Type II superspace with manifest T-duality and superstring action

Machiko Hatsuda (Juntendo & KEK)
with
Kiyoshi Kamimura & Warren Siegel

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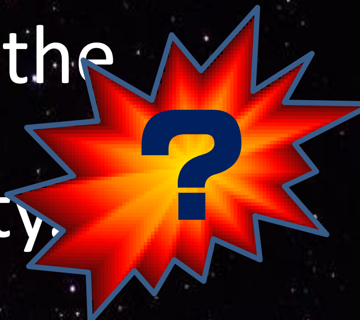
Introduction

- T-duality, originated from string, changes the picture of initial universe in Einstein gravity.



- Our approach to T-duality is

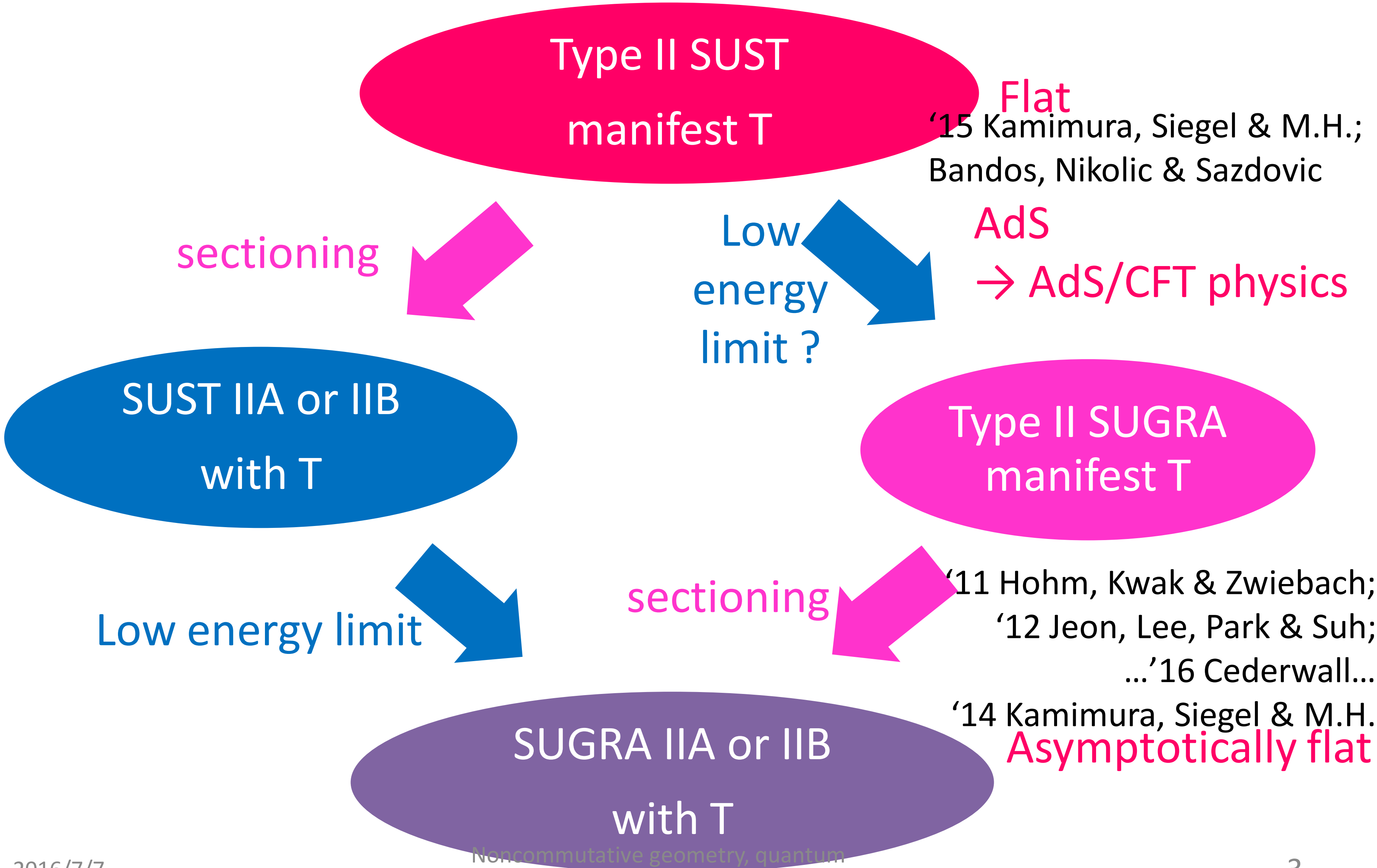
  to construct a theory

  with manifest T-duality,

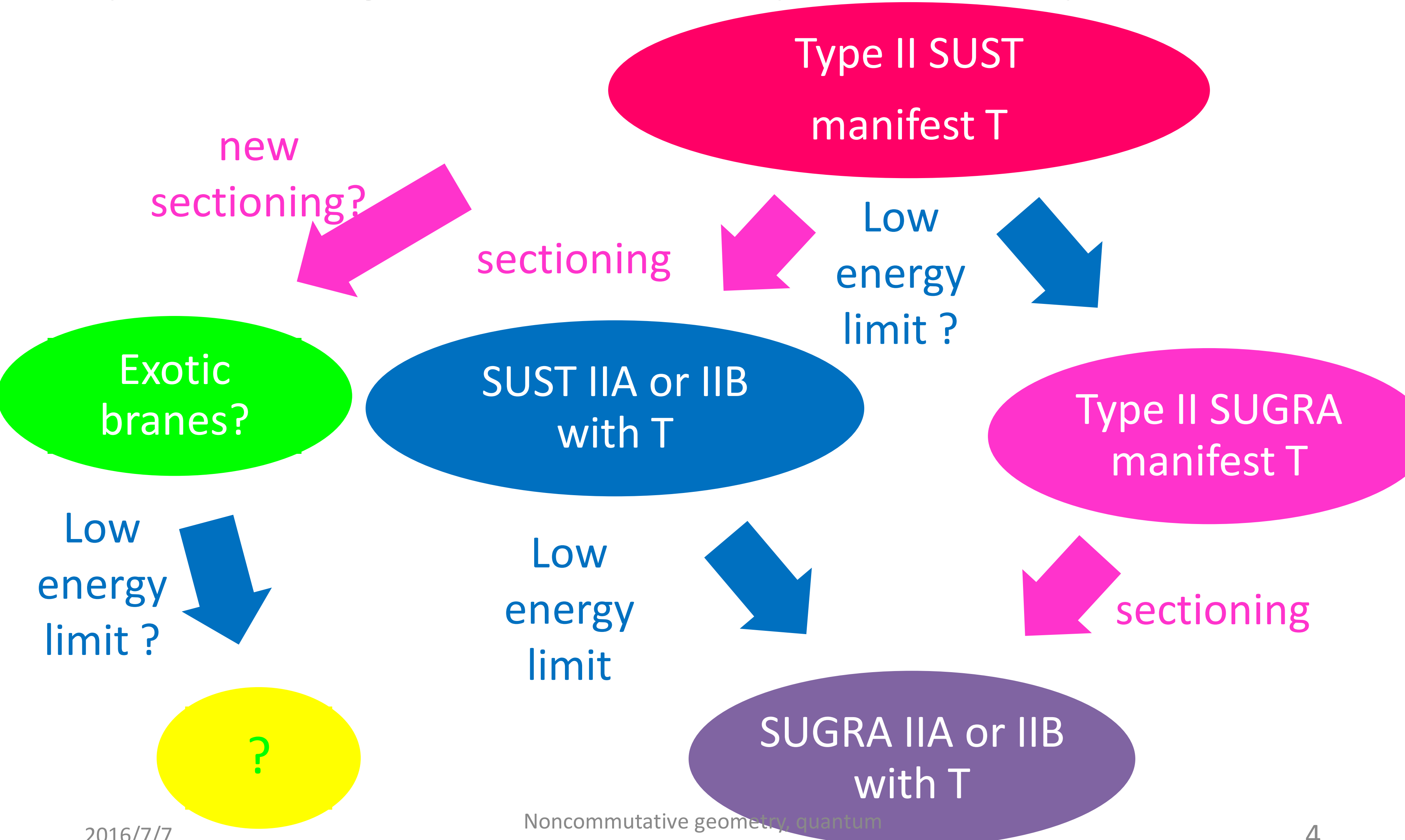
  supersymmetry &

  all gauge symmetries !

Superstring (SUST) & supergravity (SUGRA)



Superstring (SUST) & supergravity (SUGRA)



Plan of the talk

0. Introduction

I. Manifest T-duality

II. Type II superstring with manifest T-duality

III. Type II supergravity with manifest T-duality

IV. Conclusion

I. MANIFEST T-DUALITY

I-1. “Global” O(d,d) T-duality transformation

I-2. “Local” T-duality covariant general coordinate transformation

I-1. "Global" O(d,d) transf.

- Fractional linear transf.

$$(G+B)_{\rm mn} \to \frac{a(G+B)+b}{c(G+B)+d}$$

O(d,d) $\ni A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

- Vielbein: linear transf.

$$E_{\underline{\rm a}}{}^{\underline{\rm m}} \to E_{\underline{\rm a}}{}^{\underline{\rm n}} A_{\underline{\rm n}}{}^{\underline{\rm m}}$$

$$\begin{pmatrix} G^{-1} & G^{-1}B \\ -BG^{-1} & G-BG^{-1}B \end{pmatrix} = E^T \hat{\eta} E, \quad \hat{\eta} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

  – Orthonormal condition

$$E^T \eta E = \eta = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

- String currents: linear trans.

$$\mathring{\triangleright}_{\underline{\rm m}} \to A_{\underline{\rm m}}{}^{\underline{\rm n}} \mathring{\triangleright}_{\underline{\rm n}}$$

$$\mathring{\triangleright}_{\underline{\rm m}} = \begin{pmatrix} p_{\rm m} \\ \partial_\sigma x^{\rm m} \end{pmatrix} \quad , \quad \mathcal{H}_\tau = \frac{1}{2} \mathring{\triangleright} E^T \hat{\eta} E \mathring{\triangleright}$$

★"Global" gravity T-duality is originated from string T-duality.

I-2. "Local" T-cov. general coordinate transf.

- General coordinate transf. + B gauge transf.

$$\delta_{\xi^{\underline{\mathrm{m}}}}(G+B)=\delta_{\xi^{\mathrm{m}}}(G+B)+\delta_{\xi_{\mathrm{m}}}B$$

- C (Courant) bracket : $\delta_\xi E_{\underline{\mathrm{a}}}{}^{\underline{\mathrm{m}}}=[E_{\underline{\mathrm{a}}}{}^{\underline{\mathrm{m}}},\xi]_{\mathrm{T}}$ 1993 Siegel

$$[E\mathring{\triangleright}(0),\xi\mathring{\triangleright}(\sigma)]=-i\delta_\xi E\mathring{\triangleright}\ \delta(\sigma)-iE\eta\xi\ \underline{\partial_\sigma\delta(\sigma)}$$

$\exists$ ambiguity

'12 Kimura & M.H.

- Affine Lie algebra:

$$[\mathring{\triangleright}_M(0),\mathring{\triangleright}_N(\sigma)]=-if_{MN}{}^K\mathring{\triangleright}_K\delta(\sigma)-i\eta_{MN}\ \partial_\sigma\delta(\sigma)$$

– Jacobi ⇒ { η_{MN} is nondegenerate; $f_{MNL}=f_{MN}{}^K\eta_{KL}$ is totally anti-sym. }

★"Local" gravity T-duality is originated from string T-duality.

II. TYPE II SUPERSTRING WITH MANIFEST T-DUALITY

II-1. “Local ” covariant derivative
& ”Global” symmetry generator

II-2. Superstring action

II-3. AdS superstring

II-1. Covariant derivative & symmetry generator

- SUSY theory needs both **global SUSY**&

  local SUSY whose 1st class part is κ-sym.

- **Covariant derivatives:** $\mathring{\triangleright}_M\,,\ \mathring{\triangleright}_{M'}$
  - define the superspace & **local** gauge sym.
  - make a superstring Hamiltonian
- **Symmetry generators:** $\tilde{\triangleright}_M\,,\ \tilde{\triangleright}_{M'}$
  - generate **global** symmetry algebra
  - make dimensional reduction constraints

Affine Lie algebras

- Lie algebra $[G_I, G_J] = i f_{IJ}{}^K G_K$
- Affine Lie algebras
  - **Covariant derivative**

    $$[\mathring{\triangleright}_I(0), \mathring{\triangleright}_J(\sigma)] = -i f_{IJ}{}^K \mathring{\triangleright}_K(\sigma)\delta(\sigma) - i\eta_{IJ}\partial_\sigma\delta(\sigma)$$

  - **Symmetry generator**

    $$[\tilde{\triangleright}_I(0), \tilde{\triangleright}_J(\sigma)] = i f_{IJ}{}^K \tilde{\triangleright}_K(\sigma)\delta(\sigma) + i\eta_{IJ}\partial_\sigma\delta(\sigma)$$

  - They commute

    $$[\mathring{\triangleright}_I(0), \tilde{\triangleright}_J(\sigma)] = 0$$

Making nondegenerate algebra

Gomis, Kamimura & Lukierski, 2009

- For a symmetric space:

$$[h_0, h_0] = h_0 \ , \ \ [h_0, k] = k \ , \ \ [k, k] = h_0$$

- Add another Lorentz algebra: $[h_1, h_1] = h_1$
- Redefine generators :

$$\left\{ \begin{array}{l} h_0 + h_1 = h \\ h_0 - h_1 = \check{h} \\ \sqrt{2}k \to k \end{array} \right. \quad \Rightarrow \quad \left\{ \begin{array}{l} [h, h] = h \ , \ \ [h, \check{h}] = \check{h} \ , \ \ [\check{h}, \check{h}] = h \\ [h, k] = k \ , \ \ [k, k] = h + \check{h} \ , \ \ [\check{h}, k] = k \end{array} \right.$$

- Nondegenerate group metric: $\eta_{kk} = \eta_{\check{h}\check{h}} = 1$

Nondegenerate super-Poincare algebra

$$
\begin{aligned}
\{D_\mu(1), D_\nu(2)\} &= 2P_m \gamma^m_{\mu\nu}\ \delta(_{2=1}) \\
[D_\mu(1), P_n(2)] &= 2(\gamma_n \Omega)_\mu\ \delta(_{2=1}) \\
[S_{mn}(1), \Sigma^{lk}(2)] &= -i\delta^{[k}_{[m} \Sigma_{n]}{}^{l]}\ \delta(_{2=1}) \qquad +i\delta^l_{[m}\delta^k_{n]}\ \partial_\sigma(_{2=1}) \\
\{D_\mu(1), \Omega^\nu(2)\} &= -\tfrac{i}{4}\Sigma^{mn}(\gamma_{mn})^\nu{}_\mu\ \delta(_{2=1}) \qquad +i\delta^\nu_\mu\ \partial_\sigma(_{2=1}) \\
[P_m(1), P_n(2)] &= i\Sigma_{mn}\ \delta(_{2=1}) \qquad +i\eta_{mn}\ \partial_\sigma(_{2=1})
\end{aligned}
$$

Nondegenerate pairs

"maxwell algebra"

Gomis, Kamimura & Lukierski, 2009

Covariant derivative & symmetry generator

$$\mathring{\triangleright}_M = (S_{mn}, D_\mu, P_m, \Omega^\mu, \Sigma^{mn})$$

$$\begin{cases} S_{mn} &= \mathring{\nabla}_S \\ D_\mu &= \mathring{\nabla}_D + \frac{1}{2} J_D \\ P_m &= \mathring{\nabla}_P + J_P \\ \Omega^\mu &= \mathring{\nabla}_\Omega + \frac{3}{2} J_\Omega \\ \Sigma^{mn} &= \mathring{\nabla}_\Sigma + 2 J_\Sigma \end{cases}$$

$$\tilde{\triangleright}_M = (\tilde{S}_{mn}, \tilde{D}_\mu, \tilde{P}_m, \tilde{\Omega}^\mu, \tilde{\Sigma}^{mn})$$

$$\begin{cases} \tilde{S}_{mn} &= \tilde{\nabla}_S - 2\tilde{J}_S + \cdots \\ \tilde{D}_\mu &= \tilde{\nabla}_D - \frac{3}{2} \tilde{J}_D + \cdots \\ \tilde{P}_m &= \tilde{\nabla}_P - \tilde{J}_P + \cdots \\ \tilde{\Omega}^\mu &= \tilde{\nabla}_\Omega - \frac{1}{2} \tilde{J}_\Omega + \cdots \\ \tilde{\Sigma}^{mn} &= \tilde{\nabla}_\Sigma \end{cases}$$

※ overall factor $1/\sqrt{2}$ is omitted.

★ Coefficients are canonical dimensions

⇒ guarantee the closure of affine Lie algebras.

II-2. Superstring action

$$\mathcal{L} = \partial_\tau Z^{\underline{M}} \partial_{\underline{M}} - \mathcal{H},\ \mathcal{H} = \underline{\lambda_\tau} \mathcal{H}_\tau + \underline{\lambda_\sigma} \mathcal{H}_\sigma + \lambda_i \phi^i$$

1st class constraints

$$\mathcal{H}_\tau = \frac{1}{2} \mathring{\triangleright}_{\underline{M}} \hat{\eta}^{\underline{MN}} \mathring{\triangleright}_{\underline{N}} = 0,\ \mathcal{H}_\sigma = \frac{1}{2} \mathring{\triangleright}_{\underline{M}} \eta^{\underline{MN}} \mathring{\triangleright}_{\underline{N}} = 0$$

Virasoro

$$\phi_i = \begin{cases} (\not{P} D)^\mu + (\not{S}\Omega)^\mu = (\not{P}' D)^{\mu'} + (\not{S}'\Omega)^{\mu'} = 0 \\ S_{mn} = S_{m'n'} = 0 \\ \tilde{P}_m - \tilde{P}_{m'} = \tilde{\Omega}^\mu = \tilde{\Omega}^{\mu'} = \tilde{\Sigma}^{mn} = \tilde{\Sigma}^{m'n'} = 0 \end{cases}$$

κ-sym.

Local Lorentz

Dimensional reduction

$\Rightarrow$ 2d. Diffeo. inv. for κ-sym. even in chiral field action!

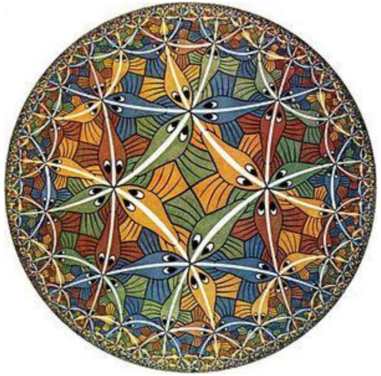
$$\mathcal{L}_0 = \underline{\sqrt{-h} h^{ij}} J_i^{P+} J_j^{P+}\ ,\quad J^{P\pm} = J^P \pm J^{P'} + \cdots$$

Winding mode

$$\mathcal{L}_{\rm WZ} = -\frac{1}{2} \epsilon^{ij} (J_i^{P+} J_j^{P-} + J_i^N J_j^M B_{MN} - J_i^{N'} J_j^{M'} B_{M'N'})$$

+boundary term $\Rightarrow$ GS SUST in simple unitary gauge

II-3. AdS superstring



- RR flux breaks double Lorentz into "*H*"

$$\langle F_{\mathrm{RR}}^{\mu\nu'}\rangle \neq 0 \Rightarrow \begin{cases} [H, F_{\mathrm{RR}}^{\mu\nu'}] = 0, \quad H = S_{mn} - S_{m'n'} \\ [K, F_{\mathrm{RR}}^{\mu\nu'}] \neq 0, \quad K = S_{mn} + S_{m'n'} \end{cases}$$

$$[H, H] = H,\ [H, K] = K,\ [K, K] = H$$

- RR gauge field is introduced by $\Upsilon_{\mu\nu'},\ \mathbf{F}^{\mu\nu'}$

$$\{D_\mu, D_{\nu'}\} \ = \ \Upsilon_{\mu\nu'}$$

$$[\mathbf{F}^{\mu\nu'}({}_1), \Upsilon_{\nu\mu'}({}_2)]$$

$$= (\delta^\mu_\nu \Sigma^{m'n'}(\gamma_{m'n'})^{\nu'}{}_{\mu'} + \delta^{\nu'}_{\mu'}\Sigma^{mn}(\gamma_{mn})^\mu{}_\nu)\delta({}_{2=1}) + \delta^\mu_\nu \delta^{\nu'}_{\mu'}\ \partial_\sigma \delta({}_{2=1})$$

III. TYPE II SUPERGRAVITY WITH MANIFEST T-DUALITY

III-1. SUGRA gauge symmetry

III-2. SUGRA field equations

III-1. SUGRA gauge symmetry

'14 Kamimura, Siegel & M.H.

- Curved space cov. der.: $\triangleright_{\underline{A}} = E_{\underline{A}}{}^{\underline{M}} \triangleright_{\underline{M}}$
- SUGRA fields:

$$
E_{\underline{AM}} = \begin{array}{c|ccccc}
 & D_\beta & P_b & \mathsf{F}^{\beta\beta'} & \Omega^\beta & \Sigma^{bc} \\
\hline
D_\alpha & B_{\alpha\beta} & B_{\alpha b} & C_\alpha{}^{\beta\beta'} & e_\alpha{}^\beta & \omega_\alpha{}^{bc} \\
P_a & & e_{ab},\, B_{ab} & C_{\mathrm{RR}} & \psi,\, \lambda & \omega,\, F_{\mathrm{NS}} \\
\mathsf{F}^{\alpha\alpha'} & & & \phi & \lambda & F_{\mathrm{RR}} \\
\Omega^\alpha & & & & F_{\mathrm{RR}}^{\alpha\beta'},\, F_{\mathrm{NS}}^{\alpha\beta} & \omega \\
\Sigma^{ab} & & & & & r
\end{array}
$$

Orthnormality

- C-bracket with gauge symmetry parameters:

$$\Lambda^{\underline{M}} = (\lambda_{mn}, \epsilon^\mu, \xi^m; \Lambda_{\mathrm{RR}}^{\mu\nu'}; \cdots)$$

III-2. SUGRA field equations

- Curved space covariant derivative algebra

$$[\triangleright_{\underline{A}}(0), \triangleright_{\underline{B}}(\sigma)] = -iT_{\underline{AB}}{}^{\underline{C}}\triangleright_{\underline{C}}\delta(\sigma) - i\eta_{\underline{AB}}\partial_\sigma\delta(\sigma)$$

'13 Polacek & Siegel

- Torsion (includes curvature tensors $T_{PP}{}^{S} = R_{ab}{}^{cd}$)

$$T_{\underline{ABC}} = T_{\underline{AB}}{}^{\underline{D}}\eta_{\underline{DC}} = \frac{1}{2}(D_{[\underline{A}}E_{\underline{B}}{}^{\underline{M}})E_{\underline{C})M} + E_{\underline{A}}{}^{\underline{M}}E_{\underline{B}}{}^{\underline{N}}E_{\underline{C}}{}^{\underline{L}}f_{\underline{MNL}}$$

'93 Siegel

- Bianchi identity

$$D_{[\underline{A}}T_{\underline{BCD})} + \frac{3}{4}T_{[\underline{AB}}{}^{\underline{E}}T_{\underline{CD})E} = 0$$

⇒field eq.

- Torsion constraints from Lorentz & κ- Virasoro

$$T_{S\bigcirc\bigcirc} = f_{S\bigcirc\bigcirc},\ T_{\alpha\beta P} = f_{\alpha\beta P},\ T_{\alpha\beta\bigcirc} = 0 \text{ for } {}_{\bigcirc\neq P} \text{ etc.}$$

↑ Lorentz ↑ κ −symmetry ↑

IV. Conclusions

- A type II superspace with manifest T-duality is proposed:
  - It is spanned by the double nondegenerate super-Poincare affine Lie algebra with RR charge.
  - κ-Virasoro gives simpler torsion constraints.
- A gauge invariant type II superstring action with manifest T-duality is proposed:
  - All terms are written in bilinears of left-inv. currents in double space and the winding mode appears.
  - 2-d. diffeo. invariance even for chiral fields allows κ-symmetry.
  - B field is product of nondegenerate metric & dilatation operator.
- Future problems
  - AdS background, D-branes & exotic branes, S&U duality extension , cosmological application