Critical Point and Onset of Deconfinement 2016

Chiral magnetic effect and chiral kinetic theory

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References:

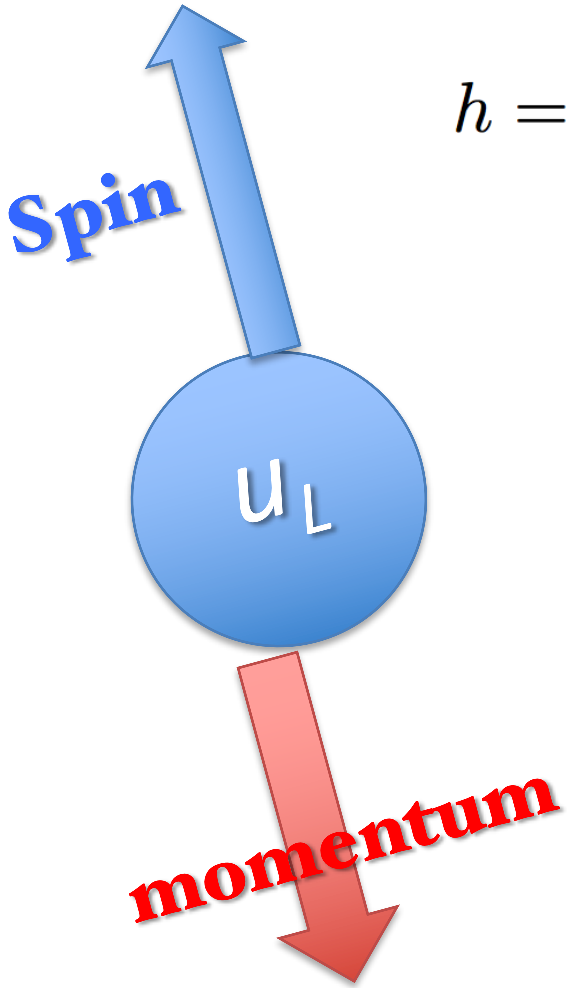
- J.H. Gao, Z.T. Liang, SP, Q. Wang, X.N. Wang, PRL 109 (2012) 232301
- J.W. Chen, SP, Q. Wang, X.N. Wang, PRL 110 (2013) 262301
- SP, S.Y. Wu, D.L. Yang, Phys.Rev. D89 (2014) 8, 085024; Phys.Rev. D91 (2015) 2, 025011
- J.W. Chen, T. Ishii, SP, N. Yamamoto, arXiv:1603.03620

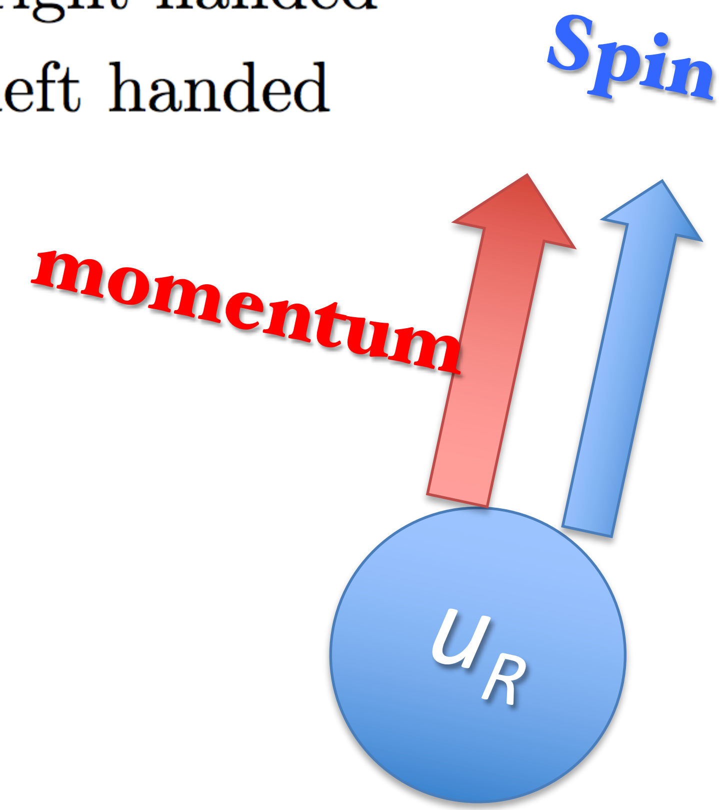
Outline

- **Chiral magnetic and vortical effects**
- **Chiral kinetic theory**
- **Recent progress**
  - Chiral Hall separation effect
  - Nonlinear chiral transport phenomena
  - Magneto-hydrodynamics
- **Summary**

Chirality of massless fermions

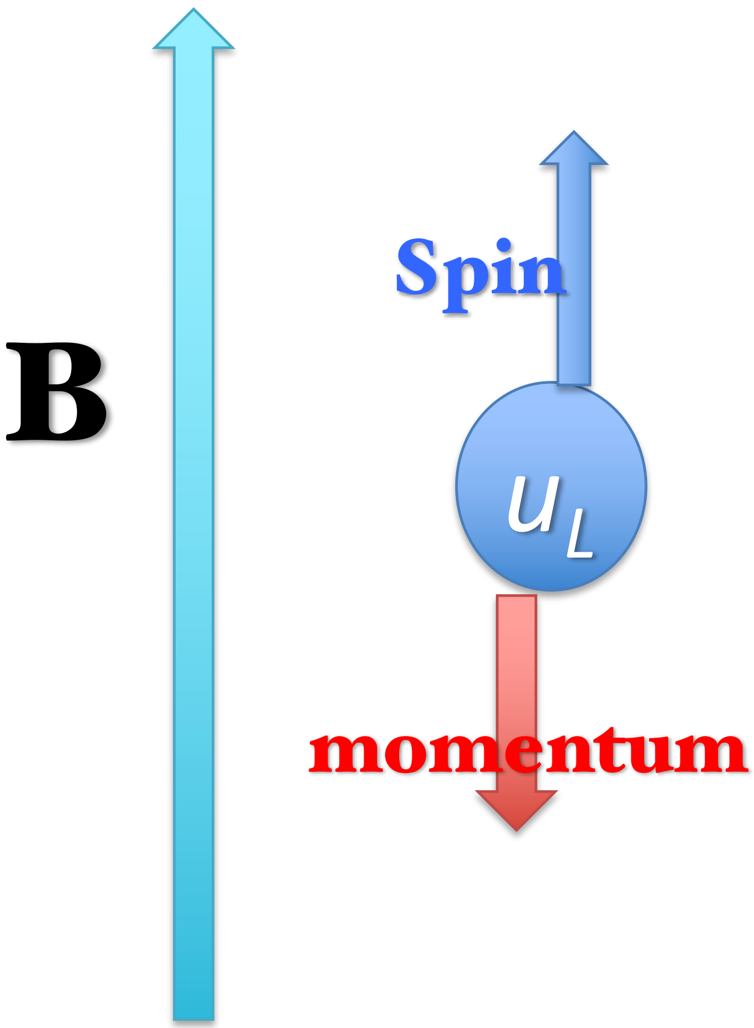
$$h = \frac{\sigma \cdot p}{|\mathbf{p}|} = \begin{cases} +1, & \text{right handed} \\ -1, & \text{left handed} \end{cases}$$

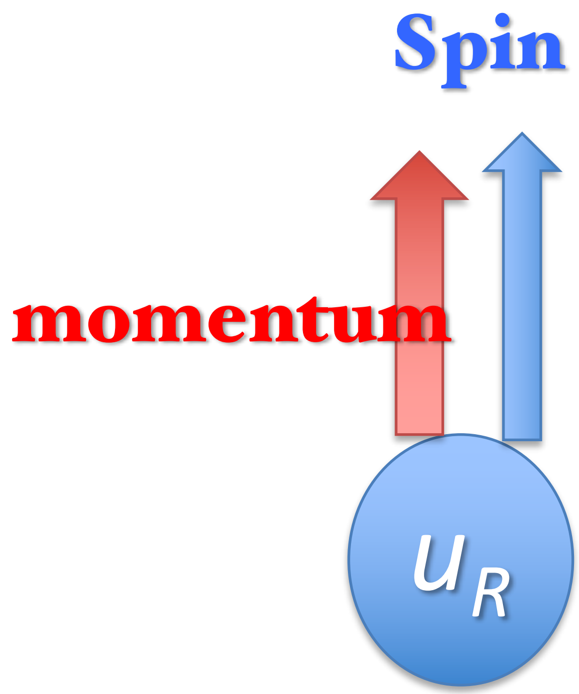




Chirality

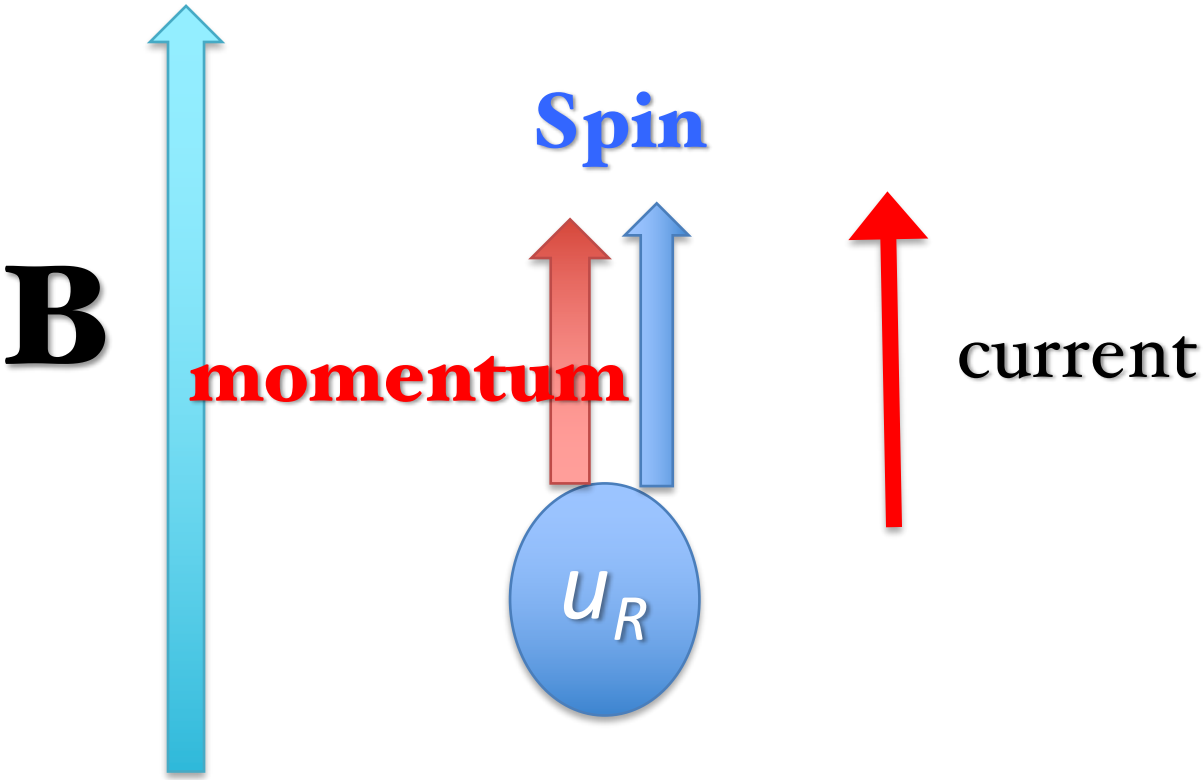
If Number of uL = Number of uR
No current is observed.



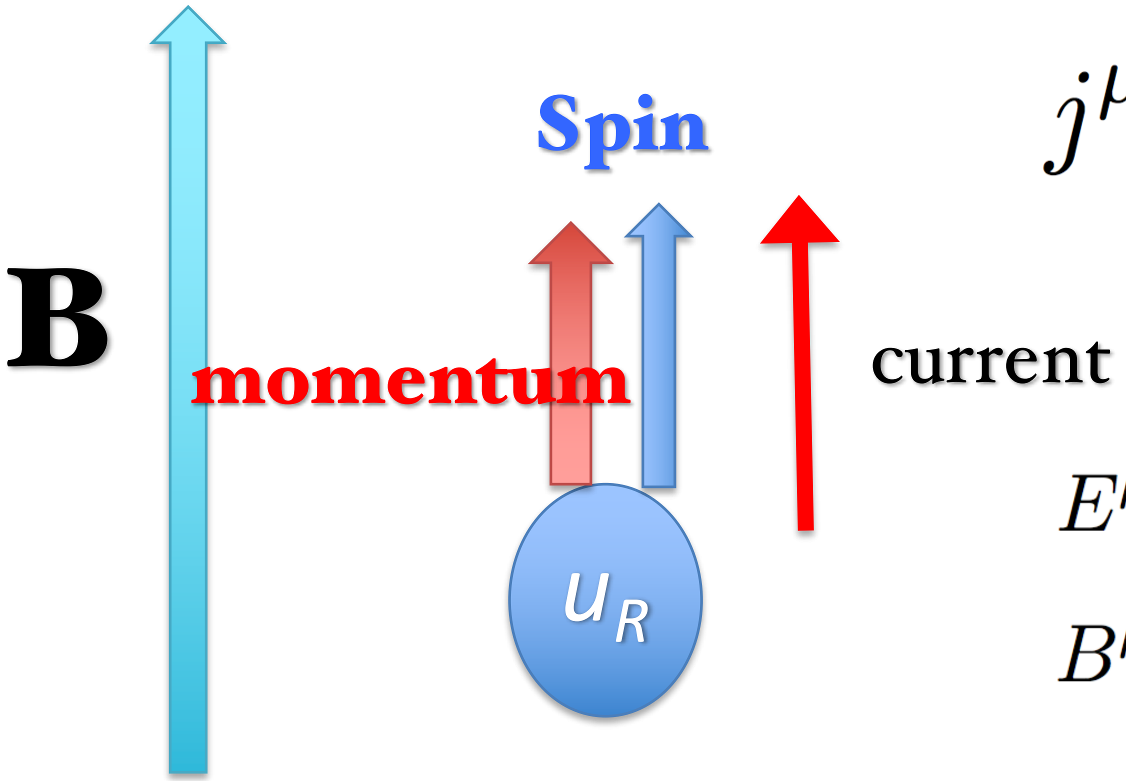


Chiral Magnetic Effect

If Number of uL ≠ Number of uR
A electric current will be observed.



Chiral Magnetic Effect (CME)



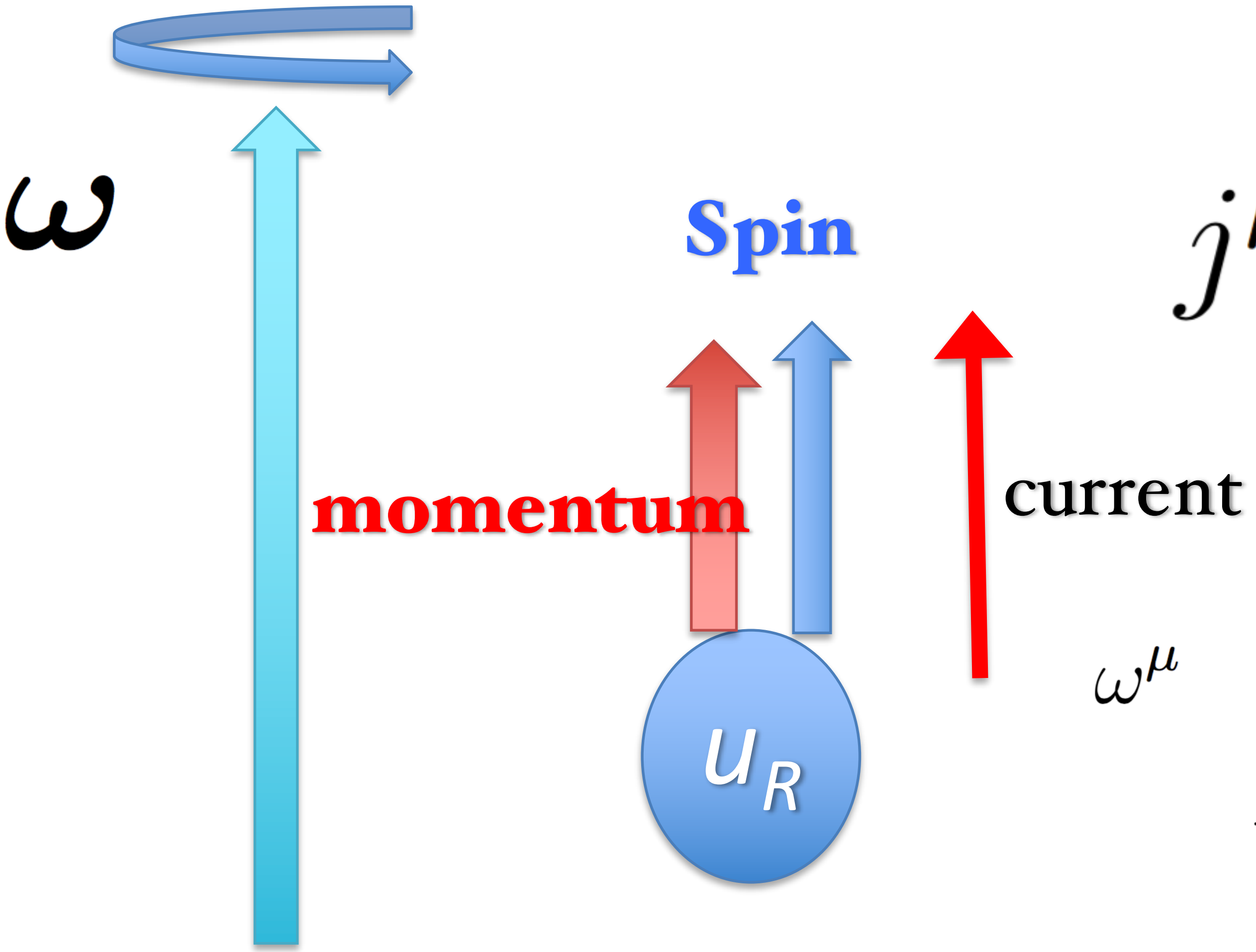
$$j^\mu = \xi_B B^\mu,$$

$$E^\mu = F^{\mu\nu} u_\nu,$$
$$B^\mu = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta} u_\nu F_{\alpha\beta}.$$

u^μ : fluid velocity

D.E. Kharzeev, L.D. McLerran, H.J. Warringa, NPA 803, 227
K. Fukushima, D. E. Kharzeev, H. J. Warringa, PRD78, 074033

Chiral Vortical Effect (CVE)



$$j^\mu = \xi \omega^\mu$$

$$\begin{aligned} \omega^\mu &= \epsilon^{\mu\nu\alpha\beta} u_\nu \partial_\alpha u_\beta \\ &\rightarrow (0, \nabla \times \mathbf{v}) \end{aligned}$$

Chiral Magnetic and Vortical Effect

Charge current

Magnetic field

Vorticity

$$j^\mu = \xi_B B^\mu + \xi \omega^\mu,$$

$$j_5^\mu = \xi_{5B} B^\mu + \xi_5 \omega^\mu,$$

Chiral current

New Transport coefficients

$$
\begin{aligned}
j^{\mu} &= \xi_B B^{\mu} + \xi \omega^{\mu}, \\
j_5^{\mu} &= \xi_{5B} B^{\mu} + \xi_5 \omega^{\mu},
\end{aligned}
$$

- Weakly coupling, Kubo formula

(Fukushima('08),Kharzeev('11),Landsteiner('11), Hou('12), …)

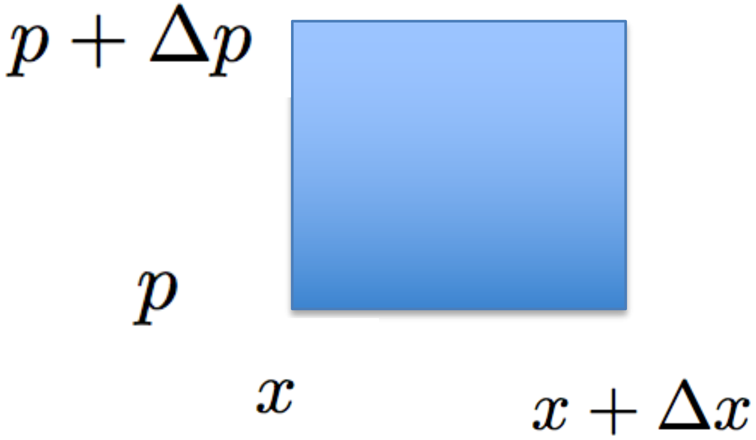
- Strong coupling, AdS/CFT duality,

(Erdmenger('09), Banerjee('11), Torabian('11), …)

Kinetic theory

- Kinetic theory: a microscopic dynamic theory for many-body system, to compute transport coefficients.

- distribution function, e.g. Fermi-Dirac distribution f(x,p)



Boltzmann equations

- Chiral Magnetic effect (quantum effect)
  VS Semi-classical Boltzmann eq.

- We try to study these chiral phenomena by Boltzmann equations, but we failed…

- It seems that one has to modify the Boltzmann equations.

SP, J.H. Gao, Q. Wang, Phys.Rev. D83 (2011) 094017

Wigner function for fermions

- Wigner function: a quantum distribution function, ensemble average, normal ordering

Vasak, Gyulassy and Elze ('86,'87,'89)

$$W(x,p) = <: \int \frac{d^4y}{(2\pi)^4} e^{-ipy} \overline{\psi}(x+\frac{1}{2}y) \otimes \boxed{PU(x,y)} \psi(x-\frac{1}{2}y) :>$$

Gauge link

$\overline{\psi}(x+\frac{1}{2}y)$ **X** $\psi(x-\frac{1}{2}y)$

Macroscopic quantities

Charge current

$$j^\mu(x) \equiv <: \overline{\psi}(x)\gamma^\mu\psi(x) :>= \int d^4p\mathrm{Tr}\ (\gamma^\mu W),$$

Chiral current

$$\begin{aligned} j_5^\mu(x) &= \lim_{\epsilon\to 0} <: \overline{\psi}(x+\frac{1}{2}\epsilon)\gamma^5\gamma^\mu e^{i\int_{x-\epsilon/2}^{x+\epsilon/2} dz\cdot A(z)}\psi(x-\frac{1}{2}\epsilon) :> \\ &= \int d^4p\mathrm{Tr}\ (\gamma^\mu\gamma^5 W) \end{aligned}$$

Master equation from Dirac Eq.

- Massless, constant external electromagnetic fields $F^{\mu\nu}_{ext}$, neglecting particles' interactions

$$[\gamma^\mu p_\mu + \frac{1}{2}i\ \gamma^\mu(\partial^x_\mu - QF^{ext}_{\mu\nu}\partial^p_\mu)]W = 0,$$

Vasak, Gyulassy and Elze ('86,'87,'89)

- First order differential equation, solve it order by order

Solve the Master equation

- Gradient expansion to Winger function W and its master equation,
  - expand all quantities at the power of derivatives

    $O(\partial_x^1), O(\partial_x^2),$
  - external fields are weak $F^{\mu\nu} \sim \partial_x^\mu A^\nu \sim O(\partial^1),$

Leading order

- 0^{th} order, non-interacting ideal gas
  - classical Fermi-Dirac distribution

- input parameters:
  - finite temperature T,
  - chemical potential $\mu = \mu_R + \mu_L$,
  - chiral chemical potential $\mu_5 = \mu_R - \mu_L$

1st order, Chiral anomaly

- It is remarkable that we obtain the chiral anomaly by Wigner function!

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} j_\lambda,$$

Energy momentum conservation

$$\partial_\mu j^\mu = 0,$$

Charge current conservation

$$\partial_\mu j_5^\mu = -\frac{e^2}{16\pi^2}\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}.$$

Chiral anomaly

Chiral magnetic and vortical effect

$$j^\mu = \xi_B B^\mu + \xi \omega^\mu,$$
$$j_5^\mu = \xi_{5B} B^\mu + \xi_5 \omega^\mu,$$

Consistent with other approaches!

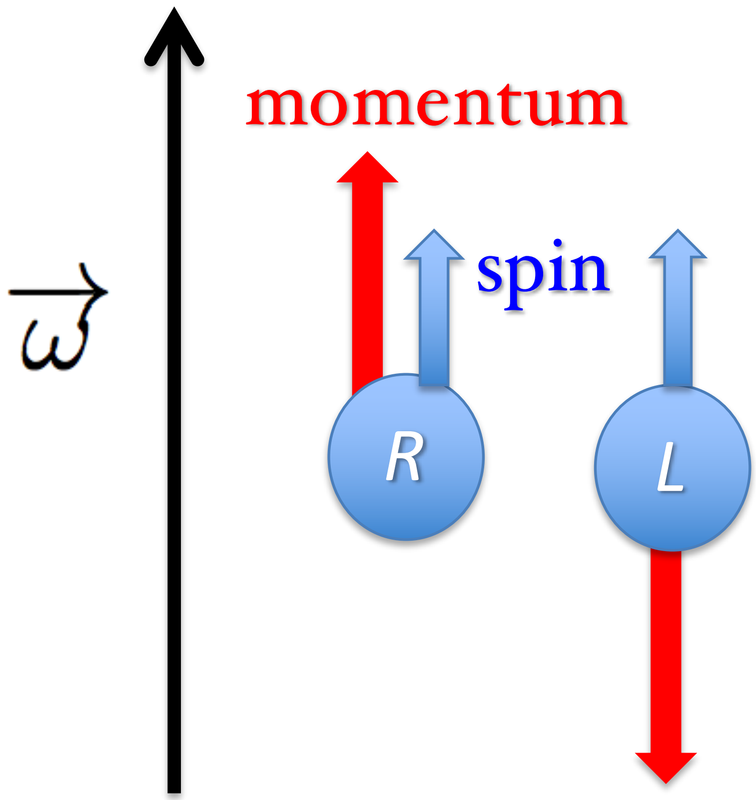
$$\xi = \frac{1}{\pi^2}\mu\mu_5,$$
$$\xi_B = \frac{Q}{2\pi^2}\mu_5,$$
$$\xi_5 = \frac{1}{6}T^2 + \frac{1}{2\pi^2}\left(\mu^2 + \mu_5^2\right),$$
$$\xi_{B5} = \frac{Q}{2\pi^2}\mu.$$

T: temperature

Chemical potentials

$$\mu = \mu_R + \mu_L,$$
$$\mu_5 = \mu_R - \mu_L,$$

Spin Local Polarization Effect



Chiral current

$$j_5^\mu \equiv j_R^\mu - j_L^\mu = \xi_5 \omega^\mu,$$

$$\xi_5 = \frac{1}{6}T^2 + \frac{1}{2\pi^2}(\mu^2 + \mu_5^2),$$

Can be observed in both high/low energy collisions

What can we learn from these results?

3-dimensional Chiral kinetic equation

- Integral over p0 and in local rest frame, we obtain the kinetic theory for chiral fermions.

$$\frac{dt}{d\tau}\partial_t f_{R/L} + \frac{d\mathbf{x}}{d\tau}\cdot\nabla_{\mathbf{x}} f_{R/L} + \frac{d\mathbf{p}}{d\tau}\cdot\nabla_{\mathbf{p}} f_{R/L} = 0,$$

$$\frac{dt}{d\tau} = 1 \pm Q\mathbf{\Omega}\cdot\mathbf{B} \pm 4|\mathbf{p}|(\mathbf{\Omega}\cdot\boldsymbol{\omega}),$$

velocity

$$\frac{d\mathbf{x}}{d\tau} = \hat{\mathbf{p}} \pm Q(\hat{\mathbf{p}}\cdot\mathbf{\Omega})\mathbf{B} \pm Q(\mathbf{E}\times\mathbf{\Omega}) \pm \frac{1}{|\mathbf{p}|}\boldsymbol{\omega},$$

force

$$\frac{d\mathbf{p}}{d\tau} = Q(\mathbf{E} + \hat{\mathbf{p}}\times\mathbf{B}) \pm Q^2(\mathbf{E}\cdot\mathbf{B})\mathbf{\Omega} \mp Q|\mathbf{p}|(\mathbf{E}\cdot\boldsymbol{\omega})\mathbf{\Omega} \pm 3Q(\mathbf{\Omega}\cdot\boldsymbol{\omega})(\mathbf{p}\cdot\mathbf{E})\hat{\mathbf{p}},$$

3-dimensional Chiral kinetic equation

- Neglect all terms proportional to Ω, it becomes the **standard** Boltzmann eq.

$$\frac{dt}{d\tau}\partial_t f_{R/L} + \frac{d\mathbf{x}}{d\tau}\cdot\nabla_{\mathbf{x}} f_{R/L} + \frac{d\mathbf{p}}{d\tau}\cdot\nabla_{\mathbf{p}} f_{R/L} = 0,$$

$$\frac{dt}{d\tau} = 1$$

velocity $$\frac{d\mathbf{x}}{d\tau} = \hat{\mathbf{p}}$$

force $$\frac{d\mathbf{p}}{d\tau} = Q(\mathbf{E} + \hat{\mathbf{p}}\times\mathbf{B})$$

$f_{R/L}$: distribution function for right or left handed fermions

Berry Phase (1)

- Firstly, let us consider an **adiabatic** process. At each time, the system is at its eigenstate e.g. U.

- Secondly, we assume the **Hamiltonian** is **time dependent** H=H(t). So do those eigenstates U=U(t),

  i.e. $U(t+\Delta t)=U(t)+\Delta t\,\Delta U$.

- Finally, the system goes to its **initial eigenstate**. Then there is an additional **phase factor** to the wave function. It is Berry phase.

- Analogy to moving a vector in a curved space.

Berry Phase (2)

- Let us consider the Hamitonlian of **Weyl** fermions in **momentum** space.

$$H = i\sigma \cdot \nabla \to \sigma \cdot \mathbf{p}, \qquad HU(p), = EU(p),$$

$$U(t+\Delta t) = U(t) + i\Delta t \frac{d\mathbf{p}}{dt} \cdot \mathbf{a}_p, \qquad \mathbf{a}_p = -iU(t)\nabla_{\mathbf{p}} U(t),$$

- If the system goes back to its initial eigenstate, then phase factor is **independent** on the **path**. So, it is **physical**!

Stokes's theorem

$$\begin{aligned} \Psi(p) &= \exp\left(i \oint_C d\mathbf{p} \cdot \mathbf{a}_p\right) U(p) \\ &= \exp\left(i \iint d\mathbf{S} \cdot \Omega_p\right) U(p) \end{aligned} \qquad \Omega_p = \nabla_p \times \mathbf{a}_p,$$

D. Xiao, M.C. Chang, Q. Niu, Rev. Mod. Phys. 82.1959

Berry Phase (3)

- In the **absence** of external fields, the Berry phase is **decoupled** to the dynamics.
- In the **presence** of electromagnetic fields,

velocity
$$\sqrt{\gamma}\frac{d\mathbf{x}}{dt} = \hat{\mathbf{p}} + e\mathbf{E}\times\Omega + e\mathbf{B}(\hat{\mathbf{p}}\cdot\Omega)$$

force
$$\sqrt{\gamma}\frac{d\mathbf{p}}{dt} = e\mathbf{E} + \hat{\mathbf{p}}\times e\mathbf{B} + e^2(\mathbf{E}\cdot\mathbf{B})\Omega$$

$$\Omega = \nabla_p \times \mathbf{a}_p = \frac{\mathbf{p}}{2|\mathbf{p}|^3},$$

$$\sqrt{\gamma} = 1 + e\mathbf{B}\cdot\mathbf{\Omega}$$

Son and Yamamoto, PRL 12', PRD 13', Stephanov and Y. Yin PRL 12',

3-dimensional Chiral kinetic equation

- We used Wigner function to obtain the chiral kinetic equation with **Berry phase**.

$$\frac{dt}{d\tau}\partial_t f_{R/L} + \frac{d\mathbf{x}}{d\tau}\cdot\nabla_{\mathbf{x}} f_{R/L} + \frac{d\mathbf{p}}{d\tau}\cdot\nabla_{\mathbf{p}} f_{R/L} = 0,$$

$$\frac{dt}{d\tau} = 1 \pm Q\boldsymbol{\Omega}\cdot\mathbf{B} \pm 4|\mathbf{p}|(\boldsymbol{\Omega}\cdot\boldsymbol{\omega}),$$

velocity

$$\frac{d\mathbf{x}}{d\tau} = \hat{\mathbf{p}} \pm Q(\hat{\mathbf{p}}\cdot\boldsymbol{\Omega})\mathbf{B} \pm Q(\mathbf{E}\times\boldsymbol{\Omega}) \pm \frac{1}{|\mathbf{p}|}\boldsymbol{\omega},$$

force

$$\frac{d\mathbf{p}}{d\tau} = Q(\mathbf{E}+\hat{\mathbf{p}}\times\mathbf{B}) \pm Q^2(\mathbf{E}\cdot\mathbf{B})\boldsymbol{\Omega} \mp Q|\mathbf{p}|(\mathbf{E}\cdot\boldsymbol{\omega})\boldsymbol{\Omega} \pm 3Q(\boldsymbol{\Omega}\cdot\boldsymbol{\omega})(\mathbf{p}\cdot\mathbf{E})\hat{\mathbf{p}},$$

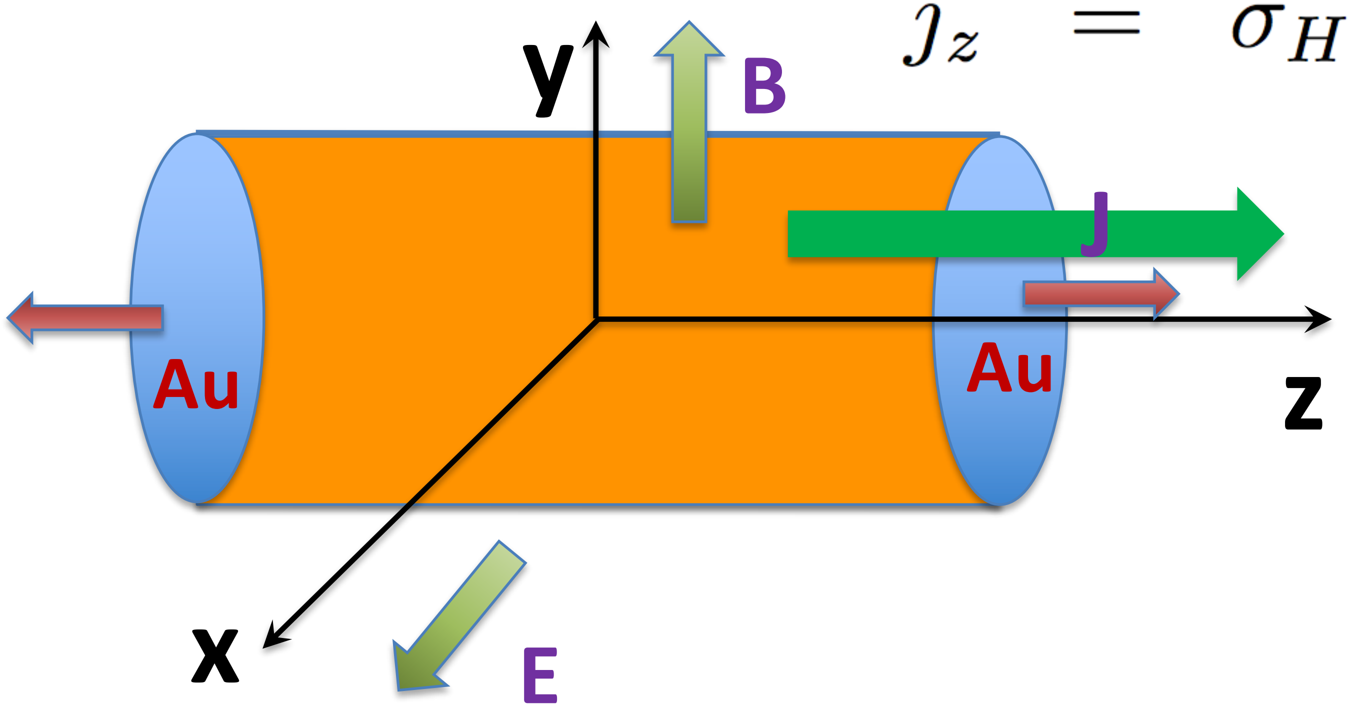
Recent progress

- **Chiral Hall separation effect**
- **Nonlinear chiral transport phenomena**
- **Magneto-hydrodynamics**

Chiral Hall separation effect

- Assuming $E \perp B$, according to **Hall effect**:

$$j_z = \sigma_H E_x B_y$$



- Charge and chirality separation in **longitudinal** direction

SP, S.Y. Wu, D.L. Yang, PRD 91 (2015) 2, 025011

Nonlinear Chiral transport effects

- An inhomogeneous chiral system
- In a large chemical potential limit
- **chiral kinetic theory** + **relaxation time approaches**

$$\mathbf{j}_e = \sigma_E \mathbf{E} + \sigma_{E\mu_5} \nabla \mu_5 \times \mathbf{E},$$

$$(\mu^2 + \mu_5^2) \frac{\sigma_{E\mu_5}}{\sigma_E} = \frac{\hbar c}{2}$$

Independent on the interactions!

J.W. Chen, T. Ishii, SP, N. Yamamoto, arXiv:1603.03620

Anomalous Magneto-hydrodynamics

- **Magneto-hydrodynamics:**
  - Relativistic hydrodynamics + Maxwell's eq.
- **Anomalous:**
  - Chiral magnetic effect + other chiral transport effects

Magneto-hydrodynamics

- **1D Bjorken + ideal Magneto-hydrodynamics:**
  - analytic solution

  V. Roy, SP, L. Rezzolla, D.H. Rischke, PLB 750 (2015) 45-52
  - with Magnetization effects

  SP, V. Roy, L. Rezzolla, D.H. Rischke, PRD 93 (2016), 074022
- **2+1 D Bjorken + ideal Magneto-hydrodynamics**
  - analytic solution:

  SP, D.L Yang, PRD93 (2016), 054042
  - Simulations:

  V. Roy, SP, L. Rezzolla, D.H. Rischke, in preparation

Summary

- We obtain the chiral magnetic and vortical effect, chiral anomaly by Wigner function.
- We derive the chiral kinetic equation (modified Boltzmann equation) related to Berry phase.
- We also made some progresses in
  - Chiral Hall separation effect,
  - nonlinear chiral transport effect
  - magneto-hydrodynamics.

Thank you!

Anomalous fluid dynamics

- We do not have those chiral transport terms in a normal fluid.

- Son and Suro´wka ('09) pointed out these terms are crucial to cancel the production of negative entropy in an anomalous fluid.

$$\partial_\mu T^{\mu\nu} = QF^{\nu\rho} j_\rho,$$

$$\partial_\mu j^\mu = 0, \qquad \partial_\mu j_5^\mu = -\frac{Q^2}{2\pi^2} E_\rho B^\rho,$$

Non-abelian Berry Phase

- For massive fermions, a particle can change its spin. In classical limit, we get,

velocity
force
spin

$$\begin{aligned} \frac{d\mathbf{x}}{dt} &= \hat{\mathbf{p}} + \frac{d\mathbf{p}}{dt} \times s_a \Omega^a, \\ \frac{d\mathbf{p}}{dt} &= e\mathbf{E} + \frac{d\mathbf{x}}{dt} \times e\mathbf{B}, \\ \frac{ds_a}{dt} &= \epsilon_{abc}\left(\frac{d\mathbf{p}}{dt} \cdot \mathbf{a}_b\right) s_c. \end{aligned}$$

J.W. Chen, J.Y. Pang, SP, Q. Wang, PRD89 (2014), 094003