Bayesian analysis for new class of hybrid star EoS with M-R observations for neutron stars

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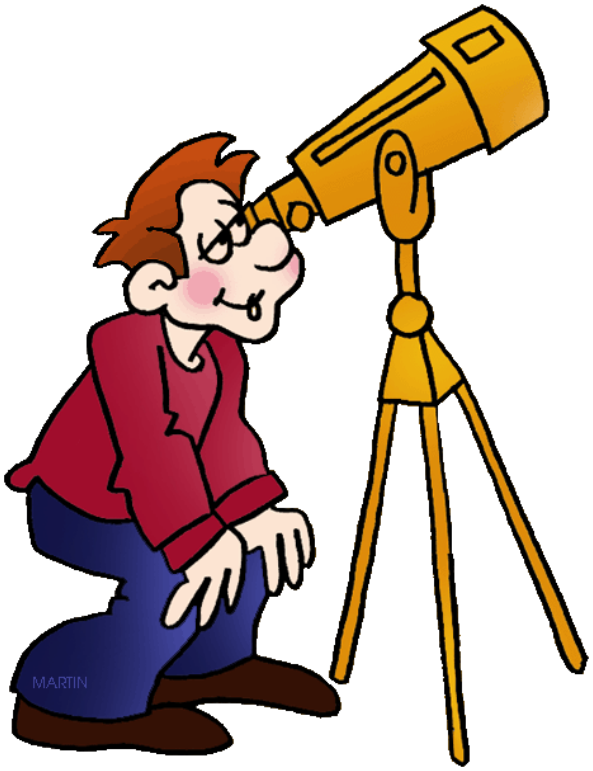
The idea is to motivate observers



to mesure radii of massive pulsars:

M[PSR J1614-2230*†] $= 1.97 \pm 0.04\ \mathrm{M}_\odot$

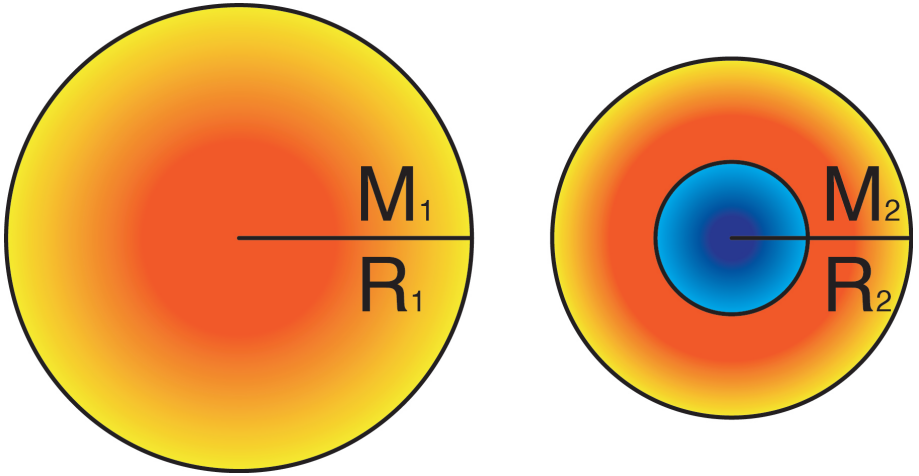
M[PSR J0348+0432*] $= 2.01 \pm 0.04\ \mathrm{M}_\odot$

*J. Antoniadis et al. Science 340 (2013), iss. 6131

*P. Demorest et al. Nature 467 (2010), pp. 1081–1083

†Recently E. Fonseca et al. made update of the PSR J1614-2230 mass: $M = 1.928(\pm 0.017)\ \mathrm{M}_\odot$ [arXiv:1603.00545]

What if we have twins

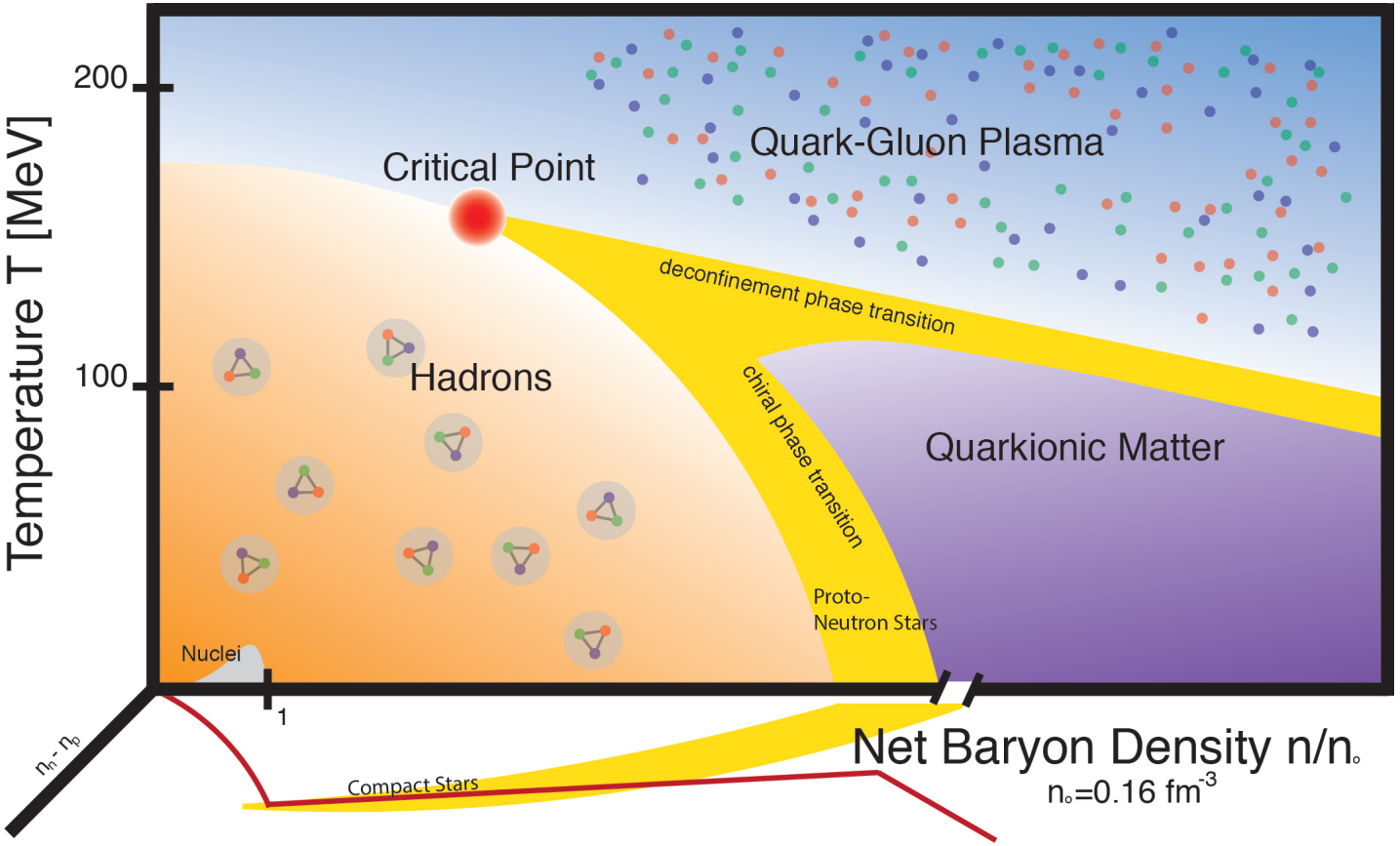


Important questions

- Does hybrid neutron star exist?
- Does NS twin exist?
- Does CEP exist on QCD phase diagram?

Picture is made by Mark Kaltenborn

Existence of CEP at the QCD Phase Diagram



Picture is made by Mark Kaltenborn

Static neutron star mass and radius

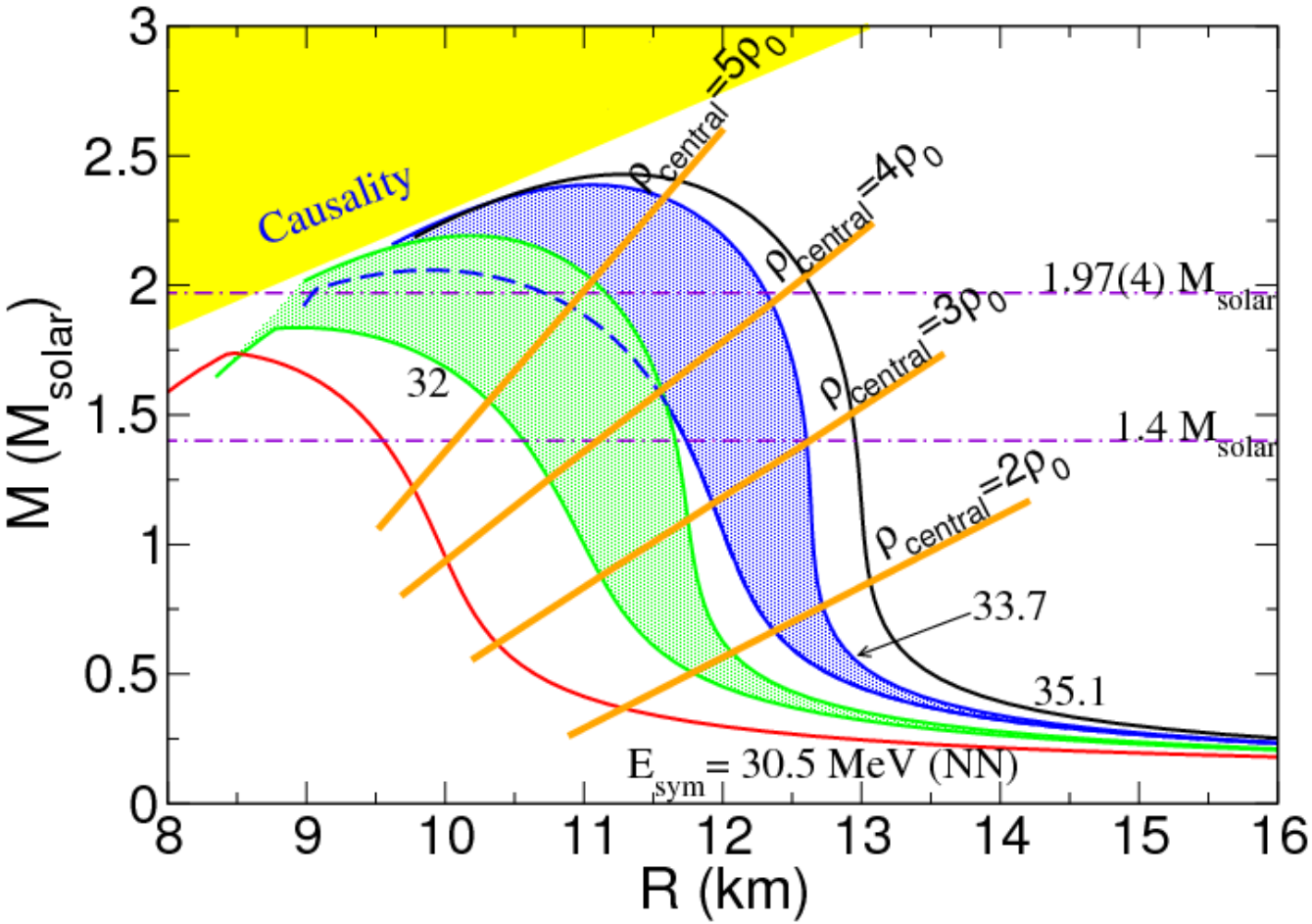
The structure and global properties of compact stars are obtained by solving the Tolman-Oppenheimer-Volkoff (TOV) equations[1,2]:

$$\begin{cases} \dfrac{dP(r)}{dr} = -\dfrac{GM(r)\varepsilon(r)}{r^2}\dfrac{\left(1+\dfrac{P(r)}{\varepsilon(r)}\right)\left(1+\dfrac{4\pi r^3 P(r)}{M(r)}\right)}{\left(1-\dfrac{2GM(r)}{r}\right)}; \\ \dfrac{dM(r)}{dr} = 4\pi r^2 \varepsilon(r); \\ \dfrac{dN_B(r)}{dr} = 4\pi r^2 \left(1-\dfrac{2GM(r)}{r}\right)^{-1/2} n(r). \end{cases}$$

[1] R. C. Tolman, Phys. Rev. **55**, 364 (1939).

[2] J. R. Oppenheimer and G. M. Volkoff, Phys. Rev. **55**, 374 (1939).

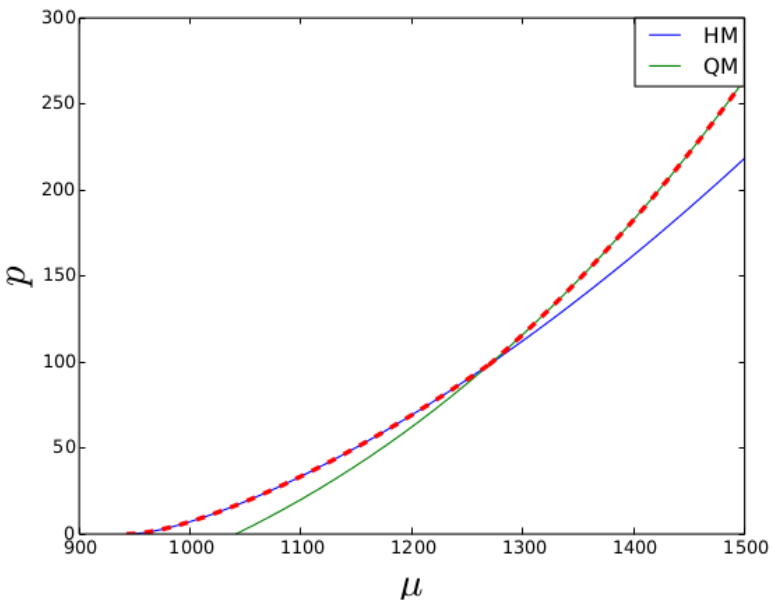
Neutron star mass and radius relation



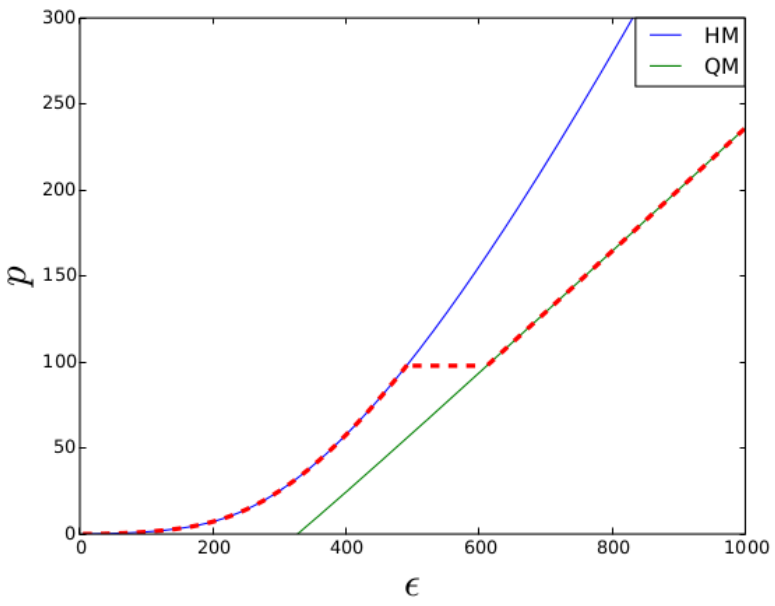
S. Gandolfi, J. Carlson, S. Reddy, Phys.Rev. C85 (2012) 032801

Hybrid equation of state

Maxwell construction of hybrid EoS



(A) $p(\mu)$ functions.

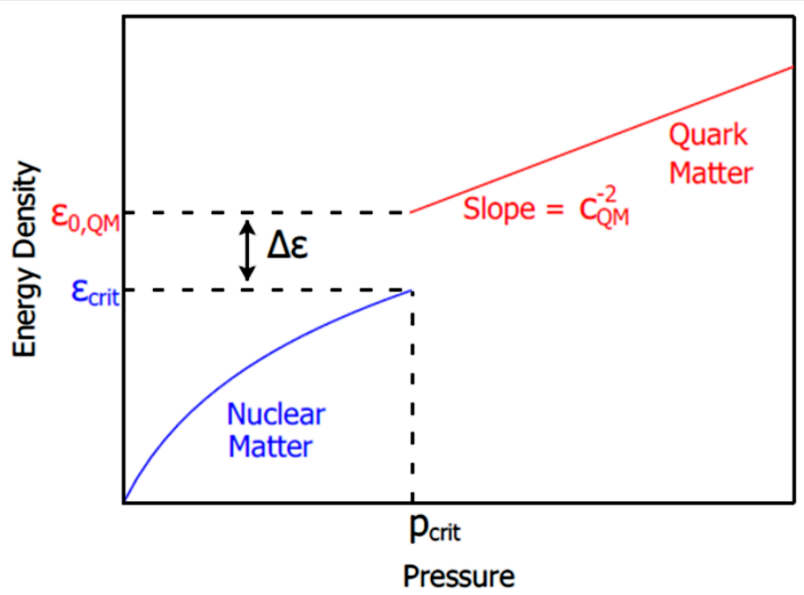


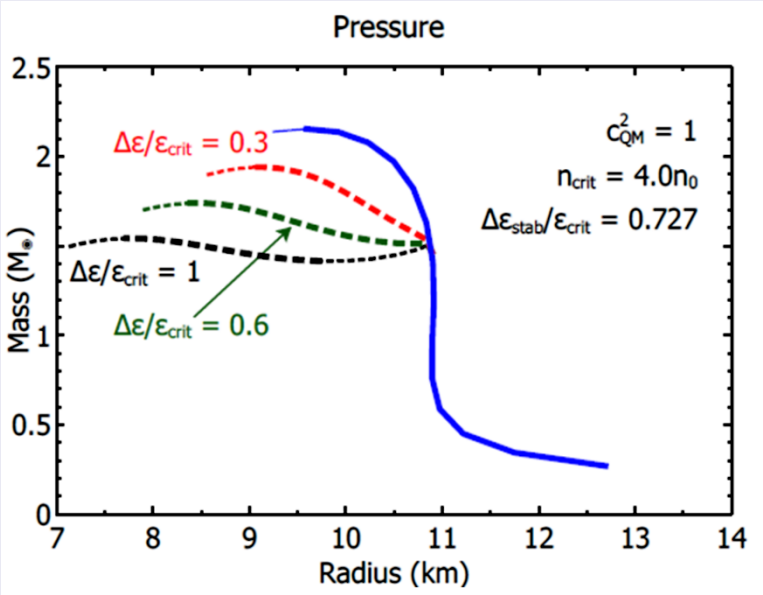
(B) $p(\epsilon)$ functions.

$$p(\mu) = \max\left[p_H(\mu), p_Q(\mu)\right]$$

Hybrid equation of state

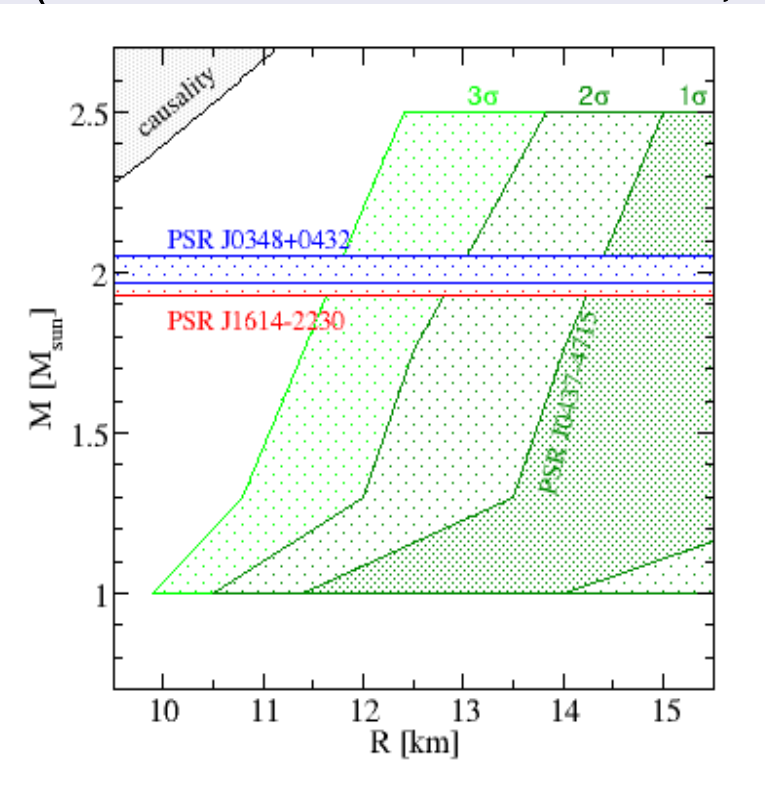
Alford-Han-Prakash scheme





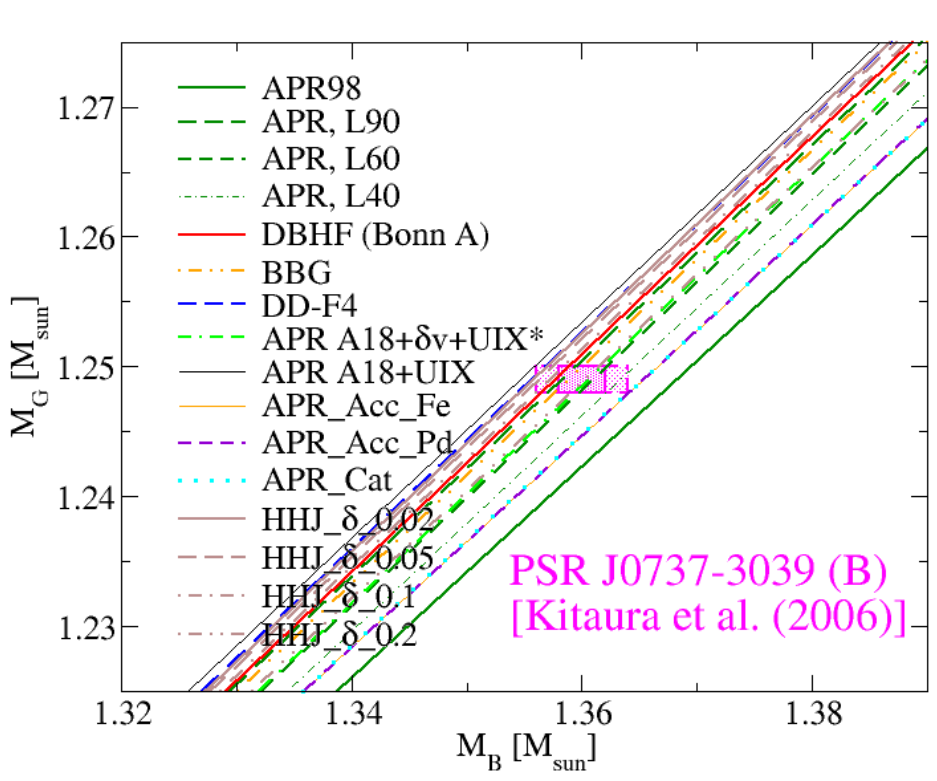
Mass and Radius Constraints

Radius and maximum mass constraints are given from PSR J0437-4715 (Bogdanov. Ast. J. **762**, 96) and PSR J0348+0432 (Antoniadis *et al.* Sci. **340**, 6131) correspondingly.

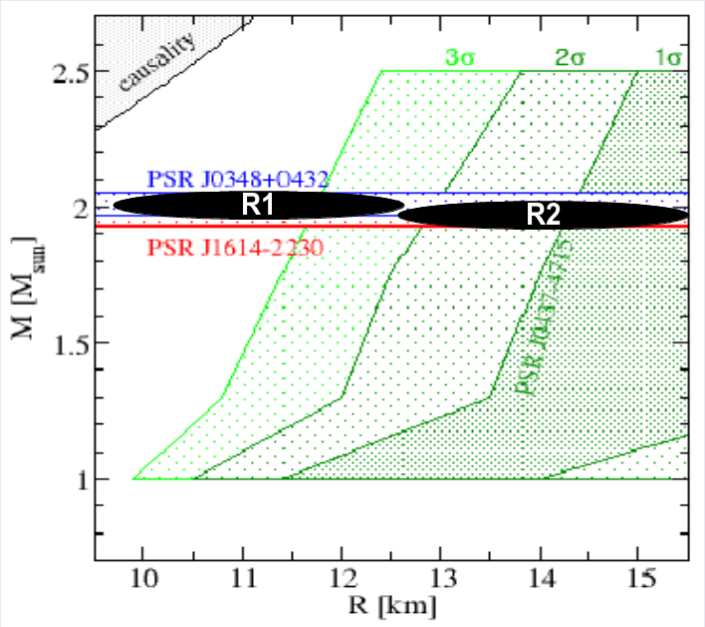


Gravitational Binding Energy Constraint

A constraint on the gravitational binding energy is taken from the neutron star B in the binary system J0737-3039 (B).



Fictitious Radius Constraints



σ	R_1	R_2	R_1	R_2	R_1	R_2	R_1	R_2
1.5	11	13	11	15	13	15	15	11
1.0	11	13	11	15	13	15	15	11
0.5	11	13	11	15	13	15	15	11

We supposed that inaccuracy of the fictitious measurements normally distributed.

Hadronic matter EoS

Excluded-volume DD2

The excluded volume correction is applied at suprasaturation densities and has the effect of stiffening the EoS. The available volume fraction Φ_N for the motion of nucleons at given density n:

$$\Phi_N = \begin{cases} 1\ , & \text{if} \quad n \leq n_{\text{sat}} \\ \exp[-v|v|(n - n_{\text{sat}})^2/2]\ , & \text{if} \quad n > n_{\text{sat}}\ , \end{cases} \tag{1}$$

with $v = 16\pi r_N^3/3$ as the van-der-Waals excluded volume corresponding to a nucleon hard-core radius r_N, where $n_{\text{sat}} = 0.16\ \text{fm}^{-3}$ is the density in the interior of atomic nuclei. We introduce the dimensionless parameter $p = 10 \times v[\text{fm}^3]$, taking values between $p = 0$ and $p = 80$ (made **by Stefan Typel**).

Hadronic matter EoS

Symmetry energy for hadronic EoS

The symmetry energy $E_s(n)$ is defined as the difference in the energy per nucleon between pure neutron matter and symmetric matter in a uniform, infinite system. In RMF models the isovector ρ meson usually represents the only contribution to the isospin dependence of the interaction. Following [**?**] we use here three parametrizations for the density-dependent ρ meson coupling ("soft", "medium" and "stiff").

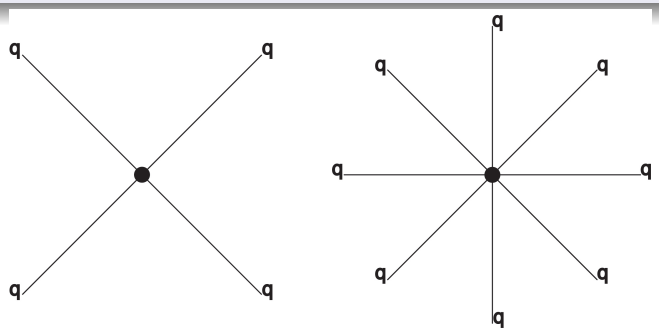
$E_s(n)$	parametrization	
stiff	DD2+	DD2F+
medium	DD2	DD2F
soft	DD2−	DD2F−

Quark matter EoS

Quark matter EoS with η_4 parameter

The quark matter was modelling by a two flavor Nambu-Jona- Lasinio (NJL) model with 8-quark interactions in the scalar and the vector channel **by S. Benic** [arXiv:1503.09145].

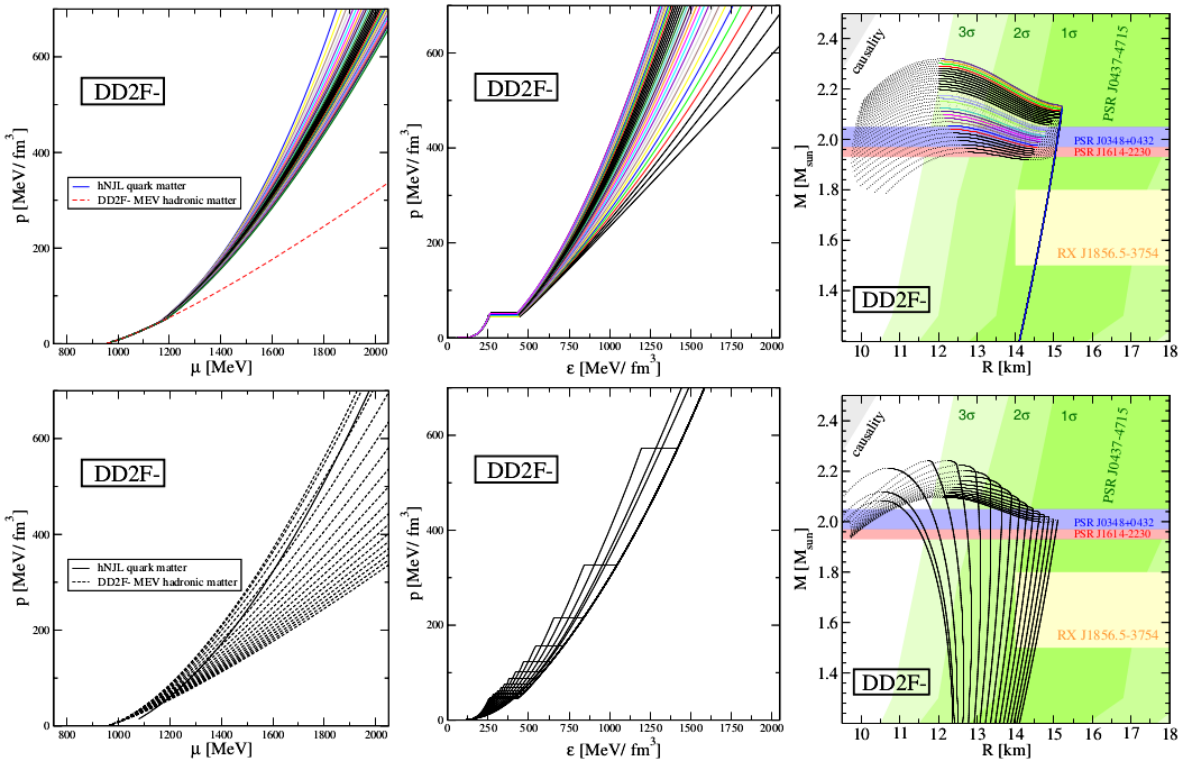
The η_2 – 4-quark vector couplings parameter was fixed (to describe hybrid stars with masses larger than $2M_{sun}$ [arXiv:1401.5380]) and the η_4 – 8-quark vector NJL couplings parameter was variated from 0 to 30 with step 1 (so, $N_2 = 31$).



Dilute System Dense System

Picture is made by Mark Kaltenborn

Hybrid matter EoS



Variations of the hybrid EoS for the DD2F$^-$ model. *Upper row*. The hadronic EoS is kept fixed while the quark EoS is allowed to vary for the parameters $\eta_4 = 0, 1, 2..30$.

Vector of Parameters

Vector of Parameters

For the BA, we have to sample the above defined parameter space and to that end we introduce a vector of the parameter values $\vec{\pi} = \{p, \eta_4\}$:

$$\vec{\pi}_i = \left\{p_{(k)}, \eta_{4(l)}\right\},$$

$i = 1 \ldots N$ (here $N = N_1 \times N_2$), $i = N_2 \times k + l$ and $k = 0 \ldots N_1 - 1$, $l = 0 \ldots N_2 - 1$

BA scheme

Formulation of set of models (set of hypothesis):
π_i here $i = 1 \ldots N$

$\downarrow$

Finding the *a priori* probabilities of the models:
$P(\pi_i) = 1/N \quad \text{for} \quad \forall\, i = 0..N-1$

$\downarrow$

Calculating the coditional probabilities of the events:
$P(E\,|\vec{\pi}_i) = \prod_\alpha P(E_\alpha\,|\vec{\pi}_i)$

$\downarrow$

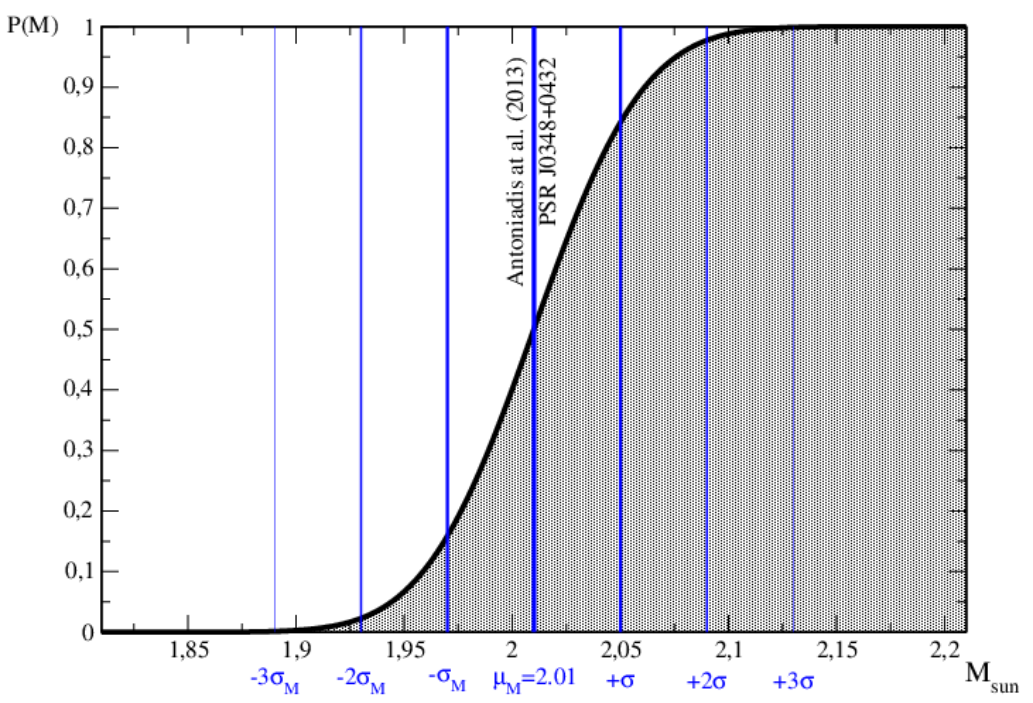
Calculating the *a posteriori* probabilities of the models:

$$P(\vec{\pi}_i\,|E) = \frac{P(E|\vec{\pi}_i)P(\vec{\pi}_i)}{\sum_{j=0}^{N-1} P(E|\vec{\pi}_j)P(\vec{\pi}_j)}$$

Calculation of Probabilities

Probability of Corresponding to Mass Constraint for π_i

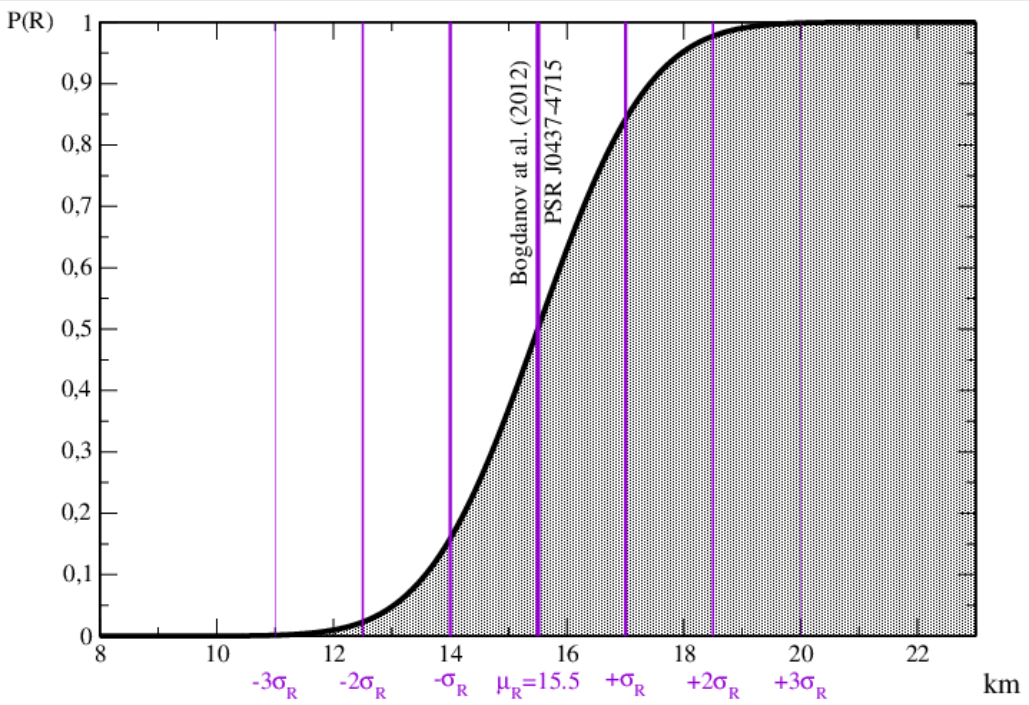
$P(E_A|\pi_i) = \Phi(M_i, \mu_A, \sigma_A)$, here M_i is max mass given by π_i. $\mu_A = 2.01\ \mathrm{M}_\odot$ and $\sigma_A = 0.04\ \mathrm{M}_\odot$ [Antoniadis et al. Sci. 340].



Calculation of Probabilities

Probability of Corresponding to Radius Constraint for π_i

$P(E_B|\pi_i) = \Phi(R_i, \mu_B, \sigma_B)$, here R_i is max radius given by π_i. $\mu_B = 15.5$ km and $\sigma_B = 1.5$ km [Bogdanov/Hambaryan et al.].



Calculation of Probabilities

Probability of M_G-M_B constraint

Using the model dependance between the masses described by $M_B = M_B(M_G; \vec{\pi}_i)$ the probability to fulfill the constraint can be calculated from the normal distribution function for that M_B values, which are in the area corresponding to the observed gravitational mass range $M_G \pm \Delta M$:

$$\begin{aligned} P(E_P|\vec{\pi}_i) &= \Phi(M_B(M_G + \Delta M; \vec{\pi}_i), \mu_B, \sigma_M) \\ &- \Phi(M_B(M_G - \Delta M; \vec{\pi}_i), \mu_B, \sigma_M). \end{aligned} \quad (2)$$

In the plots of results we show the 1σ-range of M_B values as a box on the $M_G \otimes M_B$ plane.

Calculation of Probabilities

Probability of fictitious radius constraints

To find out the best suggestion for future observations, which will be powerful for the model discrimination we employ fictitious radius measurement constraints. For our "experiment" we choose the two known objects with well measured high masses: PSR J0348 + 0432 and PSR J1614 − 2230. We assume that the possible radii of these objects will be different and in the resolution range ΔR. The masses of the objects are measured $M_{G_A} = 2.01\ M_\odot$, with error-bar $\Delta M_{G_A} = \pm 0.04\ M_\odot$ and $M_{G_D} = 1.94\ M_\odot$, with error-bar $\Delta M_{G_D} = \pm 0.04\ M_\odot$ correspondingly for the PSR J0348+0432 and PSR J1614-2230 pulsars. The fictitious radii we assumed to be R_A and R_D with statistical uncertainty σ_{R_A} and σ_{R_D} correspondingly.

Calculation of Probabilities

Probability of fictitious radius constraints

The probability to fulfil the constraint will correspond to the area where the configurations have values of radii predicted from the model when the masses are in the interval of the observational mass range $M_{G_j} \pm \Delta M_j$:

$$\begin{aligned} P\left(E_{F_\alpha} \,|\, \vec{\pi}_i\right) &= \Phi\left(R_\alpha(M_{G_\alpha} + \Delta M_\alpha; \vec{\pi}_i), \mu_{R_\alpha}, \sigma_{R_\alpha}\right) \\ &- \Phi\left(R_\alpha(M_{G_j} - \Delta M_\alpha; \vec{\pi}_i), \mu_{R_\alpha}, \sigma_{R_\alpha}\right) \end{aligned} \quad (3)$$

Because in some cases (mainly for the hybrid stars) the $R_\alpha(M_{G_\alpha}; \vec{\pi}_i)$ can be not uniquely defined functions, we have excluded possible overlaps of the boxes of the radius probability regions on the $M_{G_\alpha} \otimes R_\alpha$ plane to avoid the double counting.

Calculation of Probabilities

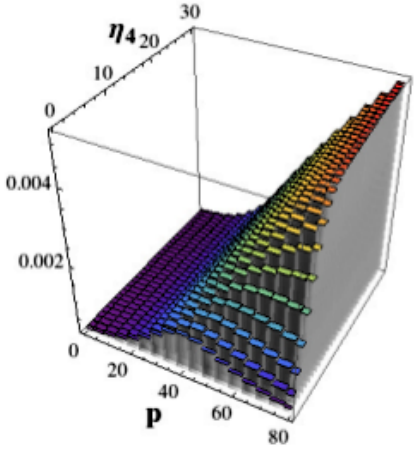
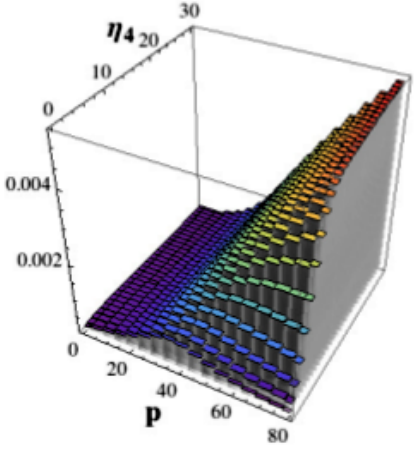
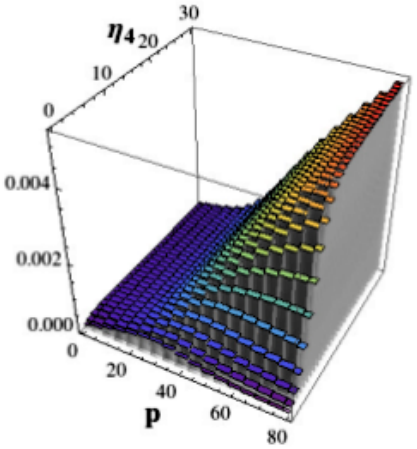
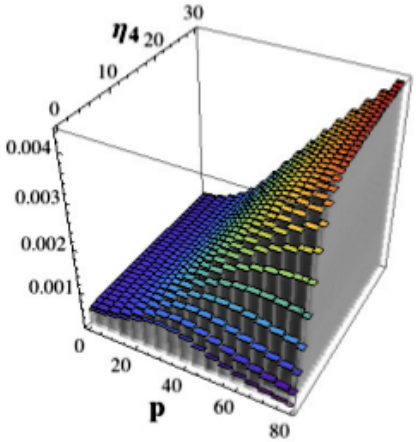
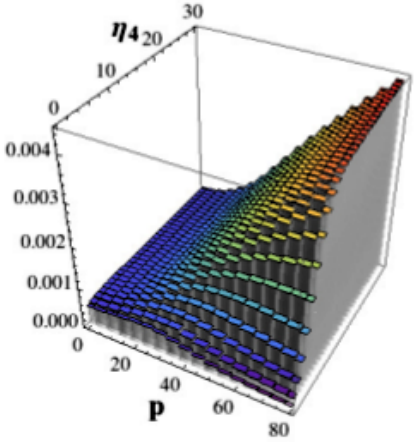
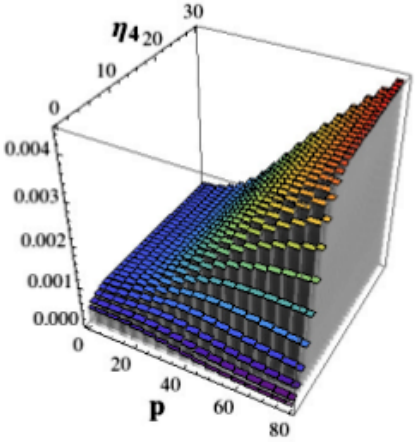
Probability of All Constraints for π_i

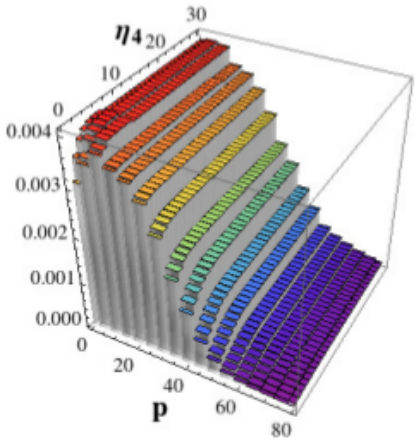
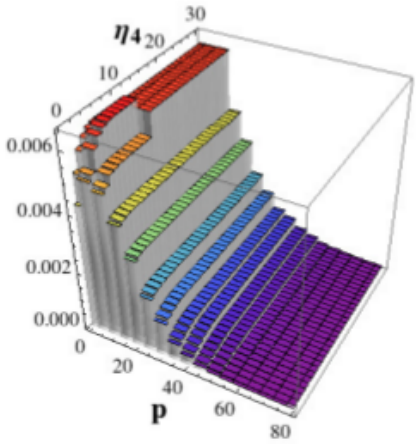
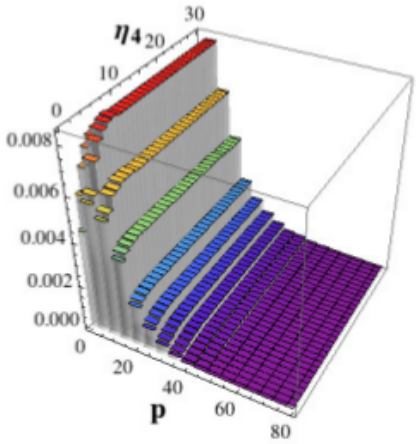
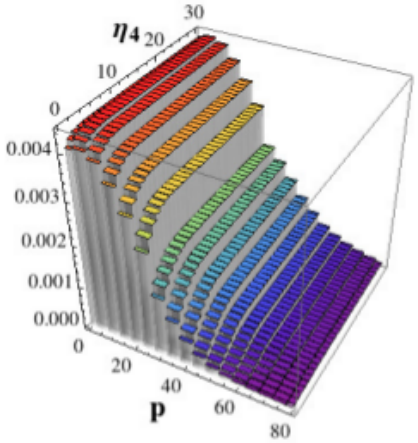
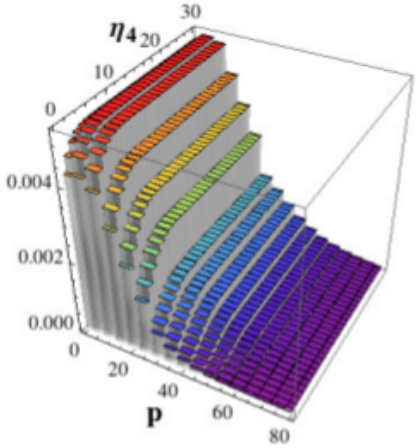
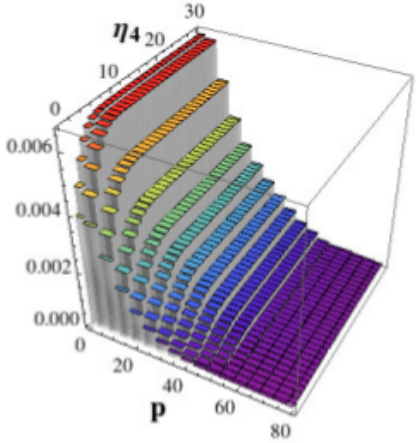
Taking to the account assumption that these measurements are independent on each other we can calculate complete conditional probability:

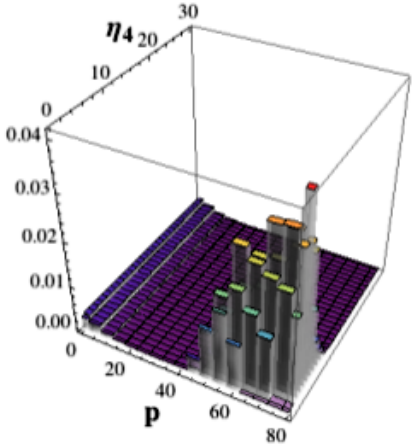
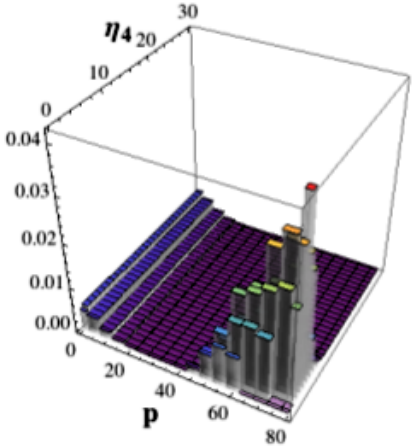
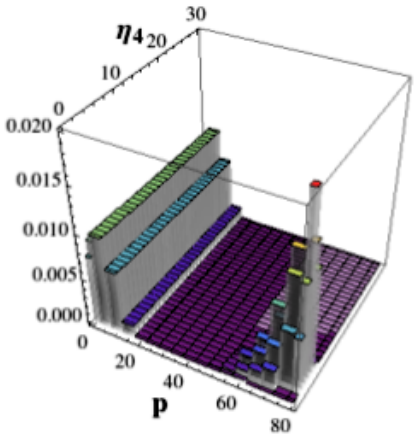
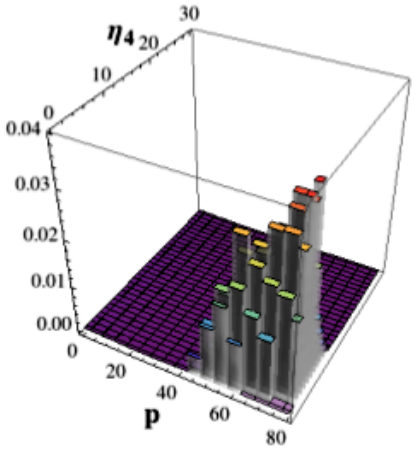
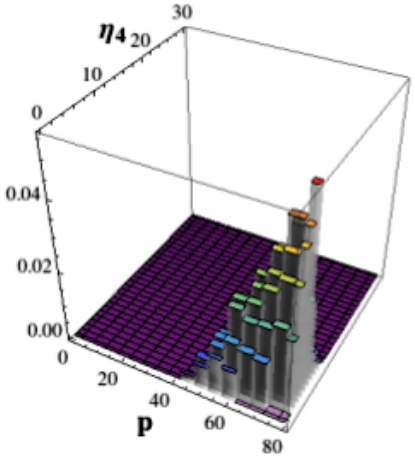
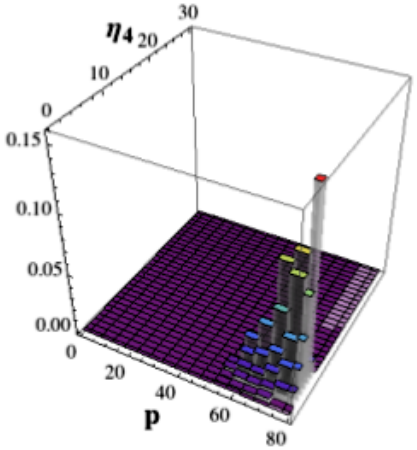
$$P(E|\vec{\pi}_i) = \prod_{\alpha} P(E_\alpha|\vec{\pi}_i)\,. \tag{4}$$

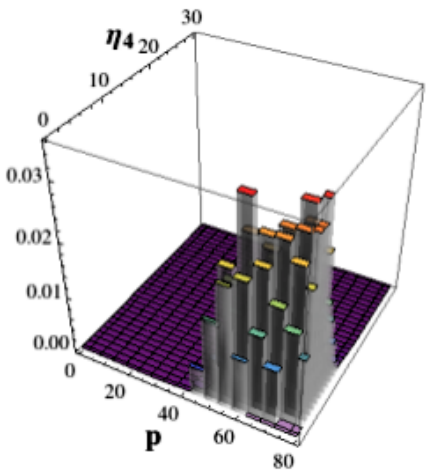
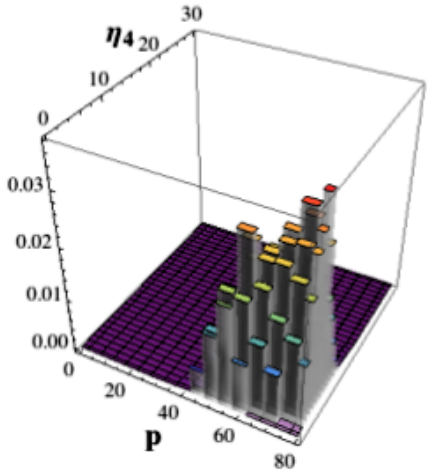
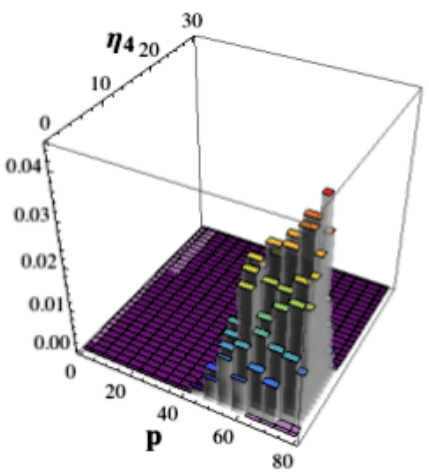
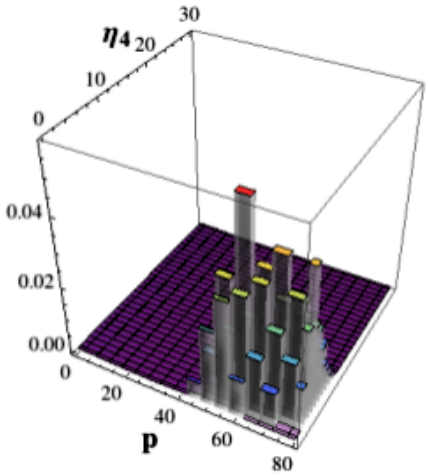
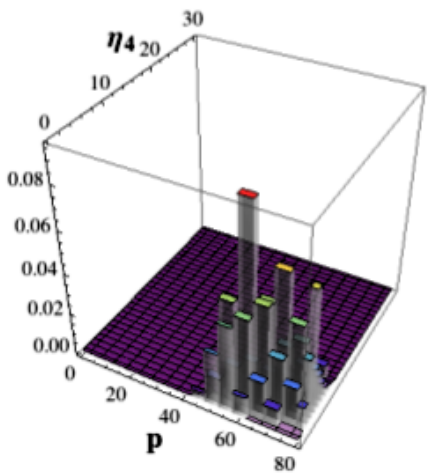
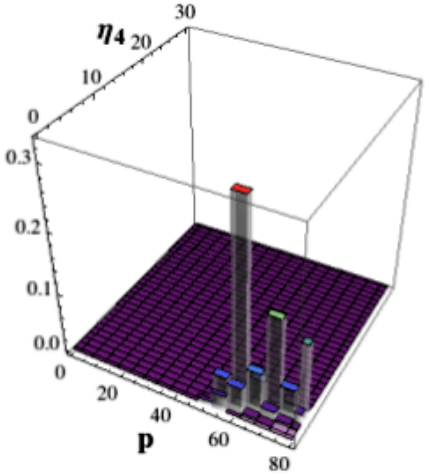
Calculation of *a posteriori* Probabilities of π_i

$$P(\vec{\pi}_i|E) = \frac{P(E|\vec{\pi}_i)\,P(\vec{\pi}_i)}{\sum_{j=0}^{N-1} P(E|\vec{\pi}_j)\,P(\vec{\pi}_j)}\,. \tag{5}$$

Symmetry energy → EoS ↓	soft	medium	stiff
DD2F (semi-soft)			
DD2 (stiff)			

Symmetry energy → EoS ↓	soft	medium	stiff
DD2F (semi-soft)			
DD2 (stiff)			

Radius error $\sigma \rightarrow$ R_A, R_D	1.5 km	1.0 km	0.5 km
11 km, 13 km			
11 km, 15 km			

13 km, 13 km			
15 km, 11 km			

Conclusions

- The region of the most probable models in the two-dimensional parameter space is sufficiently narrow, covering the ranges $40 < p < 80$ and $3 < \eta_4 < 7$.
- When fictitious radius measurements yield the smaller radius for the object with the slightly smaller mass, then the most probable value of the excluded volume parameter is lowered from $p = 80$ to the moderate $p \sim 50$, while the optimal stiffness of the quark matter remains unchanged, $\eta_4 \sim 5$.
- The developed BA tool has quote strong selective power even if the 5σ regions of the fictitious radius errors have deep overlapping.

Thanks for your attention!

- D. E. Alvarez-Castillo et al. Eur. Phys. J. A (2016) **52**:69
- A. Ayriyan et al. J. Phys. CS 668 (2016), 012038
- A. Ayriyan et al. Phys. Part. Nucl. 46(5), 2015, 854-857
- D. Blaschke et al. J. Phys. CS 496 (2014), 012002