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Event-by-event multiplicity fluctuations in relativistic heavy ion collisions

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References:

- **HJX**, Multiplicity fluctuation in statistical model, arXiv:1602.06378, arXiv:1602.07089.
- **HJX**, Jixin Li, Huichao Song, "Multiplicity fluctuation in relativistic ion collision within a Monte Carlo hadron resonance gas model", in prepare.
- Jixin Li, **HJX**, Huichao Song, "Event-by-event multiplicity fluctuations in VISHNU hybrid model", in prepare.

Outline

Q: Why previous theoretical calculations in statistical/HRG model fail to reproduce/understand the negative binomial multiplicity distribution of (net) electric charges measured in experiment?

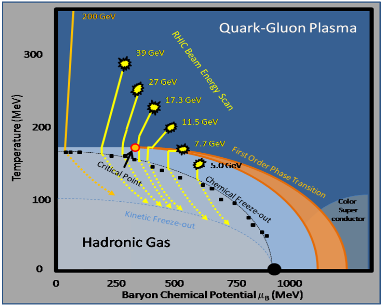
A: The previous theoretical calculations and the experimental measurements are mismatched.

1. Introduction: clarifying the mismatches
2. The basic statistical expectations: understanding the NBD
3. The HRG baselines: effect of resonance decays and acceptance cuts
4. Outlook

I. Introduction

Clarifying the mismatches between previous theoretical calculations and experimental measurements

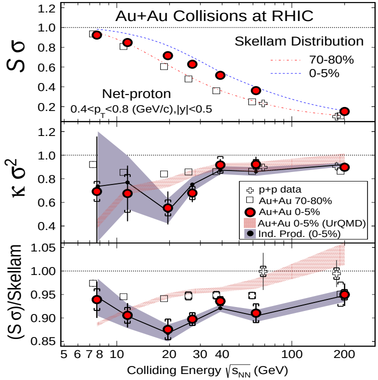
The QCD phase diagram

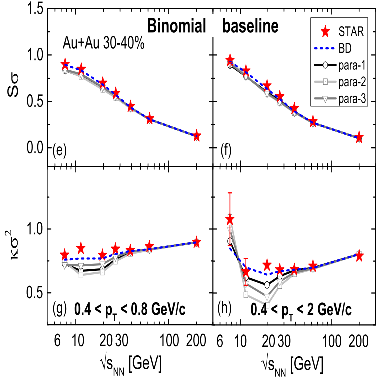


Event-by-event multiplicity fluctuations are expected to provide us crucial informations about the hot and dense QCD matter created in heavy ion collision!

- Experiment: STAR, Phys.Rev.Lett. 112, 032302;Phys.Rev.Lett. 113, 092301; Nucl.Phys. A931, 796; PHENIX, Phys. Rev., C76 , 034903; Phys.Rev., C78, 044902; Phys.Rev., C93, 011901; ...
- Theory: Stephanov, Phys.Rev., D60, 114028; Jeon, Phys.Rev.Lett., 83, 5435; Bazavov, Phys.Rev.Lett., 109, 192302; Gupta, Science 332, 1525;...

Importance of baseline studies: protons



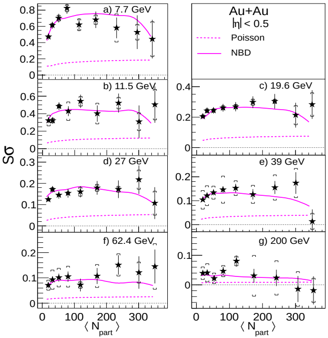


[STAR, Phys.Rev.Lett. 112, 032302]

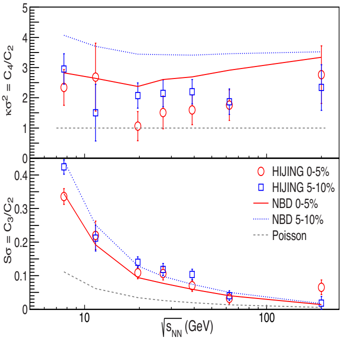
[Jiang, arXiv:1512.06164]

- The cumulant of proton/net protons distribution are closer to Poisson/Skellam baselines.
- The Binomial baselines can be used to describe the data better than the Skellam baselines. But the gap between the two baselines is small.

Importance of baseline studies: charges



[STAR, Phys.Rev.Lett. 113, 092301]



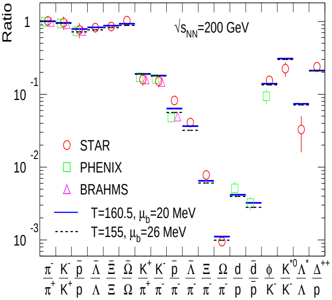
[Tarnowsky, Phys.Lett., B724, 51]

- The cumulant of charge/net charge distributions are far away from the Poisson/Skellam baselines.
- The data can be well described by the negative binomial baselines.

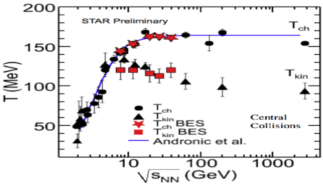
Is it possible to describe the multiplicity distributions of charges and protons systemically in statistical model? **YES**

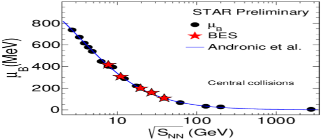
The success of the statistical model

- In relativistic heavy ion collisions, the mean multiplicity of particles can be well described by the statistical model



[Andronic, Nucl.Phys., A772, 167]





[Sabita Das, arXiv:1412.0499]

- Event-by-event multiplicity distribution?

Previous theoretical calculations (I)

- In the statistical model, the basic quantity is the partition function Z. In the grand canonical ensemble(GCE) the thermal system can be specified by a volume V, a temperature T and a chemical potential vector $\overrightarrow{\mu}(=(\mu_B, \mu_s, \mu_Q))$ [Andronic, Nucl.Phys., A772, 167]

$$\ln Z(V,T,\overrightarrow{\mu}) = \sum_{i\in\text{mesons}} \ln Z_i^+(V,T,\overrightarrow{\mu}) + \sum_{i\in\text{baryons}} \ln Z_i^-(V,T,\overrightarrow{\mu}), \quad (1)$$

- Most of previous studies are focused on the multiplicity distributions in a specific statistical ensemble with fixed (μ, T, V).
- For example, in the case of classical GCE, we obtain the Poisson/Skellam multiplicity distribution [Cleymans, Z. Phys. C, 137; Begun, Phys.Rev., C70, 034901]

$$P(q) = \frac{\lambda^q \exp(-\lambda)}{\lambda!}. \quad (2)$$

for the charges/net-conserved charges with Boltzmann statistics (neglect the contributions from multi-charge hadrons).

Previous theoretical calculations (II)

- Multiplicity distribution $P(q)$ deviate from Poisson[Karsch, Phys.Lett., B695, 136; Grag, Phys.Lett., B726, 691; Fu, Phys.Lett., B722, 144; Alba, Phys.Lett., B738, 305; Bhattacharyya, Phys.Rev., C91, 041901; Bzdak, Phys.Rev., C87, 014901; ...] :
    - Quantum effect.
    - Statistical ensemble.
    - Hadron resonance decays.
    - Global conservation law.
    - ....
- Volume fluctuations [S. Jeon, hep-ph/0304012; M. I. Gorenstein, Phys.Rev. C78, 041902; V. Skokov, Phys.Rev. C88, 034911...]:

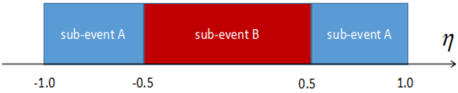
$$\mathscr{P}(q) = \int d\Omega F(\Omega) P(q;\Omega) \tag{3}$$

where Ω represents a set of principal thermodynamic variables.

Unfortunately, neither $P(q)$ nor $\mathscr{P}(q)$ are the experimental measurements.

Methods used in experiment

- Two sub-events: sub-event A for centrality definitions and sub-event B for moment-analysis.



- The conditional probability distribution for multiplicity q in given reference multiplicity bin k reads,

$$\mathscr{P}_{B|A}(q|k) = \frac{\mathscr{P}_{A\cap B}(q,k)}{\mathscr{P}_A(k)}. \tag{4}$$

- If we assume the independent production of sub-event A and B in each event, we have

$$\mathscr{P}_{B|A}(q|k) = \frac{\int d\Omega F(\Omega) P_B(q;\Omega) P_A(k;\Omega)}{\mathscr{P}_A(k)}. \tag{5}$$

Eq.(5) is the corrected statistical formula for the multiplicity distribution measured in experiment!!!

II. The basic statistical expectations

Understanding the negative binomial multiplicity distributions of (net) electric charges

- II.A Negative binomial distributions of charge distributions
- II.B Net-conserved charges
- II.C Volume boundary effect

HJX, arXiv:1602.06378, arXiv:1602.07089

Two assumptions

S1 We restrict our discussion to the classical GCE, in which $P_A(k)$ and $P_B(q)$ can be described by Poisson distributions

$$P_A(k) = \frac{\lambda^k e^{-\lambda}}{k!}, \tag{6}$$

$$P_B(q) = \frac{\mu^q e^{-\mu}}{q!}, \tag{7}$$

where the parameters $\lambda \equiv \lambda(V)$ and $\mu \equiv \mu(V) \equiv \mu(\lambda) = b\lambda$ are proportional to the volume V. We assume that the chemical potential $\overrightarrow{\mu_i}$ and system temperature are independent of centrality.

S2 The distribution of λ is a uniform distribution in the range $[0, h]$

$$f(\lambda) = \begin{cases} 1/h & \lambda \le h, \\ 0 & \lambda > h, \end{cases} \tag{8}$$

where h corresponding to the upper boundary of the system volume.

Negative binomial distribution

- From S1 and S2,the conditional probability distribution can be rewritten as

$$\mathscr{P}_{B|A}(q|k) = \frac{1}{\mathscr{P}_A(k)} \int d\lambda f(\lambda) \frac{\lambda^k e^{-\lambda}}{k!} \frac{\mu^q e^{-\mu}}{q!}, \tag{9}$$

$$= \frac{1}{\mathscr{P}_A(k)} \int_0^h d\lambda \frac{\lambda^k e^{-\lambda}}{k!} \frac{b^q \lambda^q e^{-b\lambda}}{q!}, \tag{10}$$

$$\mathscr{P}_A(k) = \int_0^h d\lambda \frac{\lambda^k e^{-\lambda}}{k!} \tag{11}$$

- For the non-central collision range, $k + q \ll h$, we have

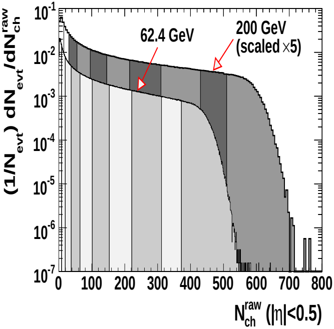
$$\mathscr{P}_{B|A}(q|k) \simeq \mathrm{NBD}(q; r, p) \equiv \frac{(r+q)!}{r!q!} p^q (1-p)^{r+1} \tag{12}$$

which mimics a standard negative binomial distribution with parameters

$$r = k + 1, \tag{13}$$

$$p = b/(1+b) \tag{14}$$

Assumptions



Distributions of reference multiplicity

[STAR, Phys.ReV., C79, 034909]

Except the boundary effect, the second assumption S2 can be replaced by

S3 The ratio

$$\frac{\mathscr{P}_A(k+m)}{\mathscr{P}_A(k)} \simeq 1., \tag{15}$$

for small m at non-central collisions.

Cumulants

- From S1 and S3 we obtain the cumulants of $\mathscr{P}_{B|A}(q|k)$

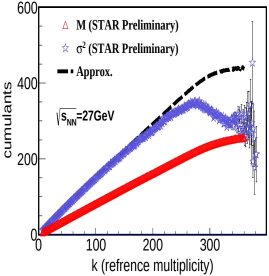
$$\begin{aligned} c_1 &= M = b(k+1), && (16)\\ c_2 &= \frac{M^2}{k+1} + M, && (17)\\ c_3 &= \frac{2M^3}{(k+1)^2} + \frac{3M^2}{k+1} + M, && (18)\\ c_4 &= \frac{6M^4}{(k+1)^3} + \frac{12M^3}{(k+1)^2} + \frac{7M^2}{k+1} + M. && (19)\end{aligned}$$

Again, we got the NBD expectations with parameters

$$\begin{aligned} r &= k+1, && (20)\\ p &= M/(M+k+1) \equiv b/(1+b). && (21)\end{aligned}$$

- For $M/(k+1) \sim 0$, we obtain the Poisson expectations (protons/anti-protons).

APP1: Total charges: AuAu 27GeV

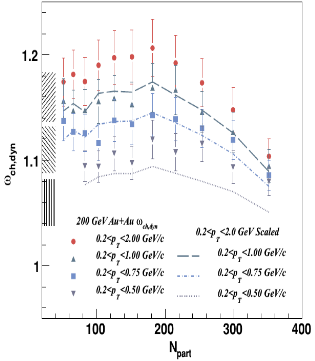


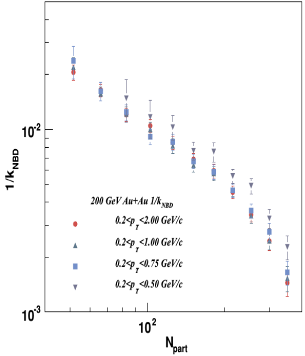
[The data are taken from Tang(STAR), J.Phys.Conf.Ser. 535, 012009]

$$\sigma^2 \equiv c_2 = M + \frac{M^2}{k+1} \tag{22}$$

- The variances of total charge distribution reported by the STAR collaboration can be well described by the approximate solution at non-central collisions.
- The deviations at most-central collision are due to volume boundary effect which make the assumption S3 becomes invalid.

APP2: p_T-dependence

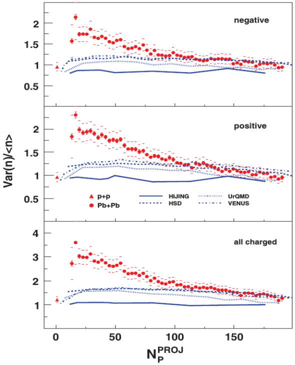


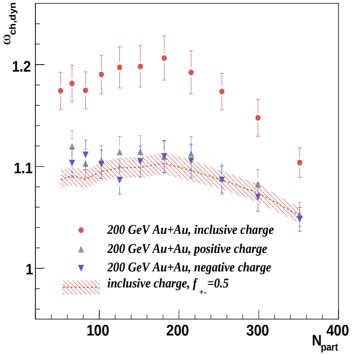


The wider p_T range correspond to the larger M. [PHENIX,PRC78,044902]

- The scale variance $\omega = c_2/c_1 = 1 + M/(k+1)$ increase with increasing M
- The NBD parameter $r = k_{NBD} = k + 1$ is independent of M

APP3: Scale variances





[NA49, PRC75,064904]

[PHENIX,PRC78,044902]

$$\omega_{\pm} = 1 + \frac{M_{\pm}}{k+1} = 1 + \frac{M_{+} + M_{-}}{k+1} = \omega_{+} + \omega_{-} - 1 \tag{23}$$

The data reported by the NA49 collaboration and PHENIX collaboration can be described by the relation Eq.(23).

Net conserved charges

S1N For the net-conserved charge, we assume $P_A(k)/P_B(q)$ is Poisson/Skellam distribution

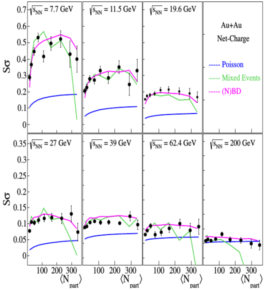
$$P_B(q) = I_q(2\sqrt{\mu_+\mu_-}) \left(\frac{\mu_+}{\mu_-}\right)^{q/2} e^{-(\mu_+ + \mu_-)}, \tag{24}$$

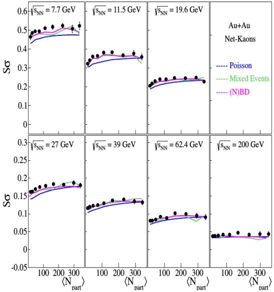
Here $\mu_\pm = b_\pm \lambda$ are the corresponding Poisson parameters of positive and negative conserved charges, and I_q is the modified Bessel function.

- With the assumptions S1N and S3, we obtain

$$\begin{aligned}
c_1^N &= M_+ - M_- \equiv b_+(k+1) + b_-(k+1), && (25)\\
c_2^N &= \frac{(M_+ - M_-)^2}{k+1} + M_+ + M_-, && (26)\\
c_3^N &= \frac{2(M_+ - M_-)^3}{(k+1)^2} + \frac{3(M_+^2 - M_-^2)}{k+1} + c_1^N, && (27)\\
c_4^N &= \frac{6(M_+ - M_-)^4}{(k+1)^3} + \frac{12(M_+ - M_-)^2(M_+ + M_-)}{(k+1)^2} \\
&\quad + \frac{6(M_+^2 + M_-^2)}{k+1} + c_2^N, && (28)
\end{aligned}$$

APP4: $S\sigma$





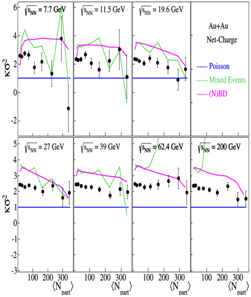
[McDonald, PhD thesis(2013)]

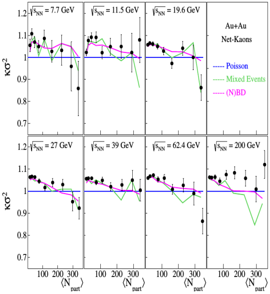
$$S\sigma = c_3/c_2 = 2\beta(1-\alpha) + \frac{\beta(1-\alpha^2)+1-\alpha}{\beta(1-\alpha)^2+1+\alpha}, \tag{29}$$

where $\alpha = M_-/M_+$, $\beta = M_+/(k+1)$. $S\sigma$ increase with increasing β and decreasing α.

- Both α and β are large in net charge case, but the magnitudes of $S\sigma$ for net charge distributions and net kaon distributions are almost the same.
- Due to smaller β in the net-kaon case, it's expectations are more closer to the Poisson/Skellam distribution.

APP5: $\kappa\sigma^2$





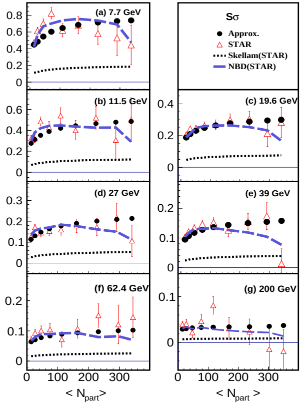
[McDonald, PhD thesis(2013)]

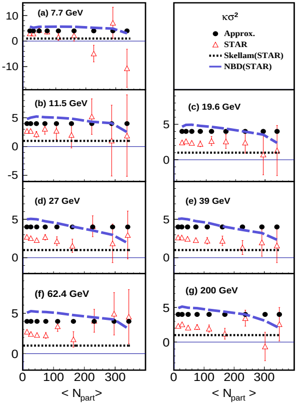
$$\kappa\sigma^2 = c_4/c_2 = 6\beta(\gamma - \frac{2\alpha}{\gamma}) + 1, \quad (30)$$

where $\gamma = \beta(1-\alpha)^2 + 1 + \alpha$. $\kappa\sigma^2$ increase with increasing α and β.

- The $\kappa\sigma^2$ in net charge case large than the one in net kaon case.
- Due to smaller β in the net-kaon case, it's expectations are more closer to the Poisson/Skellam distribution.

APP6: STAR net electric charges





[The data are taken from STAR, PRL113, 092301]

Using $(M_+ + M_-) \simeq k \gg (M_+ - M_-)$ in the net electric charges case, we obtain $S\sigma \simeq \frac{4(1-\alpha)}{1+\alpha}$ and $\kappa\sigma^2 \simeq 4$ which are about four times of the Skellam expectations. These expectations are very colse to the NBD baselines given by the STAR collaboration.

Most-central collisions

- From S1 and S2,the conditional probability distribution can be rewritten as

$$\mathscr{P}_{B|A}(q|k) = \frac{1}{\mathscr{P}_A(k)} \int d\lambda f(\lambda) \frac{\lambda^k e^{-\lambda}}{k!} \frac{\mu^q e^{-\mu}}{q!}, \quad (31)$$

$$= \frac{1}{\mathscr{P}_A(k)} \int_0^h d\lambda \frac{\lambda^k e^{-\lambda}}{k!} \frac{b^q \lambda^q e^{-b\lambda}}{q!}, \quad (32)$$

- For the most central collision range, $k \gg h$, we have

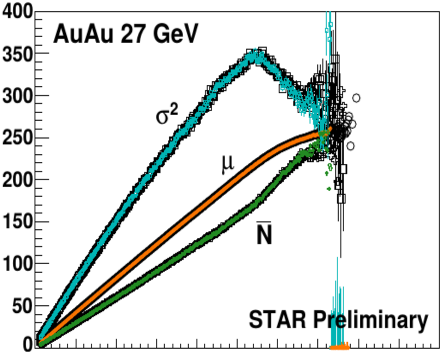
$$\mathscr{P}_{B|A}(q|k) \simeq \frac{e^{-bh}(bh)^q}{q!} \frac{k+1}{k+q+1}. \quad (33)$$

If we focus on the distribution with $q \ll k$, we have

$$\mathscr{P}_{B|A}(q|k) \sim \frac{e^{-bh}(bh)^q}{q!}. \quad (34)$$

which mimics a Poisson distribution with parameter $\tilde{\mu} = bh$.

Most-central collisions



[Tang(STAR), J.Phys.Conf.Ser. 535, 012009]

The multiplicity distribution mimic the negative binomial distributions at non-central collisions, but tend to approach the Poisson one at most central collisions due to the boundary effect from distribution of volume.

Collisional geometry

- Instead of assumption S2, we simulate the distribution of Poisson parameter λ (system volume) with optical Glauber model [Kharzeev, Z.Phys., C74, 307; Phys.Lett., B507, 121]

$$f(\lambda) = \mathscr{R} \int P(n;\zeta)[1 - P_0(\zeta)]2\pi\zeta d\zeta \qquad (35)$$

where $\mathscr{R}$ is a normalization factor and ζ is impact parameter. $P_0(\zeta)$ is the probability of no interaction among the nuclei.

- The correlation function can be written as

$$P(\lambda;\zeta) = \frac{1}{\sqrt{2\pi a\lambda}} \exp\left[-\frac{(\lambda - hn(\zeta))^2}{2a\lambda}\right] \qquad (36)$$

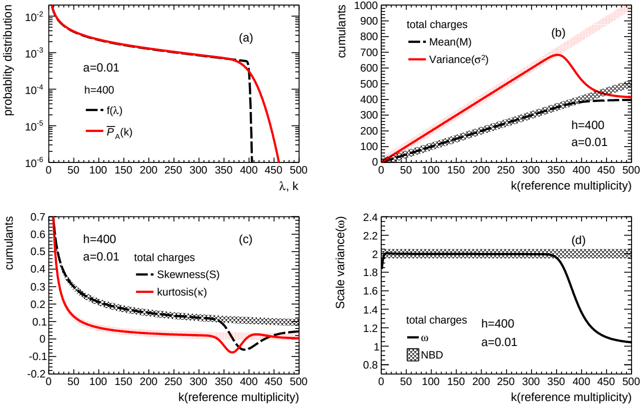
which stand for a Gaussian-type fluctuations on λ (system volume).

- In optical Glauber model

$$n(\zeta) = \left[\frac{(1-x)}{2} n_{\rm part}(\zeta) + x n_{\rm coll}(\zeta)\right] / n(0) \qquad (37)$$

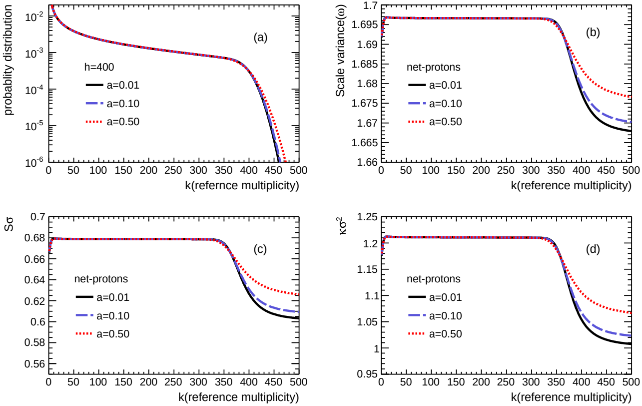
and $n_{\rm part}(\zeta)$ is the number of participant nucleons, $n_{\rm coll}(\zeta)$ is the number of binary nucleon-nucleon collisions

Numerical results: total electric charges



- The centrality dependence of skewness and kurtosis have non-monotonic behaviors in high reference multiplicity range.
- the minimum values of skewness and kurtosis can be negative.

Numerical results: net protons



- The effect of different external volume fluctuations can be only distinguished at most central collision range.
- The multiplicity distributions at most-central collision are very sensitive to the distribution of system volume (probability distribution of reference multiplicity).

Summary

1. From the corrected statistical expression for data, the basic statistical expectations of multiplicity distribution mimic the negative binomial distributions at non-central collisions, but tend to approach the Poisson one at most central collisions due to the boundary effect from distribution of volume.

2. The basic statistical baselines can be directly construced from data of $\mathscr{M}(k)$ and $\mathscr{P}_A(k)$ (For the non-central collisions, only $\mathscr{M}(k)$ is OK).

3. Due to the boundary effect, one should be careful when talking about the higher order cumulants of multiplicity distributions at $0-5\%$ centrality.

III. The HRG baselines

Effect of resonance decays and acceptance cuts on the cumulants of multiplicity distributions

HJX, Jixin Li, Huichao Song, in prepare

Why Monte Carlo approach?

- Previous studies:
  - Analytical calculations.
  - Contributions from resonance hadrons in specific statistical ensemble.
  - Average part and probabilistic part.
- Monte Carlo approach:
  - Numerical studies.
  - Distribution of system volume.
  - Fluctuations for moment-analysis particles and reference particles.
  - Correlations between reference particles in sub-event A and moment-analysis particles in sub-event B.
  - Correlations between positive and negative charges.
  - More realistic simulations of kinematic cuts.

The model: mcHRG

- The multiplicity of different particles species in each event are randomly generated by the Poisson distributions with parameters

$$\lambda_i = \frac{VTg_i m_i^2}{(2\pi)^2} \exp\left(\frac{\mu_i}{T}\right) K_2\left(\frac{m_i}{T}\right), \tag{38}$$

- We use 319 primordial particle species as inputs and 26 stable particle species after performing Monte Carlo resonance decay. [Huovinen, Nucl.Phys., A837, 26],
- The volume information is given by a Monte Carlo Glauber model [Loizides, arXiv:1408.2549],

$$V = h\left[\frac{1-x}{2} n_{\text{part}} + x n_{\text{ncoll}}\right], \tag{39}$$

- The input parameters are μ_B and T. The chemical potentials μ_S and μ_Q are calculated by

$$n_S = 0; \tag{40}$$

$$n_Q = \frac{79 n_B}{197}. \tag{41}$$

Setup: $\sqrt{s_{NN}} = 39$GeV

- $T = 0.144$ GeV and centrality-dependent $\mu_B = 0.098$ GeV.
- Spectra at kinetic freeze-out
  - The rapidity distribution is modeled by [Wong, Phys.Rev., C78, 054902]

$$\frac{dN}{dy} \propto \exp\left(\sqrt{(L^2 - y^2)}\right) \tag{42}$$

    with $L = \ln(\sqrt{s_{NN}}/m_p)$ and m_p is the mass of proton.
  - The transverse momentum spectra is modeled by Blast-wave model [Schnedermann, Phys.Rev., C48, 2462]

$$\frac{dN}{p_T dp_T} \propto \int_0^1 x dx m_\perp I_0\left(\frac{p_\perp \sinh\rho}{T_{kin}}\right) K_1\left(\frac{m_\perp \cosh\rho}{T_{kin}}\right), \tag{43}$$

    where $T_{kin} = 100$MeV, $\rho = \tanh^{-1}(\beta)$ and $\beta = \beta_S x^n$ with $\beta_S = 0.78$ and $n = 1.0$.
- Statistical errors: Delta theorem [X. Luo, JPG(2012), PRC(2015)].

Statistical baselines

- The statistical expectations for net electric charges (net protons) can be constructed by the informations of $\mathscr{P}_A(k)$ and $\mathscr{M}_\pm(k)$ with [HJX, arXiv:1602.06378, arXiv:1602.07089]

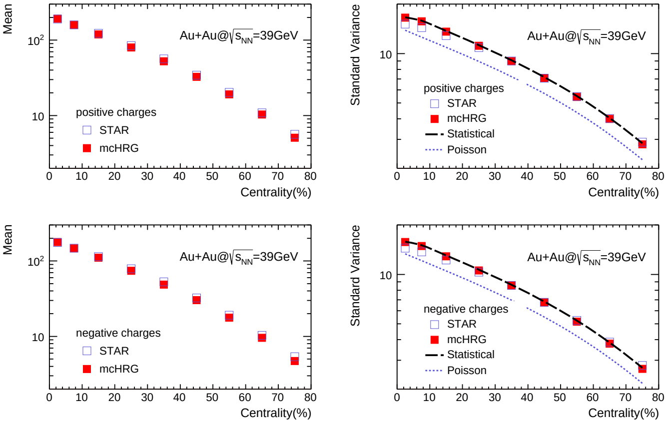
$$
\begin{aligned}
c^N_{n+1} &= m^N_{n+1} - \sum_{s=0}^{n-1} \frac{n!}{s!(n-s)!} m^N_{n-s} c^N_{s+1}, && (44)\\
m^N_1 &= \Delta b \langle \lambda \rangle, && (45)\\
m^N_2 &= \Delta b^2 \langle \lambda^2 \rangle + (b_+ + b_-)\langle \lambda \rangle, && (46)\\
m^N_3 &= \Delta b^3 \langle \lambda^3 \rangle + 3(b_+^2 - b_-^2)\langle \lambda^2 \rangle + m^N_1, && (47)\\
m^N_4 &= \Delta b^4 \langle \lambda^4 \rangle + 6\Delta b^2 (b_+ + b_-)\langle \lambda^3 \rangle \\
&\quad + 6(b_+^2 + b_-^2)\langle \lambda^2 \rangle + m^N_2, && (48)
\end{aligned}
$$

where $\Delta b = b_+ - b_-$ and

$$\langle \lambda^m_\pm \rangle = \langle \lambda^m \rangle = \frac{(k+m)!}{k!} \frac{\mathscr{P}_A(k+m)}{\mathscr{P}_A(k)}. \quad (49)$$

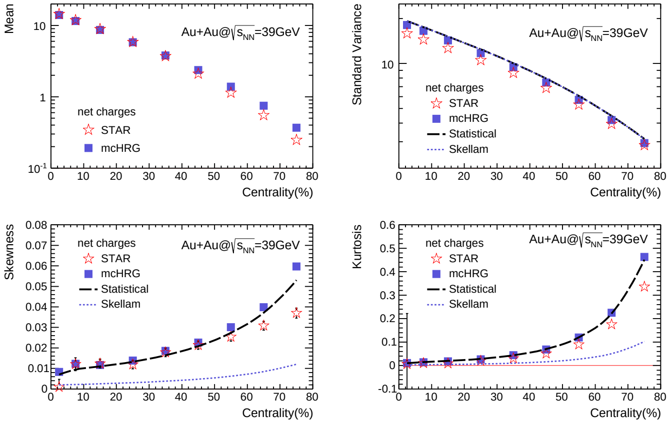
The parameters $b_\pm$ are extracted by fitting the data of mean multiplicity $\mathscr{M}_\pm(k) = b_\pm(k+1)$ in $[10\% - 50\%]$ centrality range.

mcHRG baselines: positive/negative electric charges



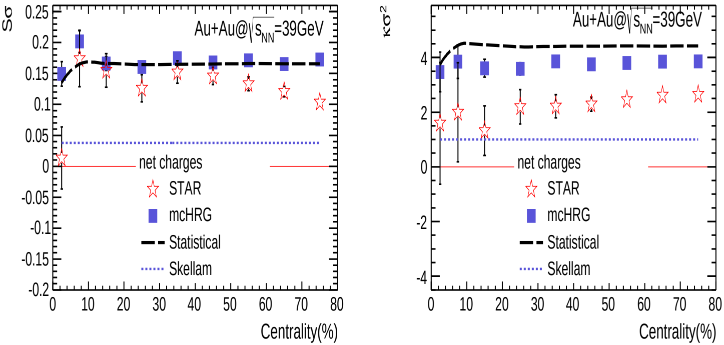
- Poisson baselines: $\mathscr{M}(k)$ from mcHRG simulations.
- Statistical baselines: $\mathscr{M}(k)$ and $\mathscr{P}_A(k)$ from mcHRG simulations.
- mcHRG baselines: event-by-event analysis.

mcHRG baselines: net electric charges (I)



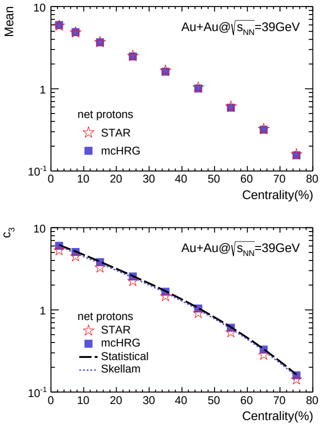
The resonance decays reduce the variances of net electric charges due to the correlations between positive and negative electric charges.

mcHRG baselines: net electric charges (II)



- $S\sigma$: Centrality-dependent model parameters.
- $\kappa\sigma^2$: Correlations between positive charges and negative charges.
- Distribution of reference multiplicity.

mcHRG baselines: net protons



- The effect of resonance decays on the net-proton distributions can be neglected.
- Negative binomial-like baselines (Statistical/mcHRG model) **VS** Binomial-like observables (data)

Summary

The mcHRG model improve the non-critical baselines **VS** The basic statistical baselines can be directly constructed from data.

1. The resonance decays reduce the variances of net electric charges due to the resulting correlations.

2. The correlations between positive charges (protons) and negative charges (anti-protons) beyond resonance decays need further investigations. (the conservation law, for example).

3. More precision centrality-dependent model parameters fixed by the data of rapidity spectra and p_T spectra and distribution of reference multiplicity $\mathscr{P}_A(k)$

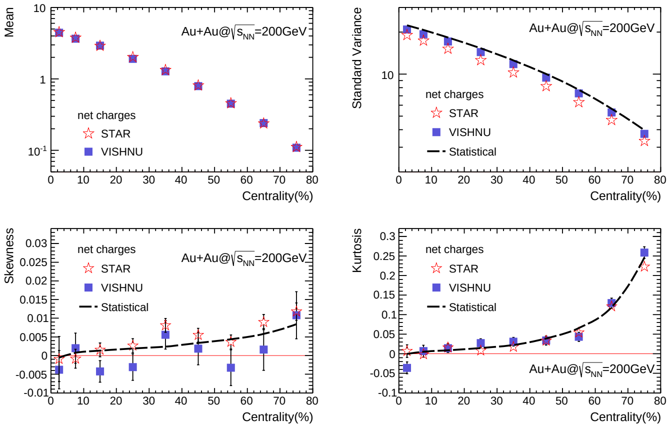
Statistical/mcHRG expectations beyond Poisson approximation?

Outlook

- Model beyond the HRG (`iEBE-VISHNU` Jixin Li, HJX, Huichao Song, in prepare)
  - Local thermal equilibrium.
  - Hadronic evolution.
  - Baryon anti-baryon annihilation.
  - Flow corrections
- Suggestions for experiment:
  - Statistical baselines: the data of $\mathscr{M}(k)$ and $\mathscr{P}_A(k)$ as function of reference multiplicity k.
  - HRG baselines: the pseudo-rapidity distributions and transverse momentum distributions of produced particles.
  - Identical methods for the moment-analysis particles and the reference particles.
  - Reduce/eliminate the uncertainties from experiment inefficiencies.

The statistical/HRG model are successful not only in describing the mean multiplicity(lowest order cumulants) of produced hadrons, but also for its high order cumulants!

VISHNU (preliminary)



[Jixin Li, HJX, Huichao Song, in prepare]

Thank you for your attention!