Outline
Low mode truncation
Left-right mixing. The chiralspin $SU(2)_{CS}$ and $SU(4)$ groups.
Zero modes and symmetries of Euclidean QCD
$SU(4)$ symmetry of $N_F = 2$ QCD at high temperature

# $SU(2N_F)$ symmetry of QCD at high temperature and its implications

L. Ya. Glozman

Institut für Physik, FB Theoretische Physik, Universität Graz

27th May 2016

Outline
Low mode truncation
Left-right mixing. The chiralspin $SU(2)_{CS}$ and $SU(4)$ groups.
Zero modes and symmetries of Euclidean QCD
$SU(4)$ symmetry of $N_F = 2$ QCD at high temperature

1. Low mode truncation

2. Left-right mixing. The chiralspin $SU(2)_{CS}$ and $SU(4)$ groups.

3. Zero modes and symmetries of Euclidean QCD

4. $SU(4)$ symmetry of $N_F = 2$ QCD at high temperature

Outline
**Low mode truncation**
Left-right mixing. The chiralspin $SU(2)_{CS}$ and $SU(4)$ groups.
Zero modes and symmetries of Euclidean QCD
$SU(4)$ symmetry of $N_F = 2$ QCD at high temperature

# Low mode truncation

Banks-Casher:

$$< \bar{q}q >= -\pi\rho(0).$$

What we do:

$$S = S_{Full} - \sum_{i=1}^{k} \frac{1}{\lambda_i} |\lambda_i\rangle\langle\lambda_i|.$$

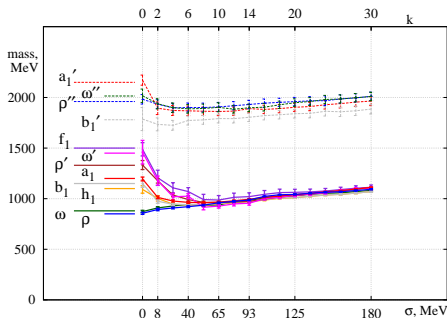What one expects for $J = 1$ mesons:

Outline
**Low mode truncation**
Left-right mixing. The chiralspin $SU(2)_{CS}$ and $SU(4)$ groups.
Zero modes and symmetries of Euclidean QCD
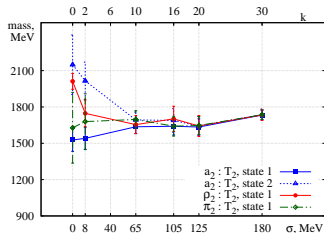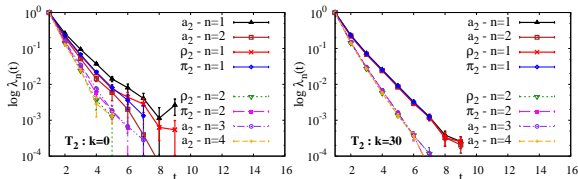$SU(4)$ symmetry of $N_F = 2$ QCD at high temperature
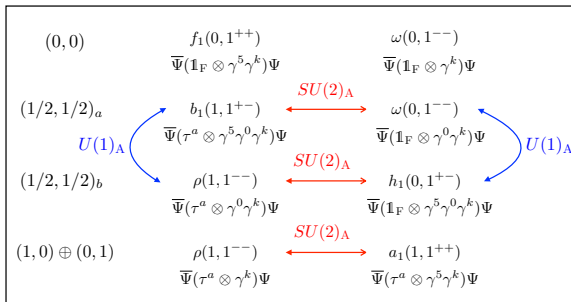
M.Denissenya, L.Ya.G., C.B.Lang, PRD 89(2014)077502; 91(2015)034505

$$J = 1$$

JLQCD overlap gauge configurations.

Outline
**Low mode truncation**
Left-right mixing. The chiralspin $SU(2)_{CS}$ and $SU(4)$ groups.
Zero modes and symmetries of Euclidean QCD
$SU(4)$ symmetry of $N_F = 2$ QCD at high temperature

# J=2 mesons: M. Denissenya, L.Ya.G, M.Pak, PRD 91(2015)114512

Outline
**Low mode truncation**
Left-right mixing. The chiralspin $SU(2)_{CS}$ and $SU(4)$ groups.
Zero modes and symmetries of Euclidean QCD
$SU(4)$ symmetry of $N_F = 2$ QCD at high temperature

# $J = 1/2$ baryons: M. Denissenya, L.Ya.G, M.Pak, PRD 92 (2015) 074508

Outline
Low mode truncation
Left-right mixing. The chiralspin $SU(2)_{CS}$ and $SU(4)$ groups.
Zero modes and symmetries of Euclidean QCD
$SU(4)$ symmetry of $N_F = 2$ QCD at high temperature
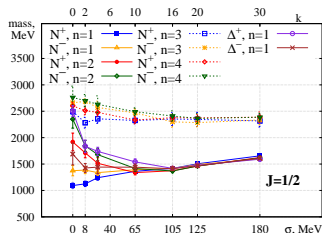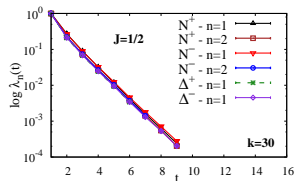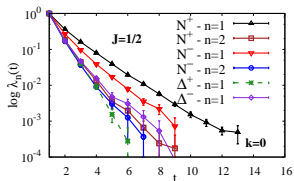
## A hidden symmetry of QCD in Euclidean space

We clearly see a larger degeneracy than the

$$SU(2)_L \times SU(2)_R \times U(1)_A$$

symmetry of the QCD Lagrangian.

What does it mean !?

There must be a hidden symmetry in Euclidean space-time.

Outline
Low mode truncation
Left-right mixing. The chiralspin $SU(2)_{CS}$ and $SU(4)$ groups.
Zero modes and symmetries of Euclidean QCD
$SU(4)$ symmetry of $N_F = 2$ QCD at high temperature

## L.Ya.G., EPJA 51(2015)27

**(i)** **(0,0):**

$$|(0,0); \pm; J\rangle = \frac{1}{\sqrt{2}}|\bar{R}R \pm \bar{L}L\rangle_J.$$

**(ii)** $(1/2, 1/2)_a$ **and** $(1/2, 1/2)_b$**:**

$$|(1/2, 1/2)_a; +; I = 0; J\rangle = \frac{1}{\sqrt{2}}|\bar{R}L + \bar{L}R\rangle_J,$$

$$|(1/2, 1/2)_a; -; I = 1; J\rangle = \frac{1}{\sqrt{2}}|\bar{R}\boldsymbol{\tau}L - \bar{L}\boldsymbol{\tau}R\rangle_J,$$

$$|(1/2, 1/2)_b; -; I = 0; J\rangle = \frac{1}{\sqrt{2}}|\bar{R}L - \bar{L}R\rangle_J,$$

$$|(1/2, 1/2)_b; +; I = 1; J\rangle = \frac{1}{\sqrt{2}}|\bar{R}\boldsymbol{\tau}L + \bar{L}\boldsymbol{\tau}R\rangle_J.$$

**(iii)** **(0,1)⊕(1,0):**

$$|(0, 1) + (1, 0); \pm; J\rangle = \frac{1}{\sqrt{2}}|\bar{R}\boldsymbol{\tau}R \pm \bar{L}\boldsymbol{\tau}L\rangle_J,$$

Outline
Low mode truncation
Left-right mixing. The chiralspin $SU(2)_{CS}$ and $SU(4)$ groups.
Zero modes and symmetries of Euclidean QCD
$SU(4)$ symmetry of $N_F = 2$ QCD at high temperature

## L.Ya.G., EPJA 51(2015)27

Consider rotations in an imaginary 3-dim space of doublets constructed from the $L$ and $R$ bispinors

$$\mathrm{U} = \begin{pmatrix} u_L \\ u_R \end{pmatrix} \qquad \mathrm{D} = \begin{pmatrix} d_L \\ d_R \end{pmatrix}$$

$$\mathrm{U} \to \mathrm{U}' = e^{i\frac{\boldsymbol{\varepsilon} \cdot \boldsymbol{\sigma}}{2}}\mathrm{U} , \qquad \mathrm{D} \to \mathrm{D}' = e^{i\frac{\boldsymbol{\varepsilon} \cdot \boldsymbol{\sigma}}{2}}\mathrm{D} ,$$

where $\boldsymbol{\sigma}$ are the standard Pauli matrices: $[\sigma^i, \sigma^j] = 2i\epsilon^{ijk}\sigma^k$ .

We refer to this imaginary three-dimensional space as the chiralspin space and denote this symmetry group as $SU(2)_{\mathrm{CS}}$

A group that contains at the same time $SU(2)_L \times SU(2)_R$ and $SU(2)_{CS}$ is $SU(4)$ with the fundamental vector

$$\Psi = \begin{pmatrix} u_{\mathrm{L}} \\ u_{\mathrm{R}} \\ d_{\mathrm{L}} \\ d_{\mathrm{R}} \end{pmatrix}$$

Outline
Low mode truncation
Left-right mixing. The chiralspin $SU(2)_{CS}$ and $SU(4)$ groups.
Zero modes and symmetries of Euclidean QCD
$SU(4)$ symmetry of $N_F = 2$ QCD at high temperature

## L.Ya.G., M. Pak, PRD 92(2015)016001

We can construct an explicit realization of the $SU(2)_{CS}$ algebra that acts on the Dirac bispinors.

Then the $SU(2)_{\rm cs}$ chiralspin rotations are generated through

$$\boldsymbol{\Sigma} = \{\gamma^0, i\gamma^5\gamma^0, -\gamma^5\} , \qquad [\Sigma^i, \Sigma^j] = 2i\epsilon^{ijk}\,\Sigma^k .$$
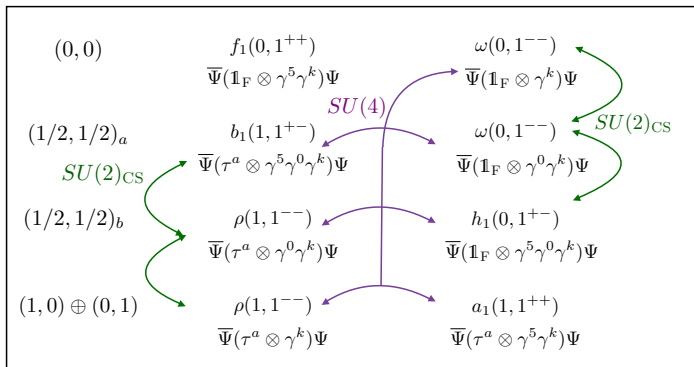
The $SU(4)$ group contains at the same time $SU(2)_L \times SU(2)_R$ and $SU(2)_{CS} \supset U(1)_A$ with the fundamental vector

$$\Psi = \begin{pmatrix} u_{\rm L} \\ u_{\rm R} \\ d_{\rm L} \\ d_{\rm R} \end{pmatrix}$$

and has the following set of generators:

$$\{(\tau^a \otimes \mathbb{1}_D), (\mathbb{1}_F \otimes \Sigma^i), (\tau^a \otimes \Sigma^i)\}$$

Outline
Low mode truncation
Left-right mixing. The chiralspin $SU(2)_{CS}$ and $SU(4)$ groups.
Zero modes and symmetries of Euclidean QCD
$SU(4)$ symmetry of $N_F = 2$ QCD at high temperature

# L.Ya.G., M. Pak, PRD 92(2015)016001

Outline
Low mode truncation
Left-right mixing. The chiralspin $SU(2)_{CS}$ and $SU(4)$ groups.
**Zero modes and symmetries of Euclidean QCD**
$SU(4)$ symmetry of $N_F = 2$ QCD at high temperature

## Zero modes and symmetries of Euclidean QCD. L.Ya.G., 1511.05857

The Lagrangian in Euclidean space-time for a massive quark in a given gauge configuration is:

$$\mathcal{L} = \Psi^\dagger(x)(\gamma_\mu D_\mu + m)\Psi(x), \tag{1}$$

with

$$D_\mu = \partial_\mu + ig\frac{t^a}{2}A_\mu^a, \tag{2}$$

The mass term $\Psi^\dagger(x)\Psi(x)$ is invariant under $U(1)_A$ chiral transformation:

$$\Psi(x) \to e^{i\alpha\gamma_5}\Psi(x); \quad \Psi^\dagger(x) \to \Psi^\dagger(x)e^{-i\alpha\gamma_5}. \tag{3}$$

The kinetic term, $\Psi^\dagger(x)(\gamma_\mu D_\mu)\Psi(x)$, breaks this symmetry ($\gamma_5\gamma_\mu + \gamma_\mu\gamma_5 = 0$).

The same is true with respect to the axial part of $SU(N_F)_L \times SU(N_F)_R$.

What are symmetry properties of both parts under the $SU(2)_{CS}$ and $SU(2N_F)$ transformations?

Outline
Low mode truncation
Left-right mixing. The chiralspin $SU(2)_{CS}$ and $SU(4)$ groups.
Zero modes and symmetries of Euclidean QCD
$SU(4)$ symmetry of $N_F = 2$ QCD at high temperature

## Zero modes and symmetries of Euclidean QCD. L.Ya.G., 1511.05857

The Euclidean $SU(2)_{CS}$ generators:

$$\Sigma = \{\gamma^4, i\gamma^5\gamma^4, -\gamma^5\} \,. \tag{4}$$

Combining the $SU(2)_{CS}$ generators with the $SU(N_F)$ flavor generators into a larger algebra we arrive at the Euclidean $SU(2N_F)$ transformations.

These symmetries are missing in the Lagrangian. Why?
Because a massless "on-shell" quark is described by the zero mode of the Dirac equation

$$\gamma_\mu D_\mu \Psi_0(x) = 0. \tag{5}$$

The zero mode is chiral, $L$ or $R$, depending on the topological charge $Q \neq 0$.
Atiyah-Singer:

$$Q = n_L - n_R$$

.
Conclusion: The zero modes break explicitly $SU(2)_{CS}$ and $SU(2N_F)$.

Outline
Low mode truncation
Left-right mixing. The chiralspin $SU(2)_{CS}$ and $SU(4)$ groups.
Zero modes and symmetries of Euclidean QCD.
$SU(4)$ symmetry of $N_F = 2$ QCD at high temperature

## Zero modes and symmetries of Euclidean QCD. L.Ya.G., 1511.05857

Zero modes are absent in the $Q = 0$ sector.

$$Z_{Q=0}^V = \int D\Psi D\Psi^\dagger e^{\int d^4 x \Psi^\dagger(x)(i\gamma_\mu D_\mu + im)\Psi(x)}. \tag{6}$$

The hermitian Dirac operator has in a finite volume $V$ a discrete spectrum:

$$i\gamma_\mu D_\mu \Psi_n(x) = \lambda_n \Psi_n(x). \tag{7}$$

We expand $\Psi(x)$ and $\Psi^\dagger(x)$ over a complete and orthonormal set $\Psi_n(x)$:

$$\Psi(x) = \sum_n c_n \Psi_n(x); \quad \Psi^\dagger(x) = \sum_k \bar{c}_k \Psi_k^\dagger(x) \tag{8}$$

$$Z_{Q=0}^V = \int \prod_{k,n} d\bar{c}_k dc_n e^{\sum \bar{c}_k c_n \int d^4 x \bar{c}_k c_n(\lambda_n + im)\Psi_k^\dagger(x)\Psi_n(x)}. \tag{9}$$

It is precisely $SU(2)_{CS}$ and $SU(2N_F)$ symmetric:

$$(U\Psi_k(x))^\dagger U\Psi_n(x) = \Psi_k^\dagger(x)\Psi_n(x). \tag{10}$$

Outline
Low mode truncation
Left-right mixing. The chiralspin $SU(2)_{CS}$ and $SU(4)$ groups.
**Zero modes and symmetries of Euclidean QCD**
$SU(4)$ symmetry of $N_F = 2$ QCD at high temperature

## Zero modes and symmetries of Euclidean QCD. L.Ya.G., 1511.05857

H. Leutwyler, A. Smilga, 1992: Up to an inessential normalization factor

$$Z_{Q=0}^{V \to \infty} \sim Z_{\theta=0}^{V \to \infty}$$

.

The rate is $1/V$.

We conclude: Classical Euclidean QCD has hidden $SU(2)_{CS}$ and $SU(2N_F)$ symmetries. The $U(1)_A$ anomaly breaks both $SU(2)_{CS}$ and $SU(2N_F)$ symmetries.

The Banks-Casher relation:

$$\lim_{m \to 0} < 0|\bar{\Psi}(x)\Psi(x)|0 > = -\pi\rho(0) . \tag{11}$$

The quark condensate in Minkowski space breaks all symmetries $U(1)_A$, $SU(N_F)_L \times SU(N_F)_R$, $SU(2)_{CS}$, $SU(2N_F)$ to $SU(N_F)_V$. Hidden classical $SU(2)_{CS}$ and $SU(2N_F)$ symmetries are broken both spontaneously and anomalously by the near-zero modes.

A remark. In $Q \neq 0$ sectors there do appear contributions from the zero modes: $\Psi_0^\dagger \Psi_0$; $\Psi_k^\dagger \Psi_0$; $\Psi_0^\dagger \Psi_n$. Their contributions vanish as $1/V$.

Outline
Low mode truncation
Left-right mixing. The chiralspin $SU(2)_{CS}$ and $SU(4)$ groups.
Zero modes and symmetries of Euclidean QCD
$SU(4)$ symmetry of $N_F = 2$ QCD at high temperature

# $SU(4)$ symmetry of $N_F = 2$ QCD at high temperature

JLQCD (2013 - 2016): $m \sim 3$ MeV, $T \sim 180$ MeV
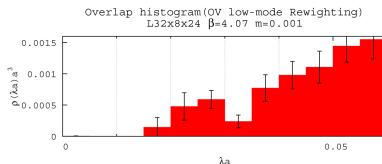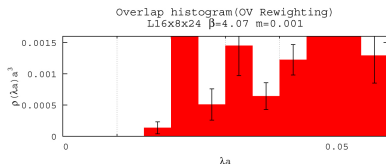


The reweighted overlap Dirac spectrum shows a gap above $T_c$, which is apparently insensitive to the volume and which increases with temperature.

Our conclusion: $N_F = 2$ QCD is above $T_c$ $SU(4)$-symmetric.

A single quark propagation in Minkowski space-time is impossible. Only $SU(4)$-symmetric confined "hadrons" can exist.

Outline
Low mode truncation
Left-right mixing. The chiralspin $SU(2)_{CS}$ and $SU(4)$ groups.
Zero modes and symmetries of Euclidean QCD
$SU(4)$ symmetry of $N_F = 2$ QCD at high temperature

## Conclusions and predictions

The Euclidean QCD has hidden $SU(2)_{CS}$ and $SU(2N_F)$ symmetries. They are broken anomalously and spontaneously by the physics that is associated with the near-zero modes of the Dirac operator. Instantons?

A consequence (L.Ya.G., arXiv 1512.06703): At high temperatures $T > T_c$, where chiral and $U(1)_A$ are restored, the matter is $SU(2)_{CS}$ and $SU(2N_F)$ symmetric. There cannot be deconfined quarks and gluons.

This statement can be verified on the lattice: Calculate correlators of operators that transform under $SU(2)_{CS}$ and $SU(2N_F)$. At $T > T_c$ they should become identical.