Bayesian analysis for new class of hybrid star EoS with M-R observations for neutron stars

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May 31, 2016
The idea is to motivate observers to measure radii of massive pulsars:

\[ M[\text{PSR J1614-2230}^{\times \dagger}] = 1.97 \pm 0.04 \, M_\odot \]
\[ M[\text{PSR J0348+0432}^{\star}] = 2.01 \pm 0.04 \, M_\odot \]

\*J. Antoniadis et al. Science 340 (2013), iss. 6131
\dagger Recently E. Fonseca et al. made update of the PSR J1614-2230 mass: \( M = 1.928(\pm 0.017) \, M_\odot \) [arXiv:1603.00545]
What if we have twins

Important questions
- Does hybrid neutron star exist?
- Does NS twin exist?
- Does CEP exist on QCD phase diagram?

Picture is made by Mark Kaltenborn
Existence of CEP at the QCD Phase Diagram

Picture is made by Mark Kaltenborn
Static neutron star mass and radius

The structure and global properties of compact stars are obtained by solving the Tolman-Oppenheimer-Volkoff (TOV) equations\(^1,2\):

\[
\begin{align*}
\frac{dP(r)}{dr} &= -\frac{GM(r)\varepsilon(r)}{r^2} \left( 1 + \frac{P(r)}{\varepsilon(r)} \right) \left( 1 + \frac{4\pi r^3 P(r)}{M(r)} \right), \\
\frac{dM(r)}{dr} &= 4\pi r^2 \varepsilon(r); \\
\frac{dN_B(r)}{dr} &= 4\pi r^2 \left( 1 - \frac{2GM(r)}{r} \right)^{-1/2} n(r).
\end{align*}
\]

\(^1\)R. C. Tolman, Phys. Rev. 55, 364 (1939).
Neutron star mass and radius relation


BA for new hybrid EoS models with M-R data for PSRs
Hybrid equation of state

Maxwell construction of hybrid EoS

\[ p(\mu) = \max [p_H(\mu), p_Q(\mu)] \]
Hybrid equation of state

Alford-Han-Prakash scheme

- Energy Density vs. Pressure
  - Nuclear Matter
  - Quark Matter
  - Slope $= c_{QM}^2$
  - $\Delta \varepsilon = \varepsilon_{QM} - \varepsilon_{crit}$

- Mass vs. Radius
  - $\Delta \varepsilon / \varepsilon_{crit} = 0.3$
  - $\Delta \varepsilon / \varepsilon_{crit} = 1$
  - $\Delta \varepsilon / \varepsilon_{crit} = 0.6$
  - $C_{QM}^2 = 1$
  - $n_{crit} = 4.0 n_0$
  - $\Delta \varepsilon_{stability} / \varepsilon_{crit} = 0.727$
Radius and maximum mass constraints are given from PSR J0437-4715 (Bogdanov. Ast. J. 762, 96) and PSR J0348+0432 (Antoniadis et al. Sci. 340, 6131) correspondingly.
A constraint on the gravitational binding energy is taken from the neutron star B in the binary system J0737-3039 (B).
We supposed that inaccuracy of the fictitious measurements normally distributed.

<table>
<thead>
<tr>
<th>$\sigma$</th>
<th>$R_1$</th>
<th>$R_2$</th>
<th>$R_1$</th>
<th>$R_2$</th>
<th>$R_1$</th>
<th>$R_2$</th>
<th>$R_1$</th>
<th>$R_2$</th>
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<tbody>
<tr>
<td>1.5</td>
<td>11</td>
<td>13</td>
<td>11</td>
<td>15</td>
<td>13</td>
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<td>15</td>
<td>11</td>
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<tr>
<td>1.0</td>
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</table>
Excluded-volume DD2

The excluded volume correction is applied at suprasaturation densities and has the effect of stiffening the EoS. The available volume fraction $\Phi_N$ for the motion of nucleons at given density $n$:

$$\Phi_N = \begin{cases} 1, & \text{if } n \leq n_{\text{sat}} \\ \exp[-\nu|\nu|(n - n_{\text{sat}})^2/2], & \text{if } n > n_{\text{sat}} \end{cases}$$

(1)

with $\nu = 16\pi r_N^3/3$ as the van-der-Waals excluded volume corresponding to a nucleon hard-core radius $r_N$, where $n_{\text{sat}} = 0.16 \text{ fm}^{-3}$ is the density in the interior of atomic nuclei.

We introduce the dimensionless parameter $p = 10 \times \nu[\text{fm}^3]$, taking values between $p = 0$ and $p = 80$ (made by Stefan Typel).
The symmetry energy $E_s(n)$ is defined as the difference in the energy per nucleon between pure neutron matter and symmetric matter in a uniform, infinite system. In RMF models the isovector $\rho$ meson usually represents the only contribution to the isospin dependence of the interaction. Following [?] we use here three parametrizations for the density-dependent $\rho$ meson coupling ("soft", "medium" and "stiff").

<table>
<thead>
<tr>
<th>$E_s(n)$</th>
<th>parametrization</th>
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<tbody>
<tr>
<td>stiff</td>
<td>DD2+ DD2F+</td>
</tr>
<tr>
<td>medium</td>
<td>DD2 DD2F</td>
</tr>
<tr>
<td>soft</td>
<td>DD2− DD2F−</td>
</tr>
</tbody>
</table>
Quark matter EoS

Quark matter EoS with \(\eta_4\) parameter

The quark matter was modelling by a two flavor Nambu-Jona-Lasinio (NJL) model with 8-quark interactions in the scalar and the vector channel by S. Benic [arXiv:1503.09145].

The \(\eta_2\) – 4-quark vector couplings parameter was fixed (to describe hybrid stars with masses larger than \(2M_{\text{sun}}\) [arXiv:1401.5380]) and the \(\eta_4\) – 8-quark vector NJL couplings parameter was varied from 0 to 30 with step 1 (so, \(N_2 = 31\)).

Picture is made by Mark Kaltenborn
Variations of the hybrid EoS for the DD2F$^-$ model. *Upper row.* The hadronic EoS is kept fixed while the quark EoS is allowed to vary for the parameters $\eta_4 = 0, 1, 2..30$. 
Vector of Parameters

For the BA, we have to sample the above defined parameter space and to that end we introduce a vector of the parameter values $\vec{\pi} = \{p, \eta_4\}$:

$$\vec{\pi}_i = \{p(k), \eta_4(l)\},$$

$i = 1 \ldots N$ (here $N = N_1 \times N_2$), $i = N_2 \times k + l$ and $k = 0 \ldots N_1 - 1$, $l = 0 \ldots N_2 - 1$
Formulation of set of models (set of hypothesis):
\[ \pi_i \text{ here } i = 1 \ldots N \]

Finding the \textit{a priori} probabilities of the models:
\[ P(\pi_i) = \frac{1}{N} \text{ for } \forall i = 0..N - 1 \]

Calculating the conditional probabilities of the events:
\[ P(E | \pi_i) = \prod_{\alpha} P(E_{\alpha} | \pi_i) \]

Calculating the \textit{a posteriori} probabilities of the models:
\[ P(\pi_i | E) = \frac{P(E | \pi_i) P(\pi_i)}{\sum_{j=0}^{N-1} P(E | \pi_j) P(\pi_j)} \]
Calculation of Probabilities

**Probability of Corresponding to Mass Constraint for** $\pi_i$

$$P (E_A | \pi_i ) = \Phi (M_i, \mu_A, \sigma_A),$$

here $M_i$ is max mass given by $\pi_i$.

$$\mu_A = 2.01 \ M_\odot$$ and $$\sigma_A = 0.04 \ M_\odot$$ [Antoniadis et al. Sci. 340].
Probability of Corresponding to Radius Constraint for $\pi_i$

$$P(E_B | \pi_i) = \Phi(R_i, \mu_B, \sigma_B),$$
here $R_i$ is max radius given by $\pi_i$. $\mu_B = 15.5$ km and $\sigma_B = 1.5$ km [Bogdanov/Hambaryan et al.].
Using the model dependence between the masses described by $M_B = M_B(M_G; \vec{\pi}_i)$ the probability to fulfill the constraint can be calculated from the normal distribution function for that $M_B$ values, which are in the area corresponding to the observed gravitational mass range $M_G \pm \Delta M$:

$$P(E_P | \vec{\pi}_i) = \Phi(M_B(M_G + \Delta M; \vec{\pi}_i), \mu_B, \sigma_M) - \Phi(M_B(M_G - \Delta M; \vec{\pi}_i), \mu_B, \sigma_M). \quad (2)$$

In the plots of results we show the $1\sigma$-range of $M_B$ values as a box on the $M_G \otimes M_B$ plane.
To find out the best suggestion for future observations, which will be powerful for the model discrimination we employ fictitious radius measurement constraints. For our “experiment” we choose the two known objects with well measured high masses: PSR J0348 + 0432 and PSR J1614 – 2230. We assume that the possible radii of these objects will be different and in the resolution range $\Delta R$. The masses of the objects are measured $M_{GA} = 2.01 \, M_\odot$, with error-bar $\Delta M_{GA} = \pm 0.04 \, M_\odot$ and $M_{GD} = 1.94 \, M_\odot$, with error-bar $\Delta M_{GD} = \pm 0.04 \, M_\odot$ correspondingly for the PSR J0348+0432 and PSR J1614-2230 pulsars. The fictitious radii we assumed to be $R_A$ and $R_D$ with statistical uncertainty $\sigma_{R_A}$ and $\sigma_{R_D}$ correspondingly.
Probability of fictitious radius constraints

The probability to fulfil the constraint will correspond to the area where the configurations have values of radii predicted from the model when the masses are in the interval of the observational mass range $M_{Gj} \pm \Delta M_j$:

\[
P(E_{F\alpha} | \vec{\pi}_i) = \Phi \left( R_\alpha (M_{G\alpha} + \Delta M_\alpha; \vec{\pi}_i), \mu_{R\alpha}, \sigma_{R\alpha} \right) - \Phi \left( R_\alpha (M_{Gj} - \Delta M_\alpha; \vec{\pi}_i), \mu_{R\alpha}, \sigma_{R\alpha} \right)
\]

Because in some cases (mainly for the hybrid stars) the $R_\alpha (M_{G\alpha}; \vec{\pi}_i)$ can be not uniquely defined functions, we have excluded possible overlaps of the boxes of the radius probability regions on the $M_{G\alpha} \otimes R_\alpha$ plane to avoid the double counting.
Calculation of Probabilities

Probability of All Constraints for $\pi_i$

Taking to the account assumption that these measurements are independent on each other we can calculate complete conditional probability:

$$P(E | \pi_i) = \prod_{\alpha} P(E_{\alpha} | \pi_i).$$  (4)

Calculation of a posteriori Probabilities of $\pi_i$

$$P(\pi_i | E) = \frac{P(E | \pi_i) P(\pi_i)}{\sum_{j=0}^{N-1} P(E | \pi_j) P(\pi_j)}. $$  (5)

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### Symmetry energy $\rightarrow$ EoS ↓

<table>
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<tr>
<th></th>
<th>soft</th>
<th>medium</th>
<th>stiff</th>
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<tbody>
<tr>
<td>DD2F (semi-soft)</td>
<td><img src="image" alt="Graph for DD2F (semi-soft)" /></td>
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<tr>
<td>DD2 (stiff)</td>
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BA results with $M-R$ data
BA results with adding $M_G-M_B$ constraint
BA results with fictitious measurements
### Observational constraints

#### Parameterization of HEoS

#### Bayesian Analysis

#### Results

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<th>medium</th>
<th>stiff</th>
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<tr>
<td>2</td>
<td>3</td>
<td>4</td>
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#### BA results with $M_R$ data

#### BA results with adding $M_G-M_B$ constraint

#### BA results with fictitious measurements

#### Conclusions

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**BA for new hybrid EoS models with M-R data for PSRs**
### Observational constraints

Parameterization of HEoS

Bayesian Analysis

Results

Conclusions

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<tr>
<th>Radius error $\sigma \rightarrow$</th>
<th>$1.5 \text{ km}$</th>
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<tr>
<td>$R_A, R_D$</td>
<td><img src="image1.png" alt="Graph" /></td>
<td><img src="image2.png" alt="Graph" /></td>
<td><img src="image3.png" alt="Graph" /></td>
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<tr>
<td>$11 \text{ km}, 13 \text{ km}$</td>
<td><img src="image4.png" alt="Graph" /></td>
<td><img src="image5.png" alt="Graph" /></td>
<td><img src="image6.png" alt="Graph" /></td>
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<tr>
<td>$11 \text{ km}, 15 \text{ km}$</td>
<td><img src="image7.png" alt="Graph" /></td>
<td><img src="image8.png" alt="Graph" /></td>
<td><img src="image9.png" alt="Graph" /></td>
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BA results with $M-R$ data

BA results with adding $M_G-M_B$ constraint

BA results with fictitious measurements

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BA for new hybrid EoS models with M-R data for PSRs
BA results with $M-R$ data
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13 km, 13 km

15 km, 11 km
The region of the most probable models in the two-dimensional parameter space is sufficiently narrow, covering the ranges $40 < \rho < 80$ and $3 < \eta_4 < 7$.

When fictitious radius measurements yield the smaller radius for the object with the slightly smaller mass, then the most probable value of the excluded volume parameter is lowered from $\rho = 80$ to the moderate $\rho \sim 50$, while the optimal stiffness of the quark matter remains unchanged, $\eta_4 \sim 5$.

The developed BA tool has quote strong selective power even if the $5\sigma$ regions of the fictitious radius errors have deep overlapping.
Thanks for your attention!

- D. Blaschke et al. J. Phys. CS 496 (2014), 012002