Report on the Ph.D. thesis entitled "Extended theories of gravity in cosmological and astrophysical applications" by Aneta Wojnar (M.Sc.)

The dark energy and dark matter problems have dominated cosmology since almost two decades. The main issue is that in fact neither of them can fully be explained by general relativity - a canonical theory which serves the explanation of the evolution and structure of the universe. This is why more and more people are studying various alternative gravity theories which can explain (both geometrically and dynamically) the above mentioned problems. The whole list of these approaches ranges from simple modifications of Newtonian gravity like famous MOND (Modified Newtonian Dynamics) up to very advanced mathematically theories such as superstring and brane.

In that context one sees the doctoral thesis of Ms. Aneta Wojnar. She considers what she calls Extended Theories of Gravity (ETG) which contain: the scalar-tensor theories of gravity in the Einstein frame, the Palatini modified gravity (with quadratic Starobinsky term and Chaplygin gas matter source) and the hybrid metric-Palatini gravity in which the Ricci curvature scalar is varied wrt metric while the terms being the function of the Palatini curvature scalar are varied wrt both the metric and the Chrisstoffell connection.

The thesis has been written as a product of a larger collaboration (A. Borowiec, S. Capozziello, L. Karpathopoulos, M. De Laurentis, F. Lobo, A. Paliathanasis, M. Paolella, A. Stachowski, M. Szydłowski, H. Velten) and I believe the statements about the Author's contribution have been presented to the Committee. It is based on three journal papers (Physical Review D, Journal of Cosmology and Astroparticle Physics, European Physical Journal C), one conference paper (Proceedings of Science) and the two e-prints. The papers are relatively new and so perhaps not yet cited intensively - the total number of citations is 17. The Hirsch index for all the scientific score of Ms. Wojnar is h=3. The thesis was written under common supervision of prof. A. Borowiec and prof. S. Capozziello who are well-known specialists in alternative gravity theories.

The thesis is 124 pages long and is composed of the Introduction, three main sections and large Appendices A and B. The bibliography is very rich and contains 197 positions. The thesis has no overall summary - instead, each of the main sections has its own summary (called conclusions or remarks).

In section 2 the Author introduces the Palatini formalism of the $f(\mathcal{R})$ theory (with $\mathcal{R}$ being the Palatini scalar and not the Ricci scalar), derives the field equations (2) and (10) by an independent variation of the action wrt the metric and the connexion. Then, conformally transforms it to the Einstein form (14), and finally shows its equivalence to the non-minimally coupled scalar-tensor theory (20) which is just Brans-Dicke conformally invariant theory with Brans-Dicke parameter $\omega = -3/2$. She omits a controversial discussion of the possible equivalence/non-equivalence of the Jordan (non-minimally coupled scalar) and the Einstein (minimally coupled scalar) frames problem of which one of her supervisors (S. Capozziello) is very keen on.

After deriving the field equations for isotropic cosmology with the scale factor $a(t)$, and the two functions b and c of the Palatini scalar $\mathcal{R}$, a source of matter in the form of the

Chaplygin gas is introduced. The cosmological solutions obtained give a generalisation of the cosmological solutions of Einstein gravity (ΛCDM) onto Palatini theory with Starobinsky term.

The obtained cosmological models are then tested against astronomical data from supernovae, by Baryon Acoustic Oscillations (BAO), cosmic microwave background (CMB) and gravitational lensing as well as by the Hubble function $f(z)$ and the Alcock-Paczyński test. After applying the Bayesian Information Criterion (BIC), it emerges that the standard ΛCDM model is of better evidence that the Palatini models under study. The Author concludes that the reason may be the fact that Palatini models have more parameters than the ΛCDM model. An interesting evolution of the universe are obtained after using the fourth parameter d of the theory defined by (50). Namely, the universe admits a standard big-bang singularity as well as a big-freeze singularity (of finite-time) which characterises of blowing up of the energy density and pressure accompanied by a non-vanishing value of the scale factor. A nice detector of a big-freeze singularity is that it shows up whenever the term $b + d/2$ diverges. The dynamical system studies of these Palatini models show that there exist oscillating models as well as the models which start with a big-bang, reach a big-freeze (which is not necessarily a strong or geodesically incomplete) singularity, finally undergoing the de Sitter phase. The big-freeze has also an interesting property that it drives inflation in this Palatini theory.

Section 3 is devoted to the so-called hybrid extended gravity theory which is composed of the action which contains standard Ricci scalar plus the function of the Palatini scalar. Such an approach generalises eq. (3) in a way that it adds extra the Einstein tensor $G_{\mu\nu}$ producing eq. (78). All other tricks with the conformal transformation as well as with introducing extra scalar field of Brans-Dicke type give path to formulate hybrid gravity cosmology in subsection 3.1.1. Then, the Noether symmetries method is used to generate cosmological solutions both in hybrid gravity and in conformal hybrid gravity. This method roots in quantum cosmology and so there is a brief discussion of the advantages of it. Some quantum cosmological solutions in the Wheeler-deWitt formalism for hybrid gravity are presented in subsection 3.3.1.

In section 4 the scalar-tensor gravity of Brans-DIcke type is studied. Firstly, one uses Noether symmetries method to study some homogeneous solutions of Bianchi type both in the context of classical (Einstein) and quantum cosmological (Wheeler-deWitt) formulation. Then, there is a jump into the considerations related to the equilibrium configurations for the relativistic stars both in extended gravity theory represented by the field equations (250) and to the scalar-tensor gravity theory in the Einstein frame (minimal coupling).

My criticism refers to the following issues:

- The presentation of singularities for isotropic cosmologies in the subsection 2.2.4 is imprecise and incomplete. Type IV is not a big-brake singularity since big-brake is the limit of type II or sudden singularity (I understand that this flaw is due to Ref. [80] which is a follower of [79]) - this is the only case when there is an exact relation between the pressure and the energy density for type II. Namely, as it is seen from (32), the Chaplygin gas example gives $p \to \infty$ when $\rho \to 0$, while for a sudden singularity $p \to \infty$

when $\rho \rightarrow$ const. which is not necessarily zero. Besides, type IV (if we stick to Ref. [79]) has additionally that the barotropic index $w \rightarrow \infty$. Apart from that, there is no mention of the $w-$singularity (cf. Dąbrowski and Denkiewicz PRD 79, 063521 (2009)) as well as the little-rip, the pseudo-rip and related singularities (Frampton, Ludwick and Scherrer, PRD 84, 063003 (2011); Frampton, Ludwick and Scherrer, PRD 85, 083001 (2012)). Most detailed classification was presented in Dąbrowski "Mathematical Structures of the Universe", M. Eckstein, M. Heller, S.J. Szybka (eds.), Copernicus Center Press, Kraków, Poland (2014), p. 99 – arXiv: 1407.4851.

- It is also worth mentioning again that type III singularity is characterised by the divergent pressure and the energy density accompanied by a finite value of the scale factor - this is why there exists a different name for it - finite scale factor singularity (cf. Dąbrowski and Denkiewicz, AIP Conference Proceedings **1241**, 561 (2010); Denkiewicz, JCAP **07**, 036 (2012)). It is a weak singularity according to the definition of Tipler (Phys. Lett. A64, 8 (1977)) and strong singularity according to the definition of Królak (CQG 3, 267 (1986)).
- I would like the Author to explain what is the physical reason that type III singularity appears in the Palatini theory while other types of singularities (type II, IV) do not. In particular, why it drives inflation. What special physical characteristics are provided by the Palatini formulation in that context?
- The last but one sentence on p. 31 in the subsection 2.2.6 which says that "A freeze singularity, as well as three other ones, has a well-defined acceleration $\dot{a}$ while in our case it diverges" must be wrong!? First of all, $\dot{a}$ is velocity and not acceleration, and secondly the acceleration $\ddot{a}$ obviously diverges for type I and II at least.
- The subsection 3.2 contains a mix of classical and quantum hybrid gravity cosmologies and it is not clear if one discusses either and why. The solution of the Wheeler-deWitt equation (176) given by (183) is not developed in the standard context of boundary conditions as it is the case in the "creation from nothing" scenario, for example. The sole solution is not enough study in quantum cosmology. I understand that there is relation to a Noether symmetry method on both classical and quantum levels and this was perhaps the main point here, but one feels some need for more discussion.
- Section 4 is too diverse. It contains elements of scalar-tensor gravity, quantum cosmology and astrophysics (star configurations). Perhaps better would be to split it into the two sections: one about the scalar tensor gravity and its relation to Noether symmetries, and another about the star equilibrium configurations.
- The above note also refers to the section 4.1.1 where quantum cosmology of hybrid gravity is studied - e.g. the solution (229) is only the first step to study the physically interesting points of the universe creation "from nothing" etc.

- There is a lack of the common conclusion section for the paper. There are conclusions/remarks for each section, but not the resume of all the thesis. It makes the impression that the thesis contains three independent topics though glued together as "extended theories of gravity". Section 4 seems to be an extra and not smoothly fitting to the previous ones.

- In relation to the above - the whole thesis does not seem to have expressed an overall idea/main conclusion which could serve a kind of a "breakthrough point" related to the issue of extended gravity theories.

- The appendices contain 28 pages which is almost one third of the main body of the thesis. This overbalances the whole work.

Smaller points are as follows:

- The meaning of the parameter κ in the formula (1) is not explained. From (3) it seems to be the Einstein constant.

- Is R in equation (39) the Ricci or Palatini scalar? If Palatini (as it can be concluded from the following text), it should be written as $\mathcal{R}$. Besides, what is the unit of the Palatini scalar if γ has a unit of mass? I mean $\mathcal{R}$ and $\gamma\mathcal{R}^2$ should have the same units, should not they?

- In the 2nd sentence after (39) it says "the square root term gains an importance ...". Should not be "the quadratic term ..."?

- The sentence after fromula (60). The case $\alpha = 0$ (giving $p =$ const.) seems to be excluded from the definition of Chaplygin gas (37) (there, we have the condition $0 < \alpha \leqslant 1$).

- Little comment to the last sentence of section 2.2.6: some physicists think that it is quite the opposite - the singularities are necessary also in quantum gravity though their avoidance in this theory is different (the wave function should vanish at a point of a classical singularity, for example).

Being aware of the above mentioned drawbacks one should still say that the thesis is well-written, sound, with good explanations of the theory of extended gravities under study. It is based on already published papers which appeared in high impact-factor journals. It also contains the explanation of mathematical tools used in the two large appendices.

In conclusion, despite some weaknesses and inconsistencies, the thesis fulfils requirements which are necessary to present for the doctoral degree so that I recommend the admittance of Ms. Aneta Wojnar for the defence.

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