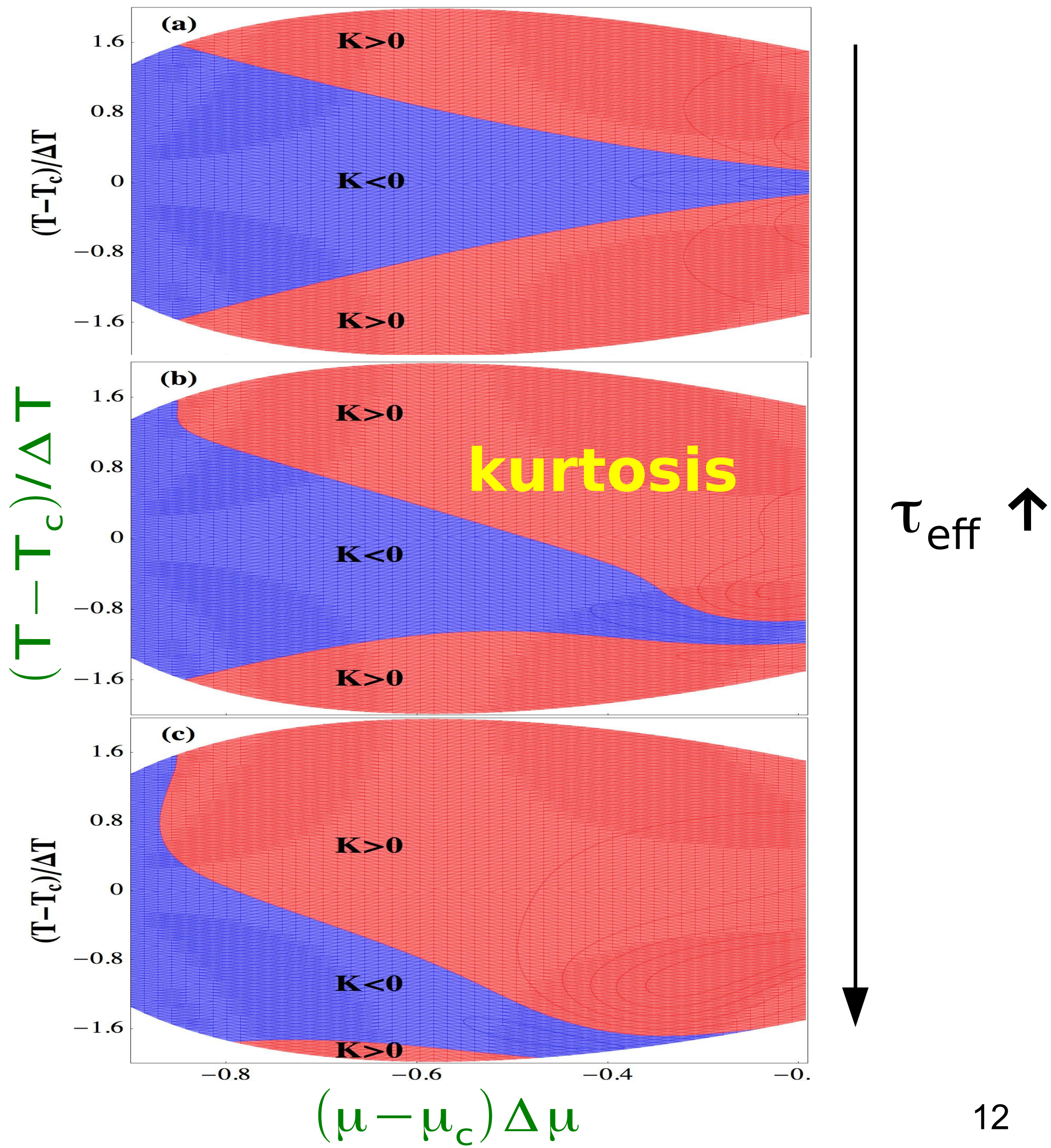
$\tilde{\tau} = \tau - \tau_c$

non-Gaussian cumulants do not follow growth of the correlation length

unlike equilibrium expectation

universality lost …

sign can be different
from equilibrium one



new idea: Kibble-Zurek (KZ) dynamics

SM, R. Venugopalan, Y. Yin:
arXiv:1605.09341

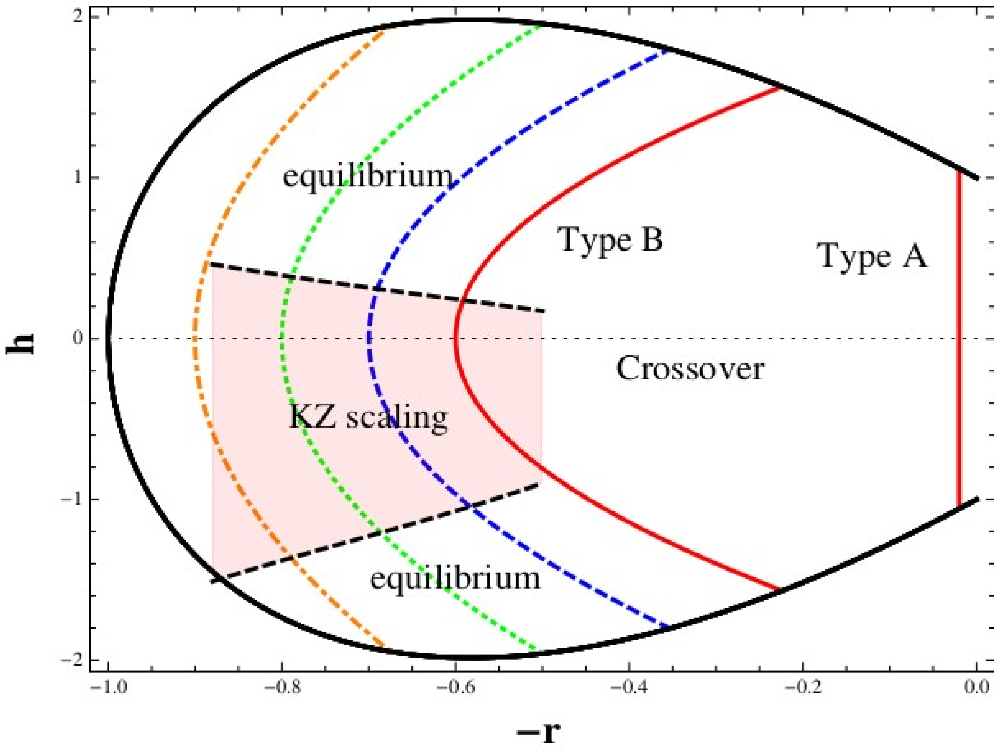
two competing time scales

$$\tau_{\text{eff}}(\tau) \qquad \tau_{\text{quench}}(\tau)$$

$$\tau_{\text{quench}} = \min\left(\tau^{\xi}_{\text{quench}},\ \tau^{\theta}_{\text{quench}}\right)$$

$$\tau^{\xi}_{\text{quench}} = \left|\frac{\xi_{\text{eq}}(\tau)}{\partial_\tau \xi_{\text{eq}}(\tau)}\right|$$

$$\tau^{\theta}_{\text{quench}} = \left|\frac{\theta(\tau)}{\partial_\tau \theta(\tau)}\right|$$



$$(r, h) \leftrightarrow (\xi, \theta)$$

emergent scales

$$\tau_{KZ} = \tau_{eff}(\tau^*) = \tau_{quench}(\tau^*)$$

$$l_{KZ} = \xi_{eq}(\tau^*) \qquad \theta_{KZ} = \theta_{eq}(\tau^*)$$

$$\tau_{eff}(\tau) \gg \tau_{quench}(\tau)$$

↓

frozen fluctuation
& magnetization

emergent scaling

$$\kappa_n(\tau;\Gamma) \sim l_{KZ}^{-\frac{1}{2}+\frac{5}{2}(n-1)} \bar{f}_n^{l}(t;\theta_{KZ})$$

$$t = \tilde{\tau}/\tau_{KZ} \qquad \tilde{\tau} = \tau - \tau_c$$

$$\kappa_n^{eq} \sim \xi_{eq}^{-\frac{1}{2}+\frac{5}{2}(n-1)} f_n^{eq}(\theta)$$

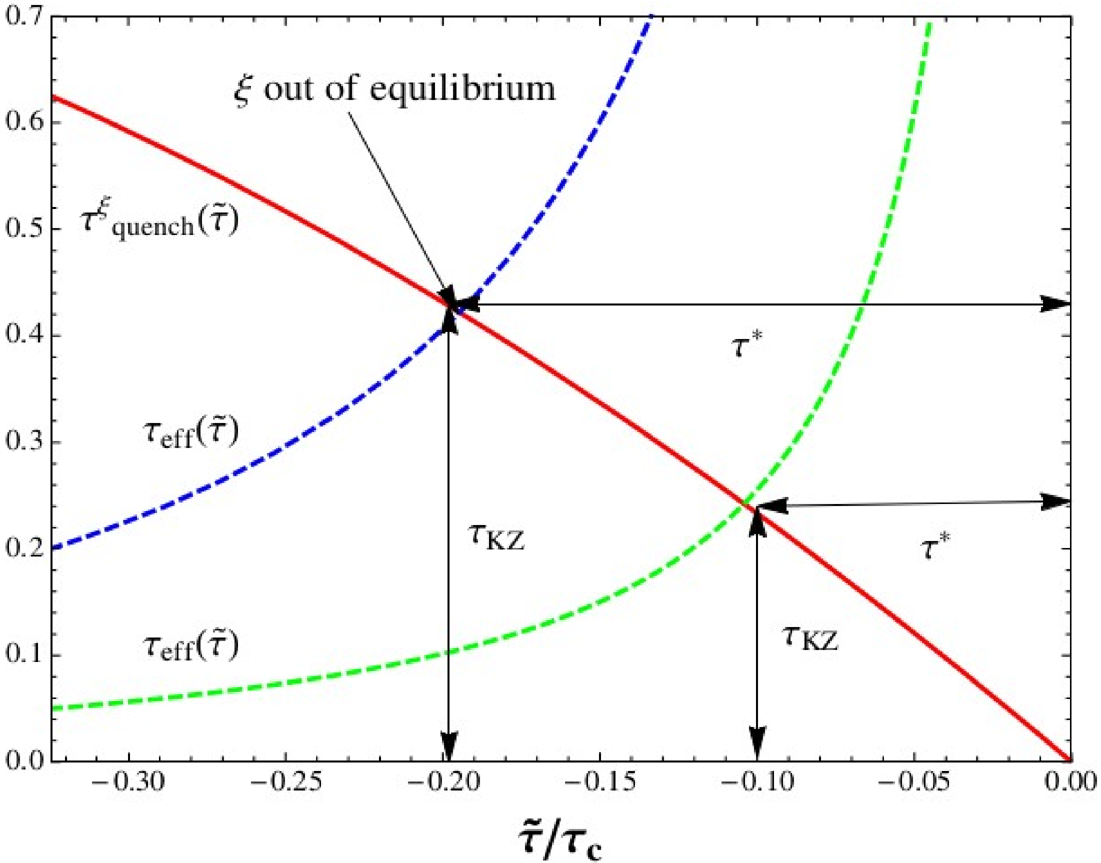
l ← class of trajectories

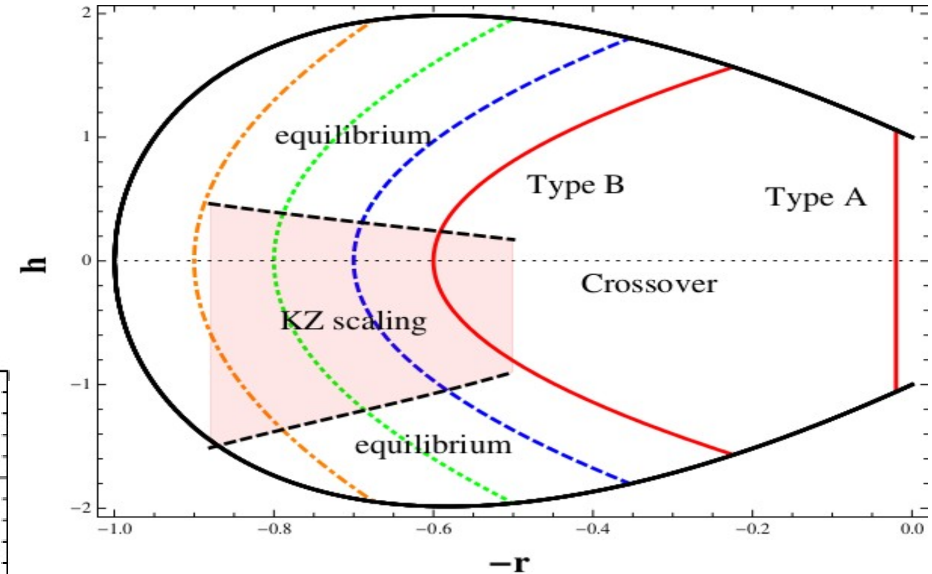
$$\tau_{KZ}(\Gamma),\ l_{KZ}(\Gamma),\ \theta_{KZ}(\Gamma)$$

Γ ← non-universal variables

example:

trajectory Type-A, vary τ_{eff}



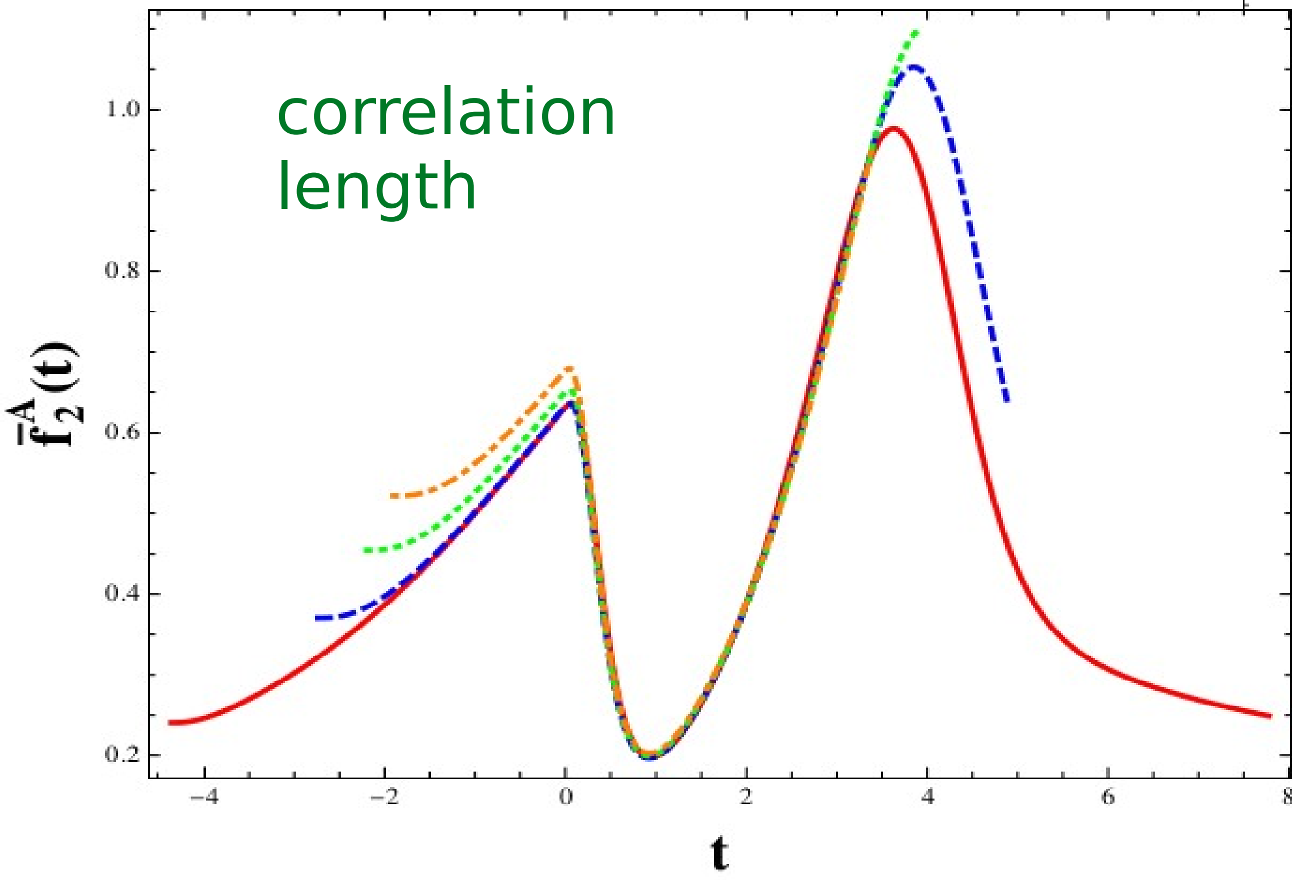


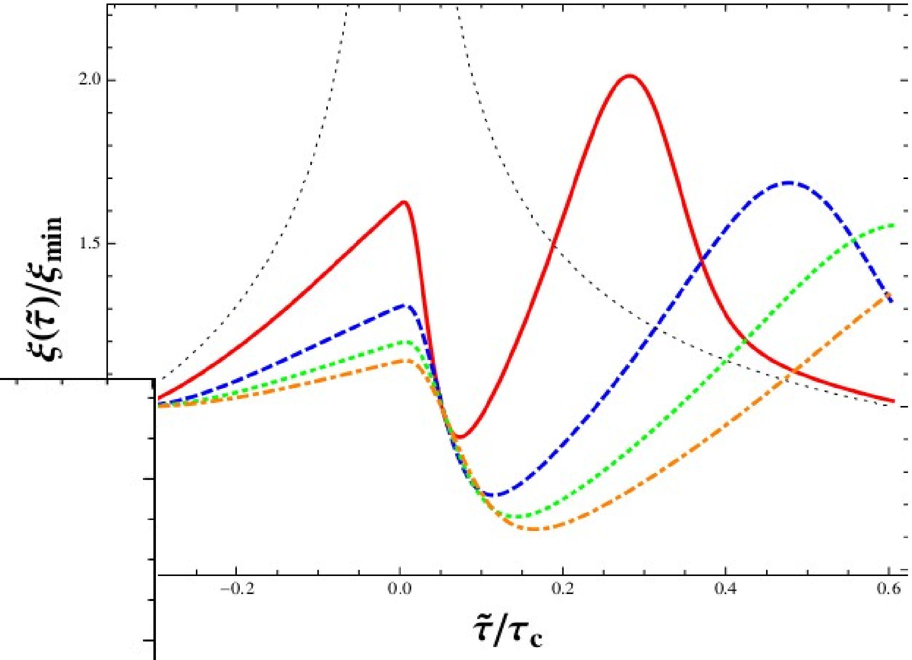
$$\tau_{quench} = \tau^{\xi}_{quench}$$

$$\tau^{\xi}_{quench} = \left|\frac{\xi_{eq}(\tau)}{\partial_\tau \xi_{eq}(\tau)}\right|$$

universality regained ...

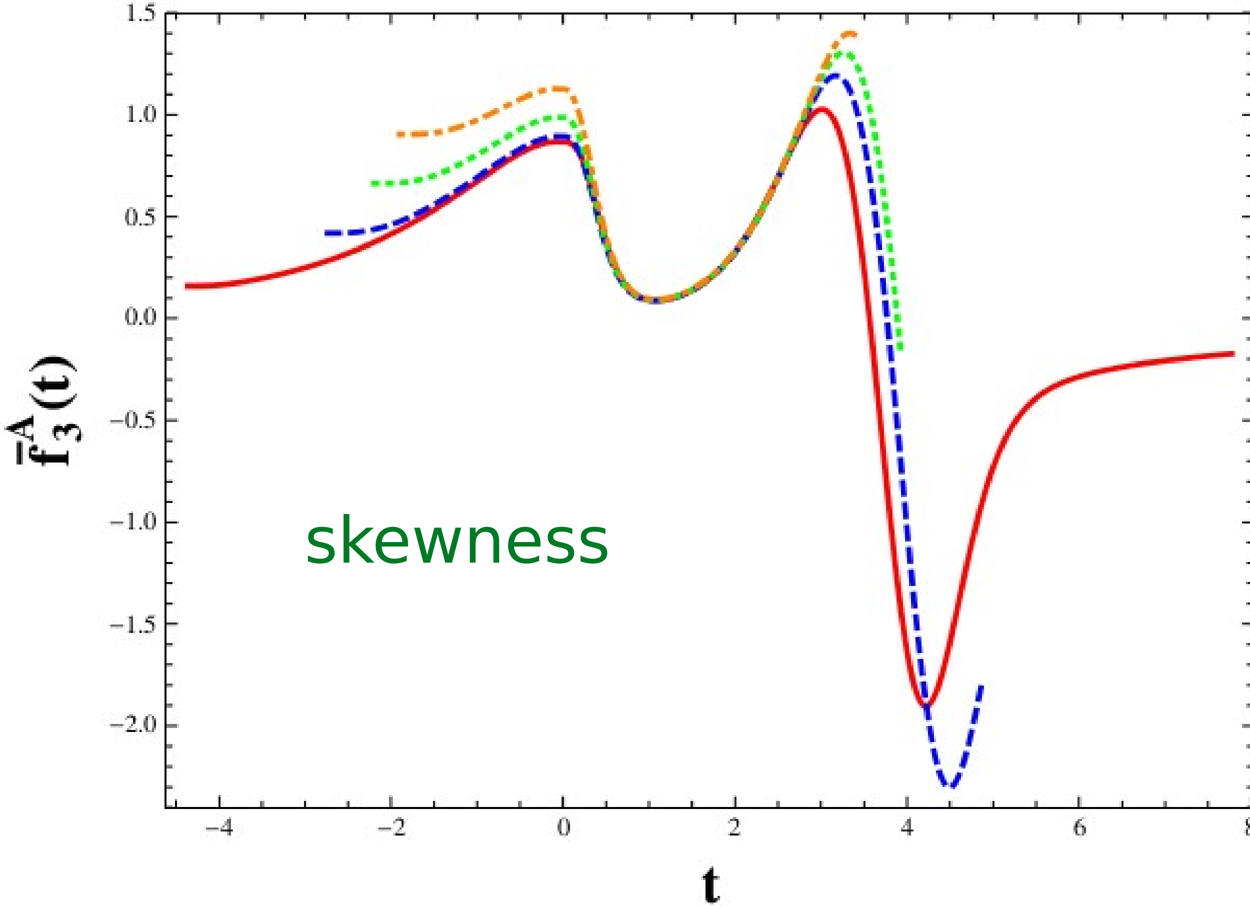
$$\kappa_n(\tau;\Gamma) \sim l_{KZ}^{-\frac{1}{2}+\frac{5}{2}(n-1)}\ \bar{f}^{l}_n(t;\theta_{KZ})$$

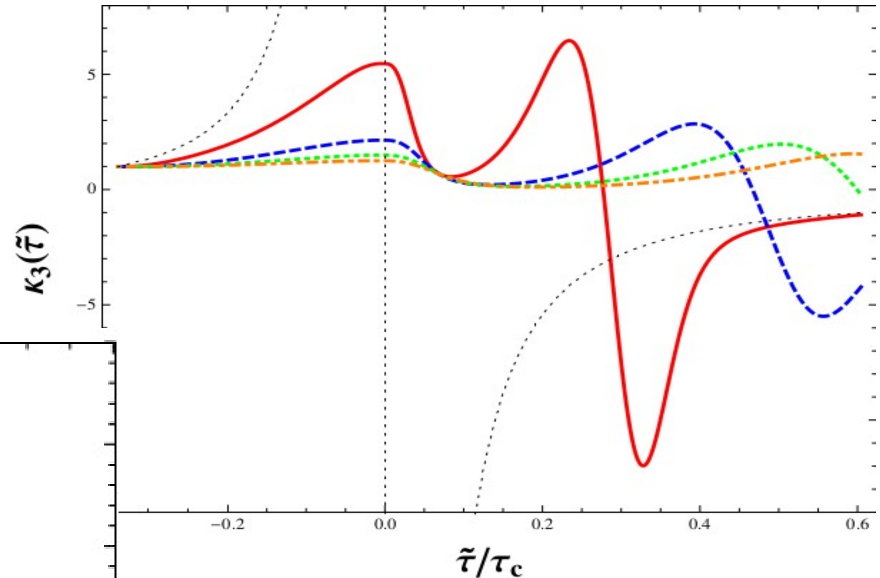




universality regained …

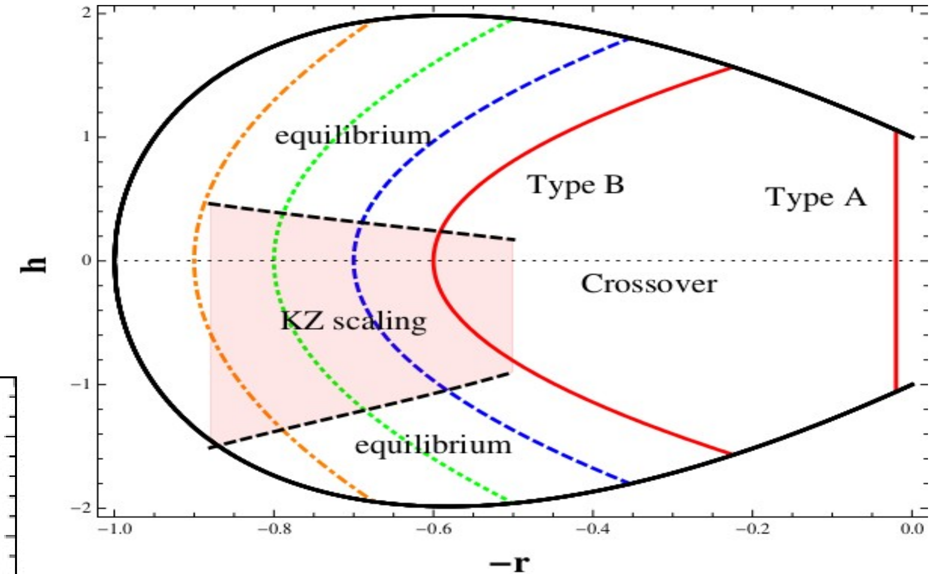
$$\kappa_n(\tau;\Gamma) \sim l_{KZ}^{-\frac{1}{2}+\frac{5}{2}(n-1)}\ \bar{f}_n^{l}(t;\theta_{KZ})$$

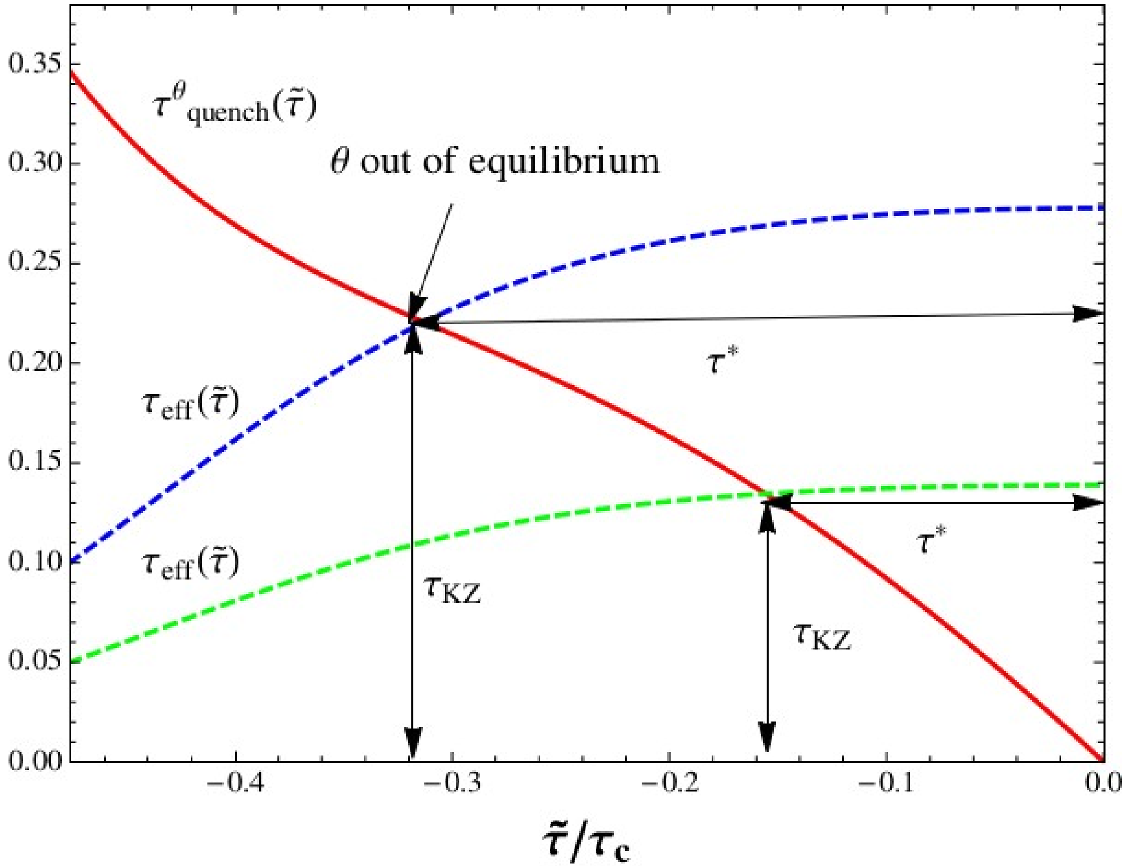




more general example:

trajectory Type-B

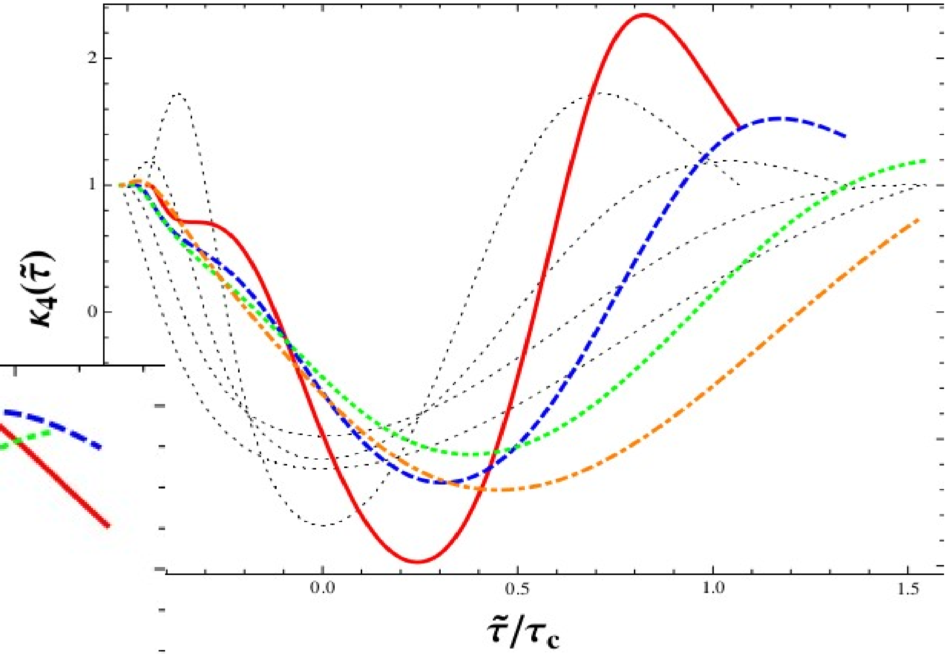


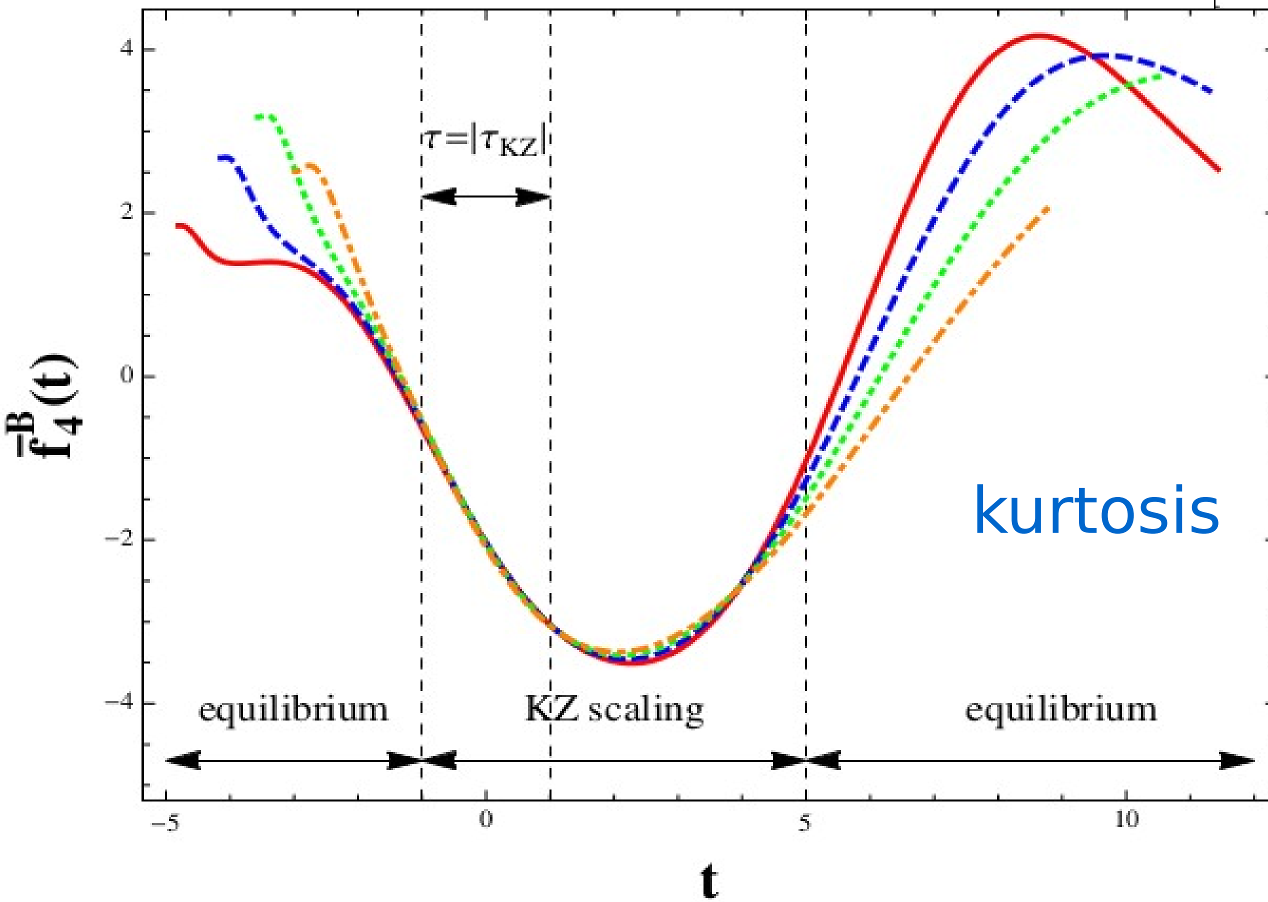


$$\tau_{\text{quench}} = \tau^{\theta}_{\text{quench}}$$

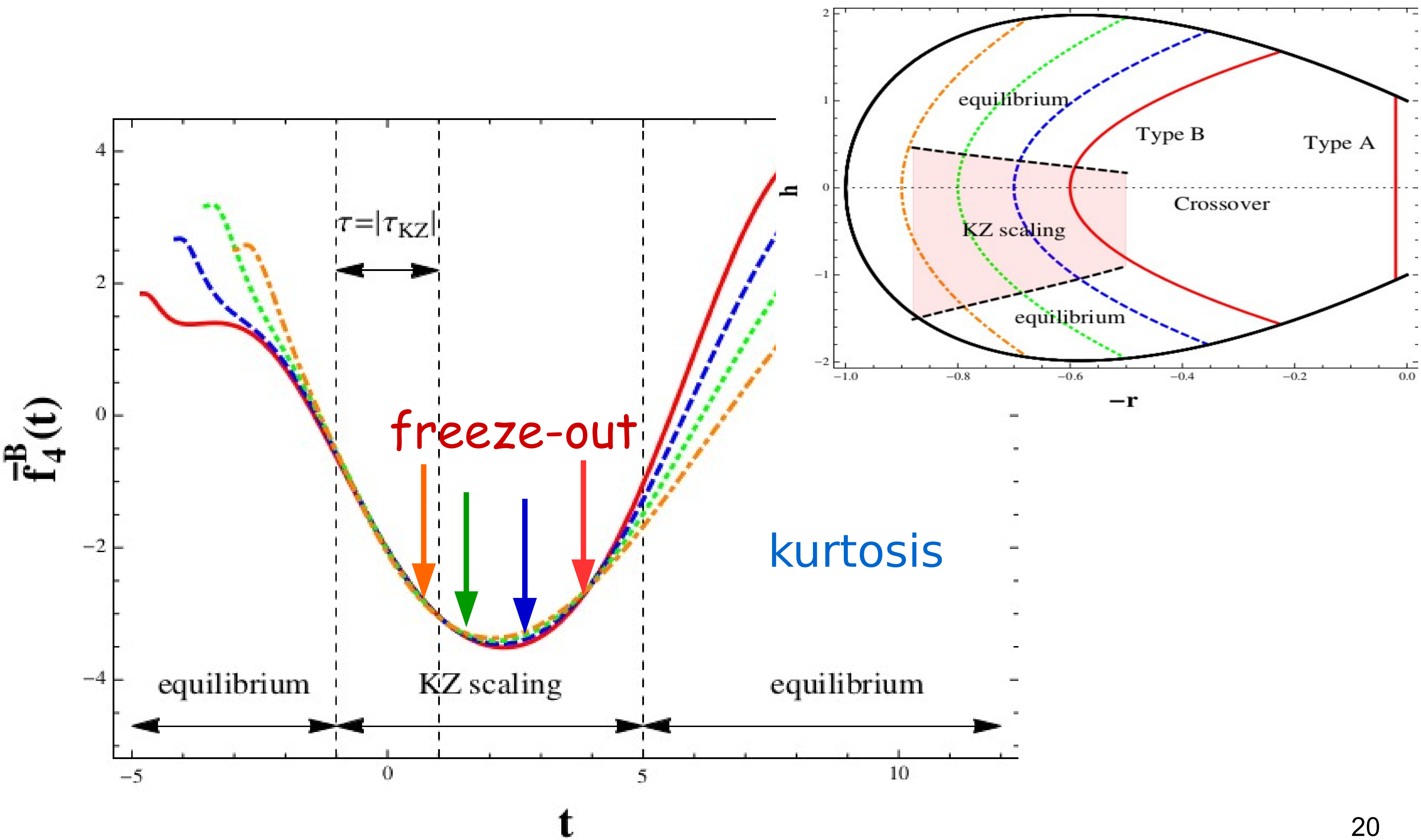
$$\tau^{\theta}_{\text{quench}} = \left|\frac{\theta(\tau)}{\partial_\tau \theta(\tau)}\right|$$

$$\kappa_n(\tau;\Gamma) \sim l_{KZ}^{-\frac{1}{2}+\frac{5}{2}(n-1)}\,\bar{f}_n^{l}(t;\theta_{KZ})$$

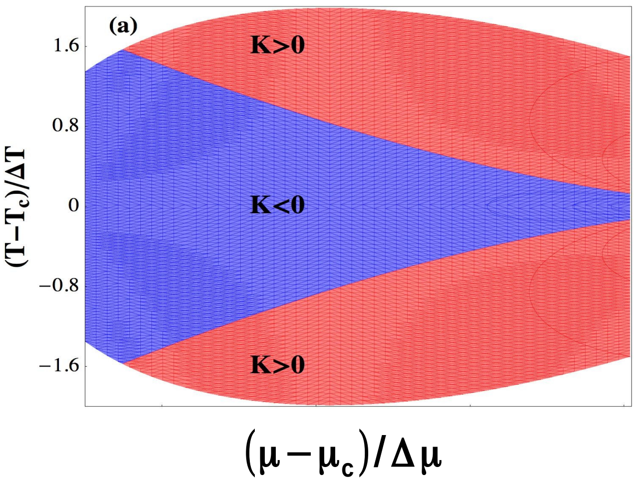




kurtosis

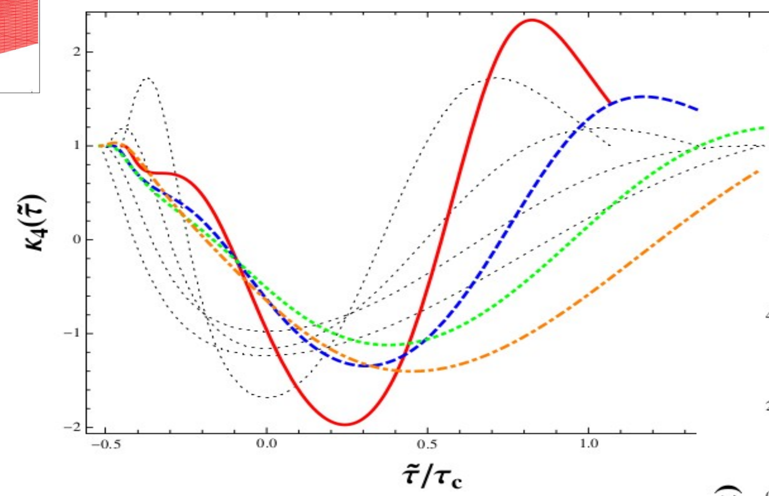

freeze-out
kurtosis
τ=|τ_KZ|
equilibrium
KZ scaling
equilibrium
t
equilibrium
Type B
Type A
Crossover
KZ scaling
equilibrium
h
−r

summary



equilibrium
criticality

off-equilibrium
complexity



off-equilibrium
universality