QCD thermodynamics in the crossover/freeze-out region

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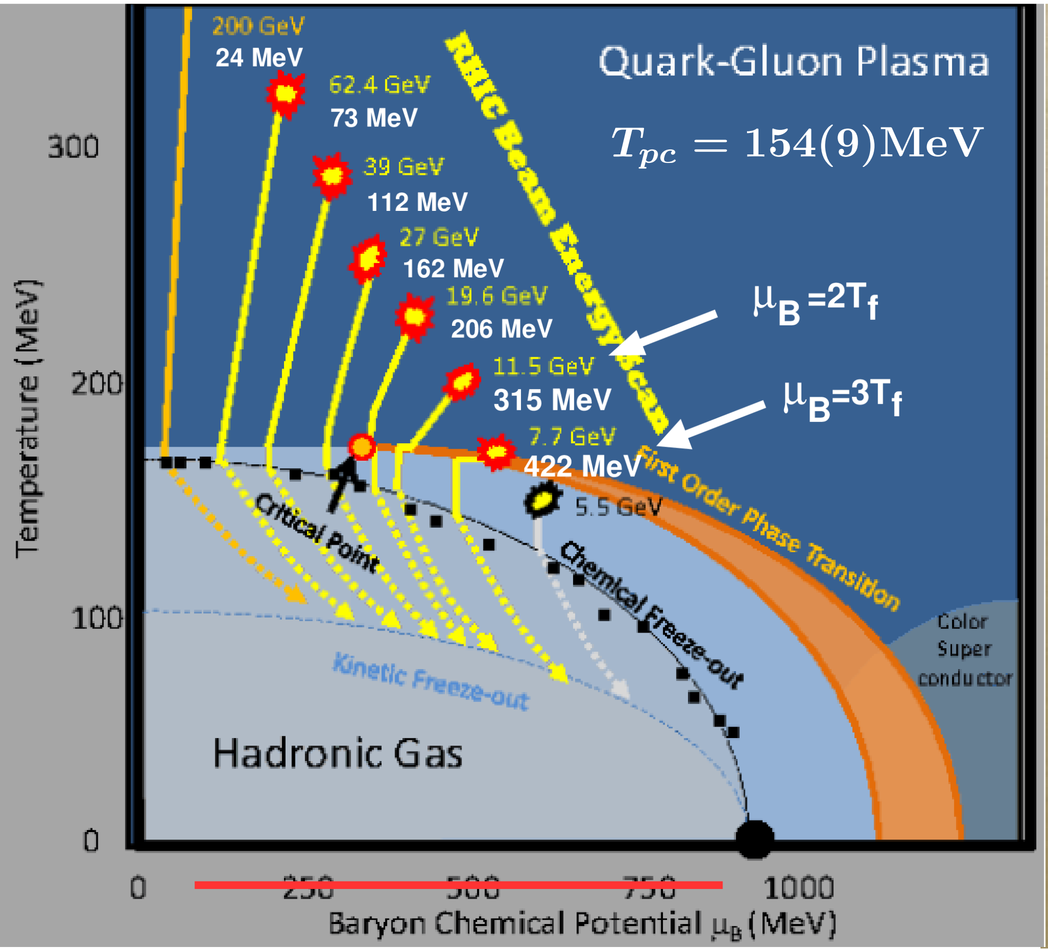
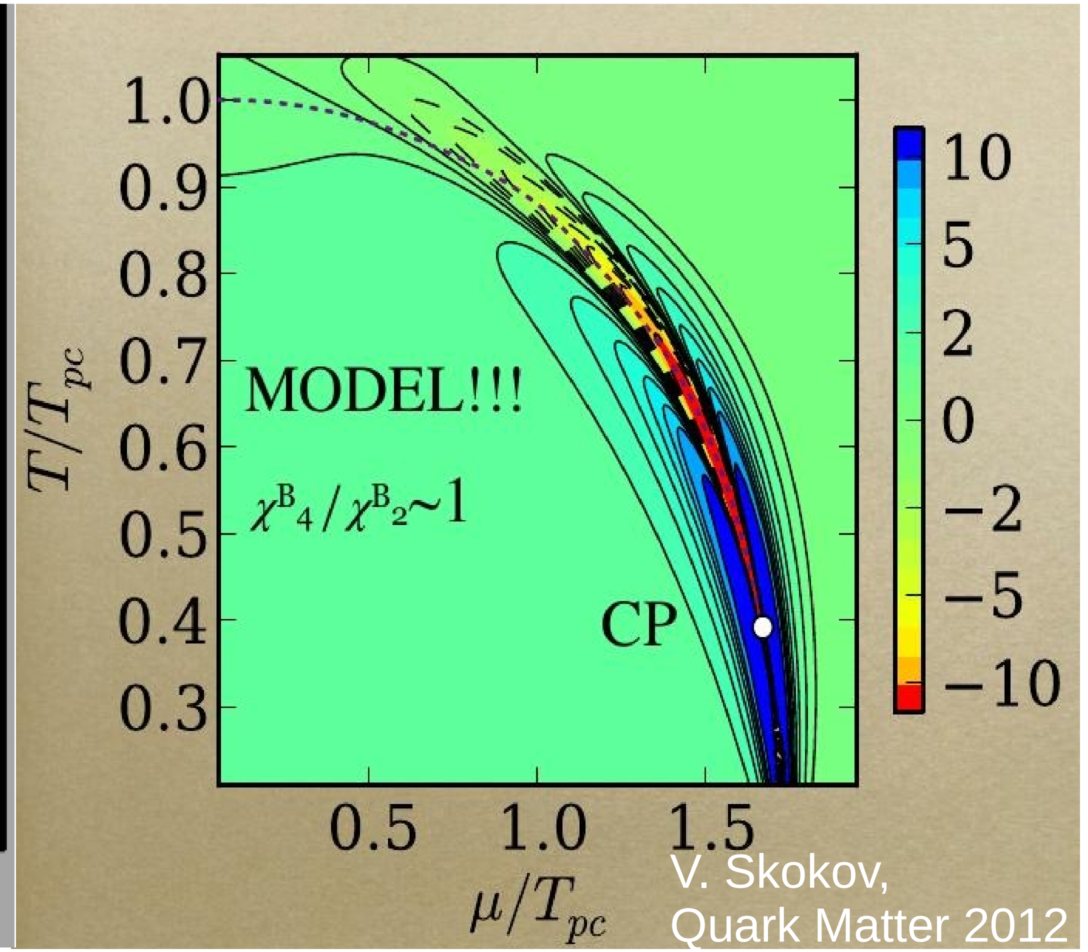
- characterization of bulk thermodynamics and fluctuations of conserved charges in the crossover region
- Taylor expansion of the equation of state
- constraining the location of the critical point

BEST COLLABORATION

May 2016

Probing the properties of matter through the analysis of conserved charge fluctuations





Where is the critical point?

Using Taylor expansions in the range $0 \leq \mu_B/T \leq 3$ to

– explore structure of the phase diagram in the experimentally accessible regime

– characterize the properties of matter in this region

Probing the properties of matter through the analysis of conserved charge fluctuations

Taylor expansion of the **QCD** pressure: $\frac{P}{T^4} = \frac{1}{VT^3} \ln Z(T, V, \mu_B, \mu_Q, \mu_S)$

$$\boxed{\frac{P}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{ijk}^{BQS}(T) \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k}$$

cumulants of net-charge fluctuations and correlations:

$$\chi_{ijk}^{BQS} = \left. \frac{\partial^{i+j+k} P/T^4}{\partial \hat{\mu}_B^i \partial \hat{\mu}_Q^j \partial \hat{\mu}_S^k} \right|_{\mu_{B,Q,S}=0} \quad , \quad \hat{\mu}_X \equiv \frac{\mu_X}{T}$$

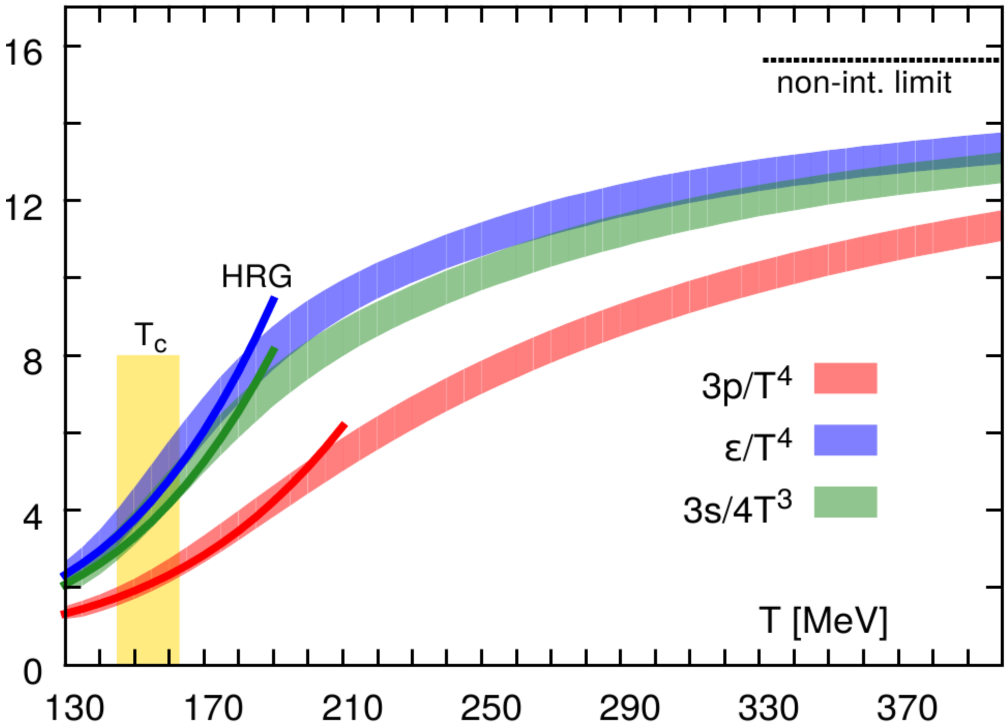
the pressure in hadron resonance gas (**HRG**) models:

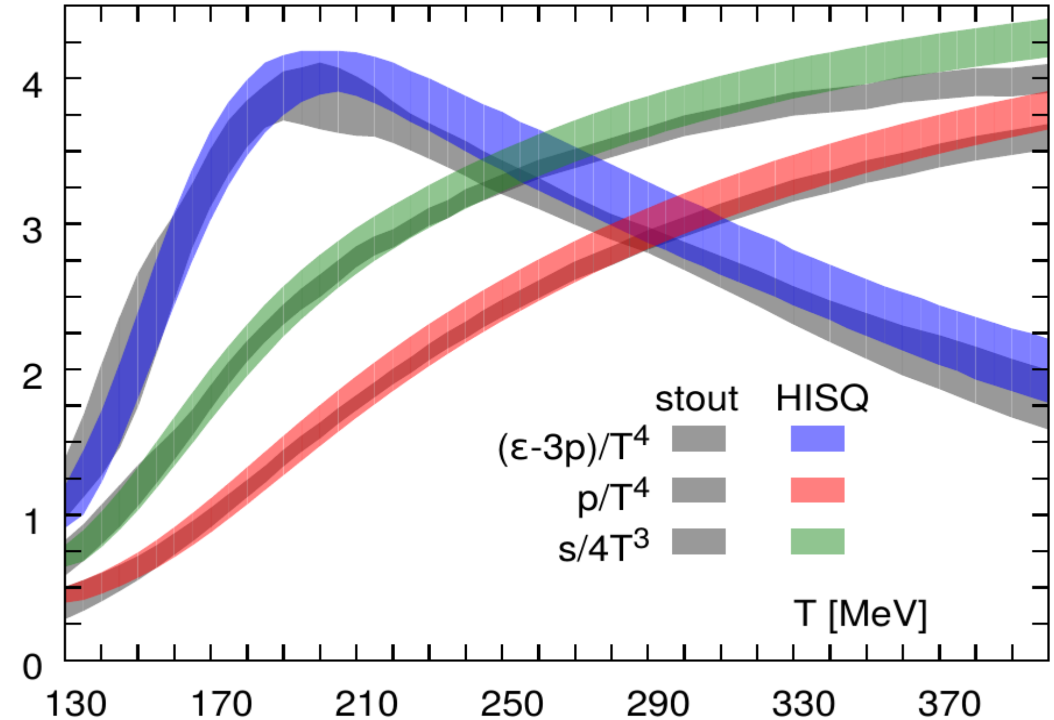
$$\boxed{\frac{p}{T^4} = \sum_{m \in meson} \ln Z_m^b(T, V, \mu) + \sum_{m \in baryon} \ln Z_m^f(T, V, \mu)}$$

$$\sim \mathrm{e}^{-m_H/T} \mathrm{e}^{(B\mu_B + S\mu_S + Q\mu_Q)/T}$$

Equation of state of (2+1)-flavor QCD: $\mu_B/T = 0$

pressure, **entropy** & **energy** density





A. Bazavov et al. (hotQCD) ,
Phys. Rev. D90 (2014) 094503

– improves over earlier hotQCD calculations: A. Bazavov et al., Phys. Rev. D80, 014504 (2009)

– consistent with results from Budapest-Wuppertal (stout): S. Borsanyi et al., PL B730, 99 (2014)

– up to the crossover region the QCD EoS agrees quite well with hadron resonance gas (HRG) model calculations; **However**, QCD results are systematically above HRG

Equation of state of (2+1)-flavor QCD: $\mu_B/T > 0$

$$\frac{P}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{i,j,k}^{BQS}(T) \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

the simplest case: $\mu_S = \mu_Q = 0$

$$\frac{P(T,\mu_B)}{T^4} = \frac{P(T,0)}{T^4} + \frac{\chi_2^B(T)}{2} \left(\frac{\mu_B}{T}\right)^2 + \frac{\chi_4^B(T)}{24} \left(\frac{\mu_B}{T}\right)^4 + \mathcal{O}((\mu_B/T)^6)$$

Hadron resonance gas (HRG) in Boltzmann approximation:

$$\frac{P(T,\mu_B)}{T^4} = \frac{P(T,0)}{T^4} + \sum_{m_i \in \mathrm{baryons}} f(m_i/T) \left(\cosh(\mu_B/T) - 1\right)$$

Equation of state of (2+1)-flavor QCD: $\mu_B/T > 0$

$$\frac{P}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{i,j,k}^{BQS}(T) \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

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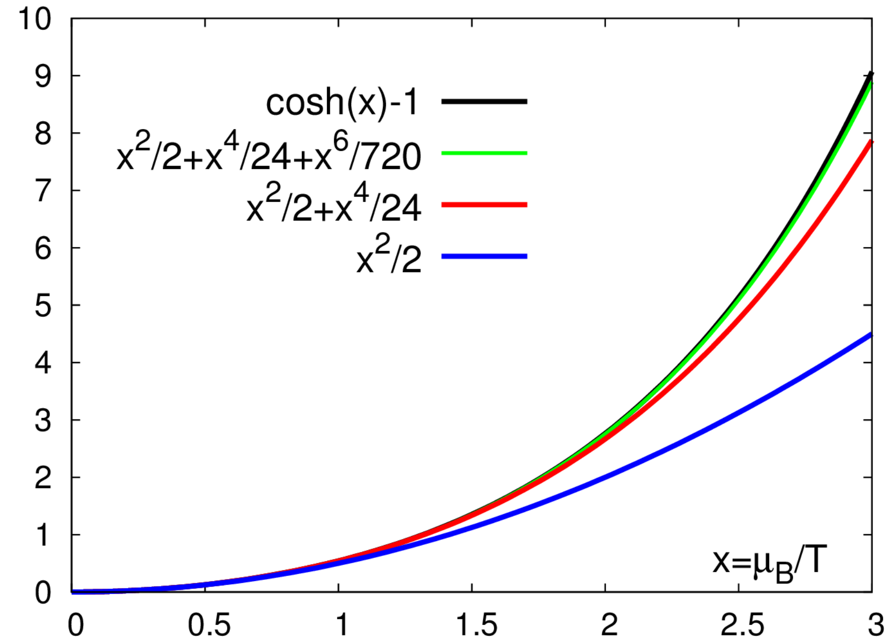
An $\mathcal{O}((\mu_B/T)^4)$ expansion is exact in a QGP up to $\mathcal{O}(g^2)$

HRG vs. QCD:

$\mathcal{O}((\mu_B/T)^4)$: difference is less than 3% at $\mu_B/T = 2$

$\mathcal{O}((\mu_B/T)^6)$: difference is less than 2% at $\mu_B/T = 3$



Equation of state of (2+1)-flavor QCD: $\mu_B/T > 0$

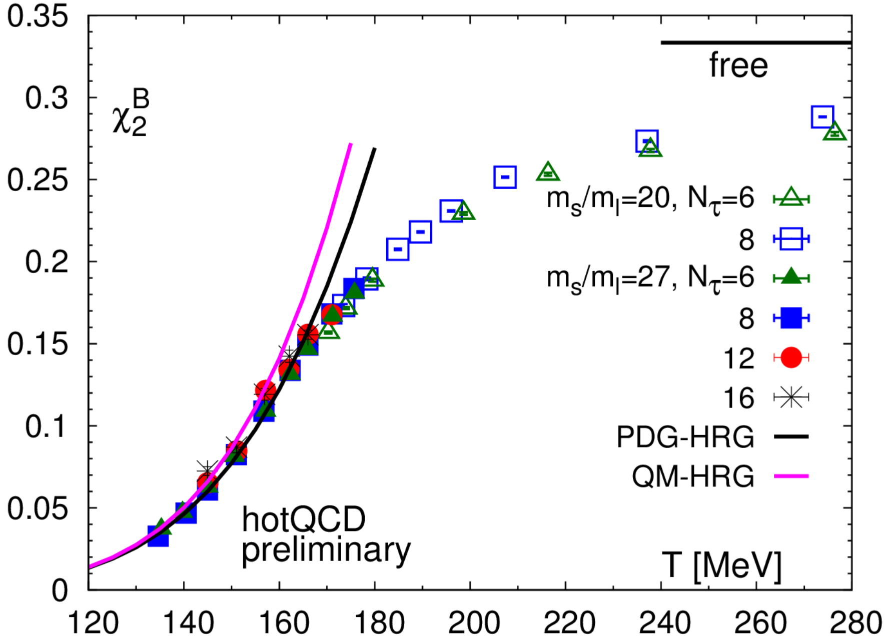
$$\frac{\Delta(T,\mu_B)}{T^4} = \frac{P(T,\mu_B)-P(T,0)}{T^4} = \frac{\chi_2^B}{2}\left(\frac{\mu_B}{T}\right)^2\left(1+\frac{1}{12}\frac{\chi_4^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^2\right)$$

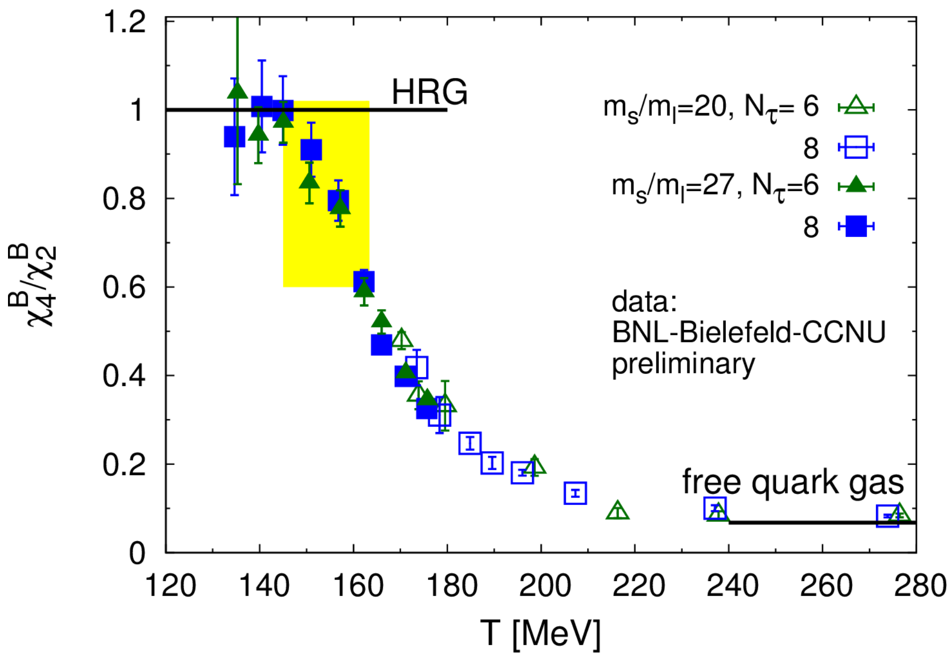
variance of net-baryon number distribution

kurtosis*variance

$\kappa_B\sigma_B^2$

P. Hegde (BNL-Bielefeld-CCNU), arXiV:1412.6727





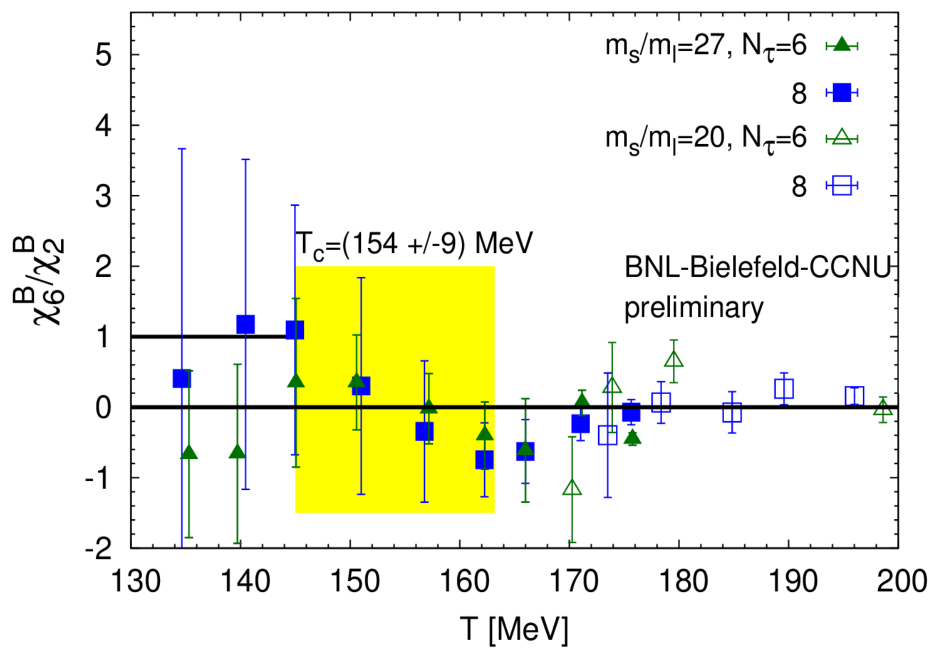
– leading and next-to-leading order corrections agree well with HRG for T<150 MeV

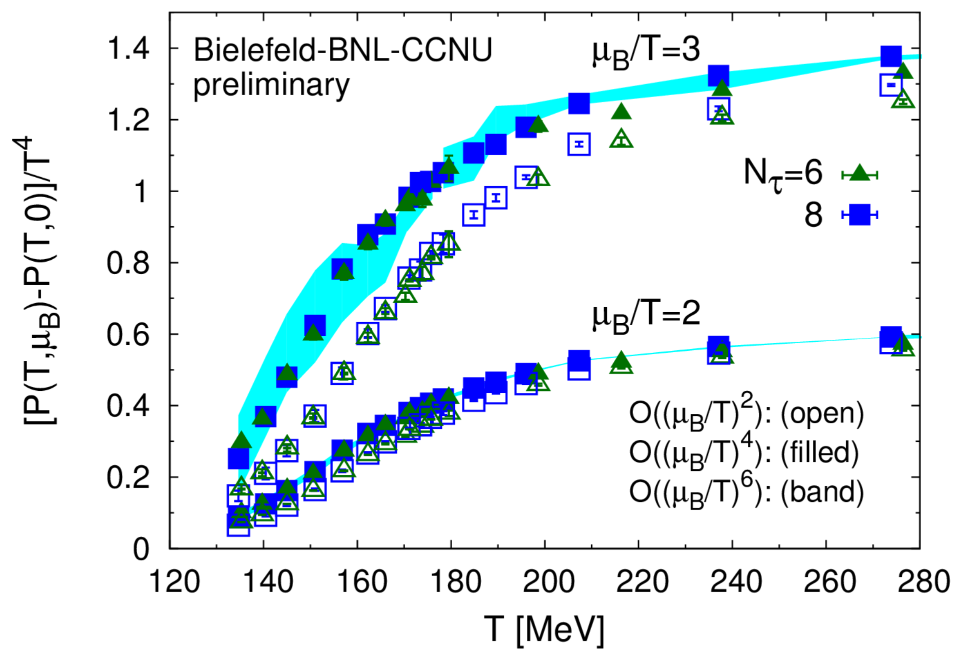
– already in the crossover region deviations from HRG can reach ~40% for T~165 MeV

Equation of state of (2+1)-flavor QCD: $\mu_B/T > 0$

$$\frac{\Delta(T,\mu_B)}{T^4} = \frac{P(T,\mu_B)-P(T,0)}{T^4} = \frac{\chi_2^B}{2}\left(\frac{\mu_B}{T}\right)^2\left(1+\frac{1}{12}\frac{\chi_4^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^2\right)$$

estimating the $\mathcal{O}((\mu_B/T)^6)$ correction: $\sim \frac{1}{720}\frac{\chi_6^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^6$



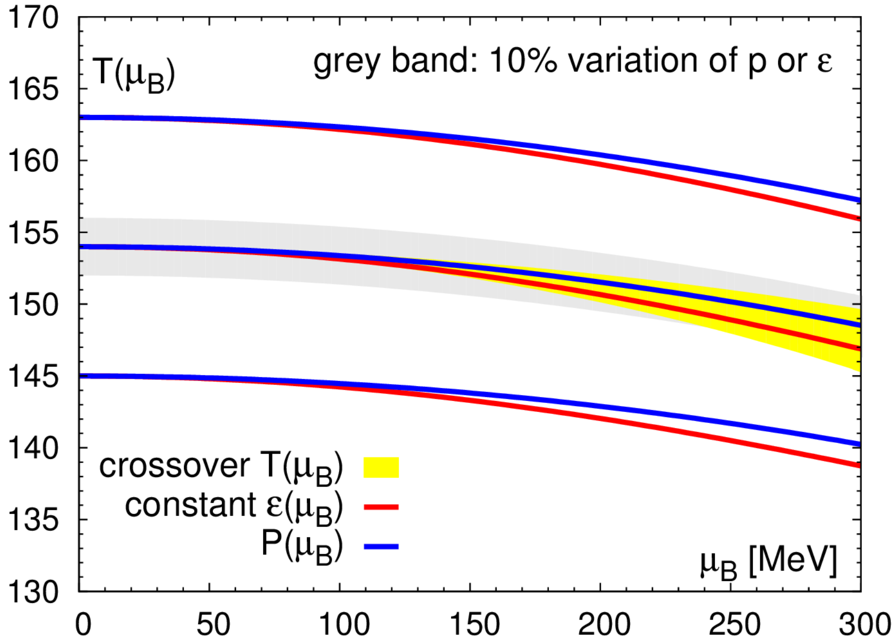


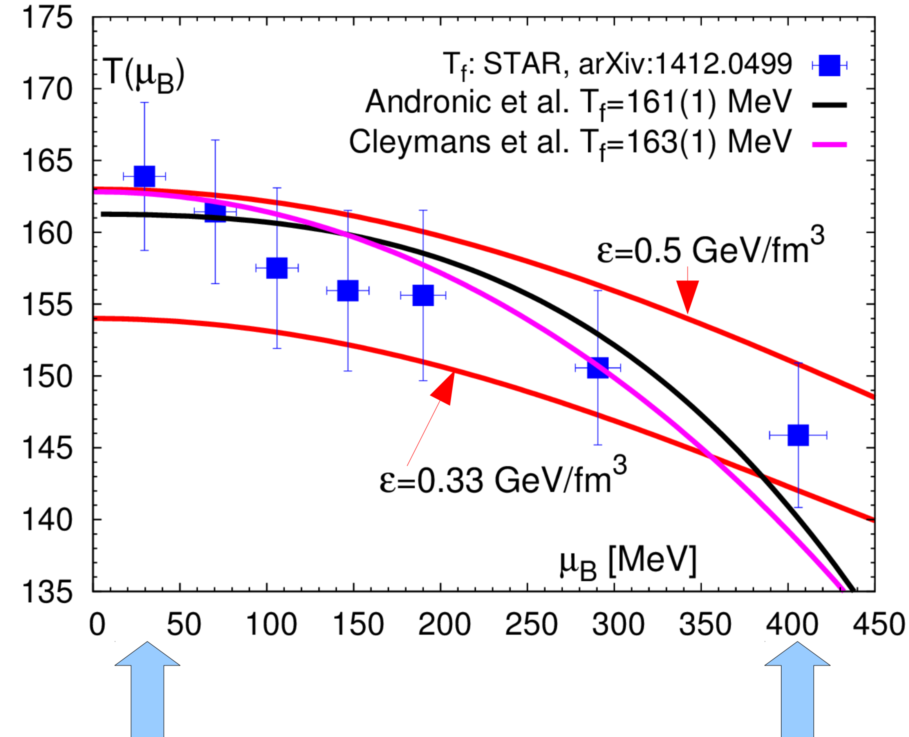
The EoS is well controlled for $\mu_B/T \leq 2$
or equivalently $\sqrt{s_{NN}} \geq 20$ GeV

P. Hegde (BNL-Bielefeld-CCNU),
arXiV:1412.6727

Lines of constant physics and freeze-out

$$T_f(\mu_B) = T_f(0)\left(1 - \kappa_{f,2}\hat{\mu}_B^2 - \kappa_{f,4}\hat{\mu}_B^4\right)$$





200 GeV $\sqrt{s_{NN}}$ **7.7 GeV**

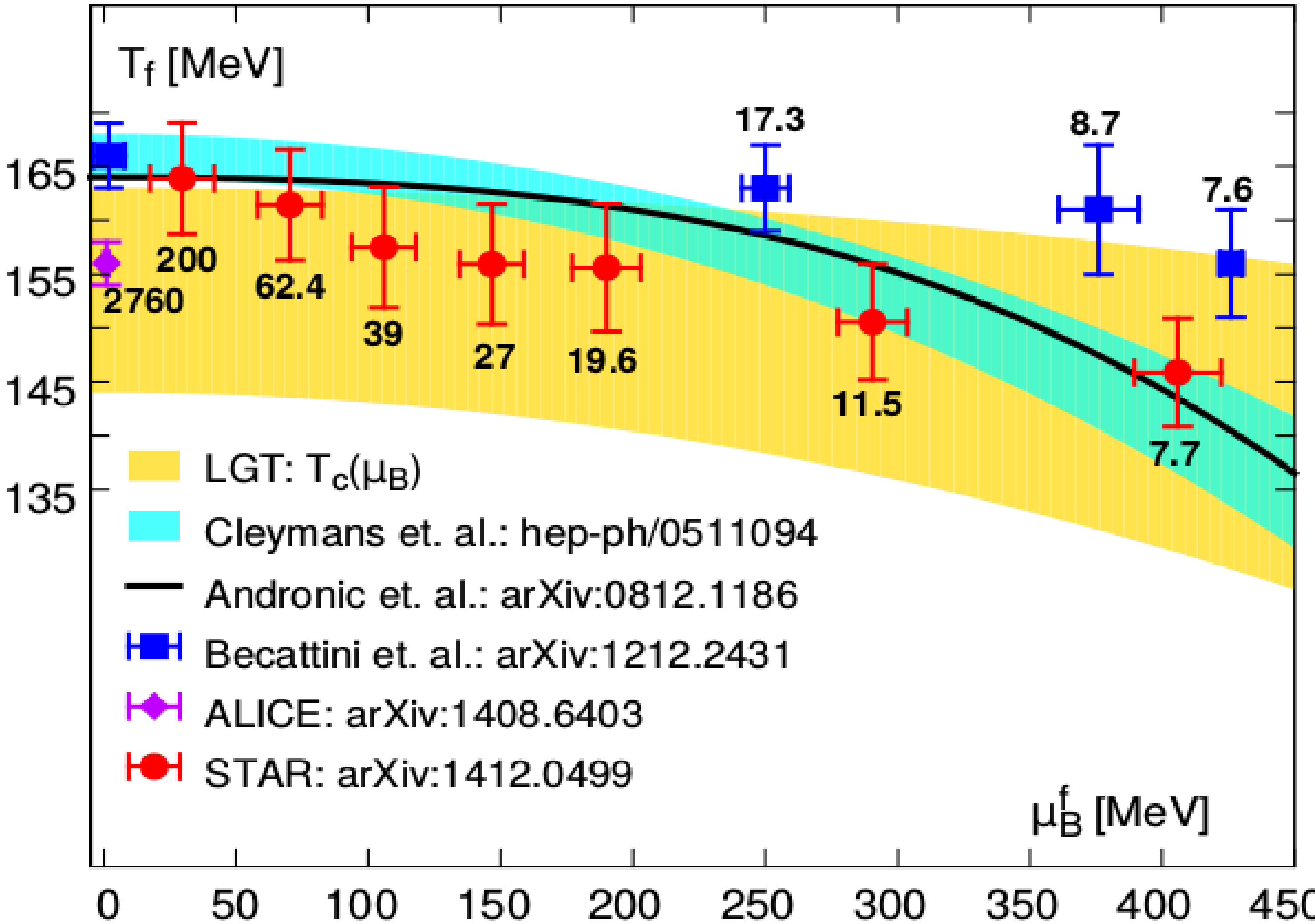
$\underline{\mu_Q = \mu_S = 0:}$

constant pressure: $\kappa_{2,p} \simeq 0.011$

constant energy density: $\kappa_{2,\epsilon} \simeq 0.013$

crossover line: $\kappa_{2,c} \simeq 0.006 - 0.013$

Chiral transition and freeze-out



phenomenological freeze-out / hadronization curve, QCD transition line and experimental data (obtained by assuming the validity of the HRG model) are consistent for

$$\mu_B/T \lesssim 3$$

HOWEVER

physics is quite different at lower and upper end of the current error bar on Tc

LGT: $T_c(\mu_B) = 154(9)(1 - 0.0066(7)(\mu_B/T)^2)\mathrm{MeV}$

Chiral transition and freeze-out

– chiral transition temperature: Tc= 154 (8) (1) MeV

scale uncertainty

statistical uncertainty

– error band on Tc is mainly statistical;
physics is quite different at lower and upper end of the current error bar

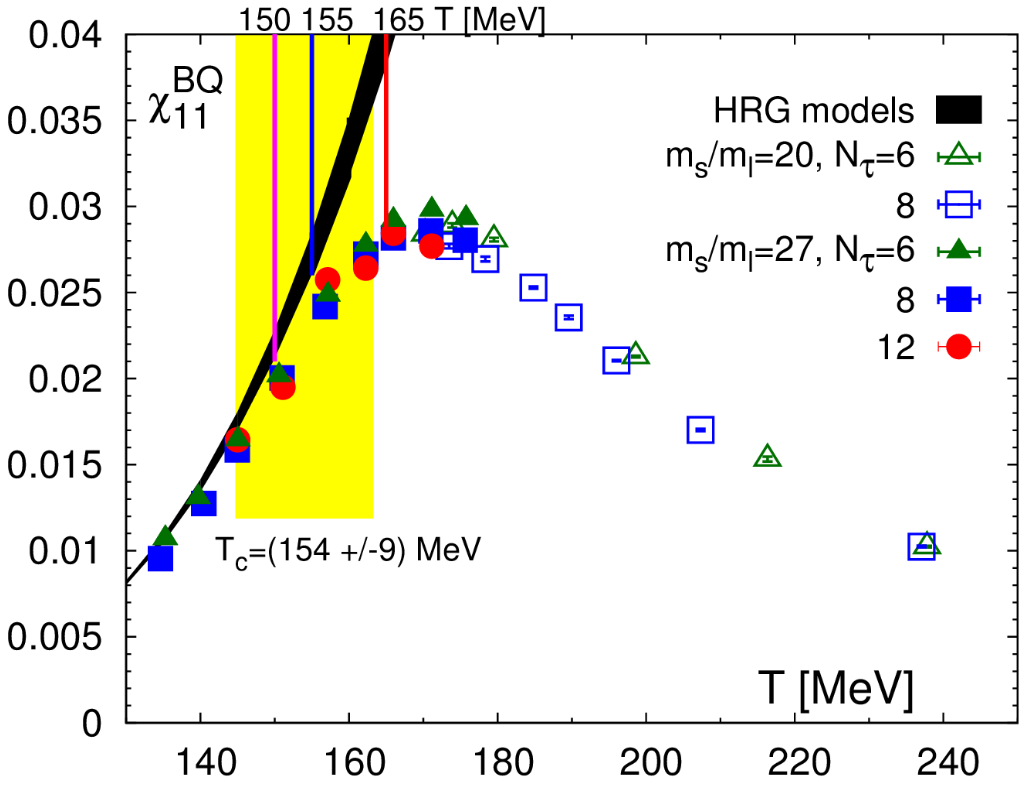
baryon number – electric charge correlations

$T \simeq 150$ MeV :

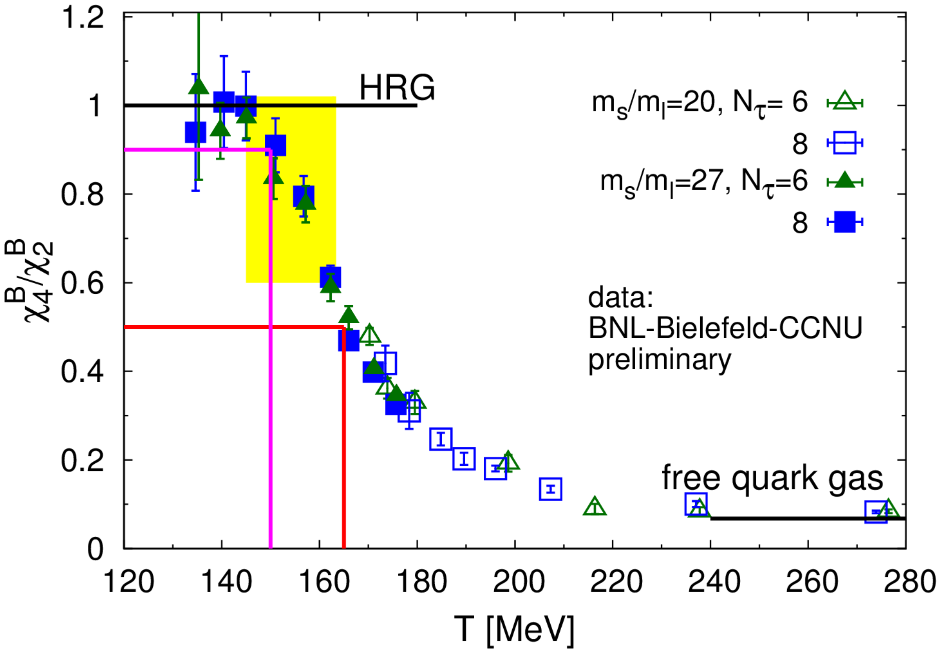
correlations **similar to HRG**

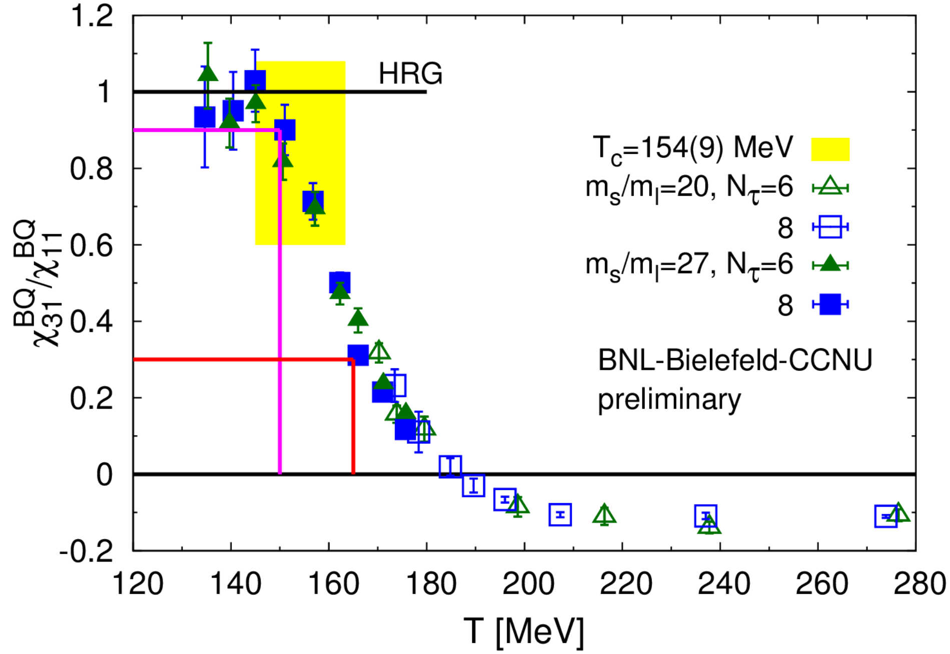
$T \simeq 165$ MeV :

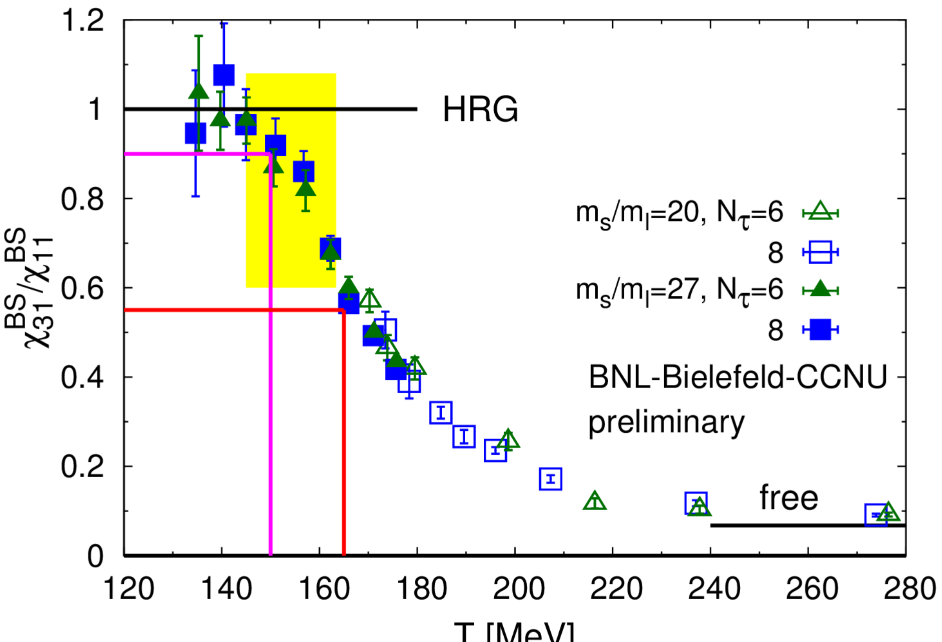
correlations **very different from HRG**

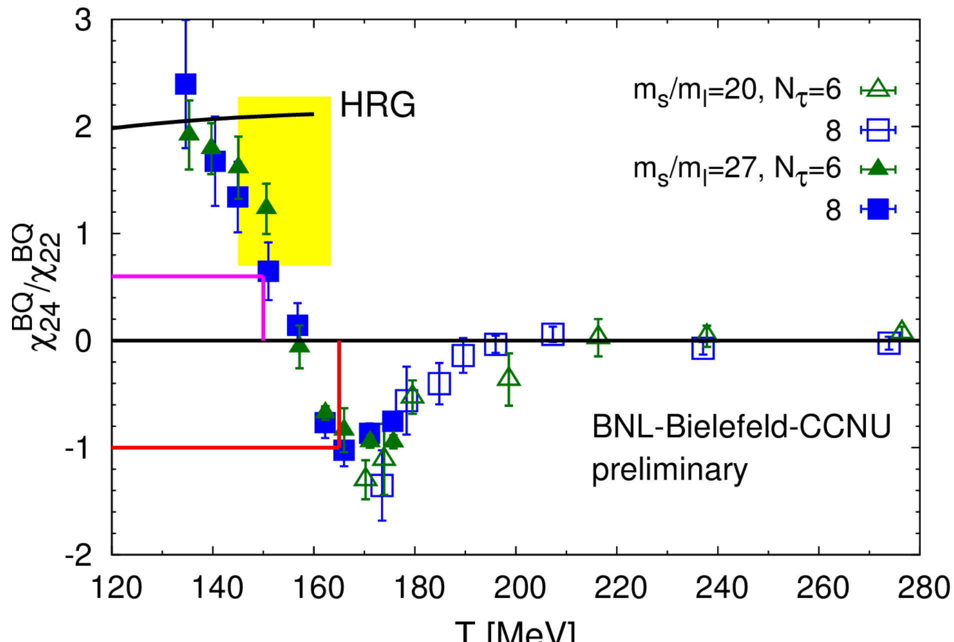


Some 4th and 6th order cumulants

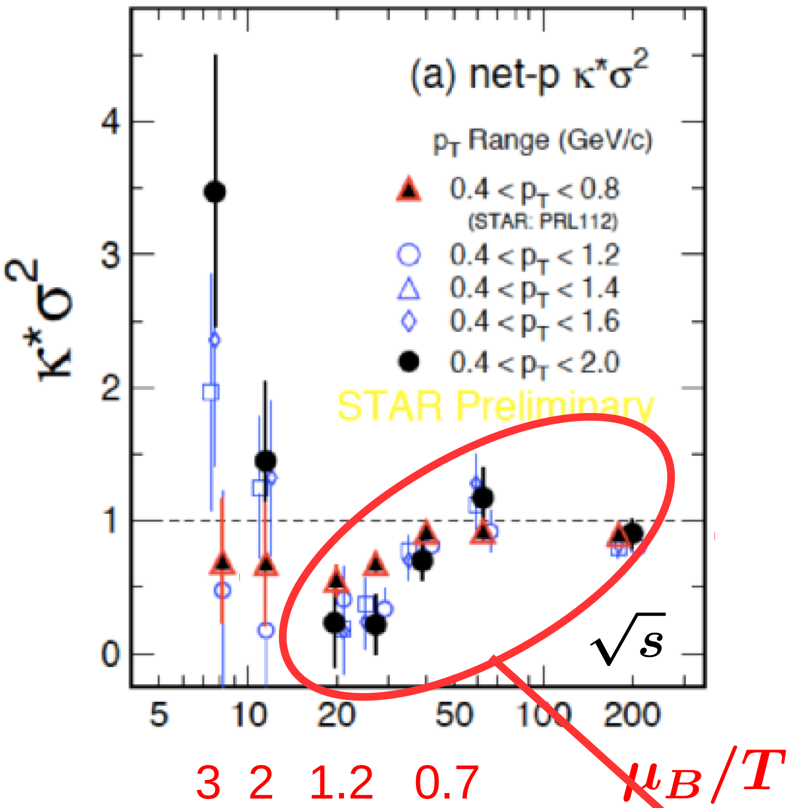








Exploring the QCD phase diagram



More moderate questions:

- Can we understand the systematics seen in cumulants of charge fluctuations in terms of QCD thermodynamics ?
- How far do we get with low order Taylor expansions of **QCD** in explaining the obvious deviations from HRG model behavior ?

?

- For $\sqrt{s} \geq \mathbf{20\ GeV}$ **:** Structure of (net-electric charge and) net-proton cumulants is inconsistent with HRG thermodynamics, but can eventually be understood in terms of **QCD thermodynamics in a next-to-leading order Taylor expansion**

Conserved charge fluctuations and freeze-out

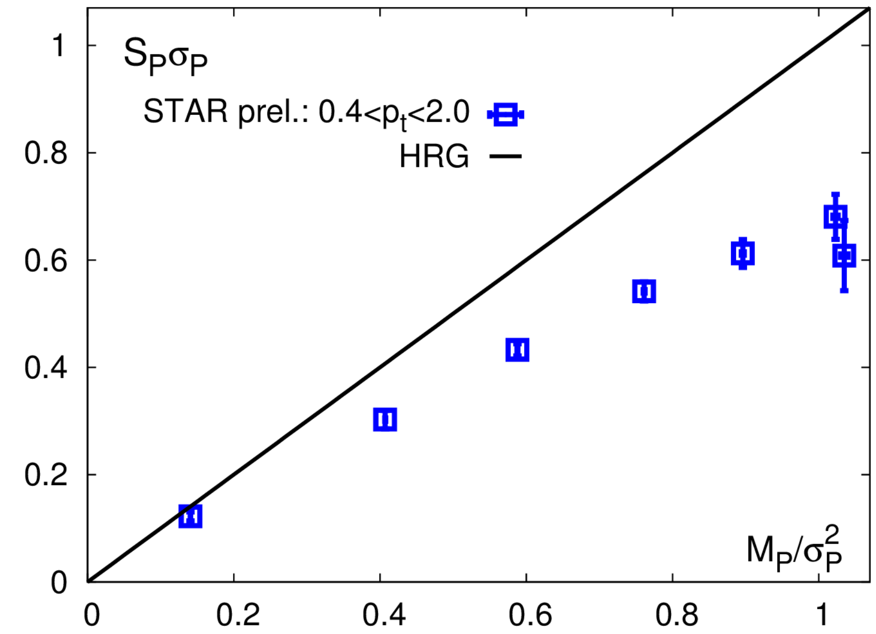
$$\frac{\Delta(T,\mu_B)}{T^4} = \frac{P(T,\mu_B)-P(T,0)}{T^4} = \frac{\chi_2^B}{2}\left(\frac{\mu_B}{T}\right)^2\left(1+\frac{1}{12}\frac{\chi_4^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^2\right)$$

$$\frac{M_B}{\sigma_B^2} = \frac{\mu_B}{T} + \mathcal{O}(\mu_B^3)$$

$$S_B\sigma_B = \frac{\mu_B}{T}\frac{\chi_4^B}{\chi_2^B} + \mathcal{O}(\mu_B^3)$$

no need for talking about a chemical potential

$$\boxed{S_B\sigma_B = \frac{M_B}{\sigma_B^2}\frac{\chi_4^B}{\chi_2^B} + \mathcal{O}(\mu_B^3)}$$



$$\frac{\chi_4^B}{\chi_2^B} < 1$$

slope < 1

Conserved charge fluctuations and freeze-out

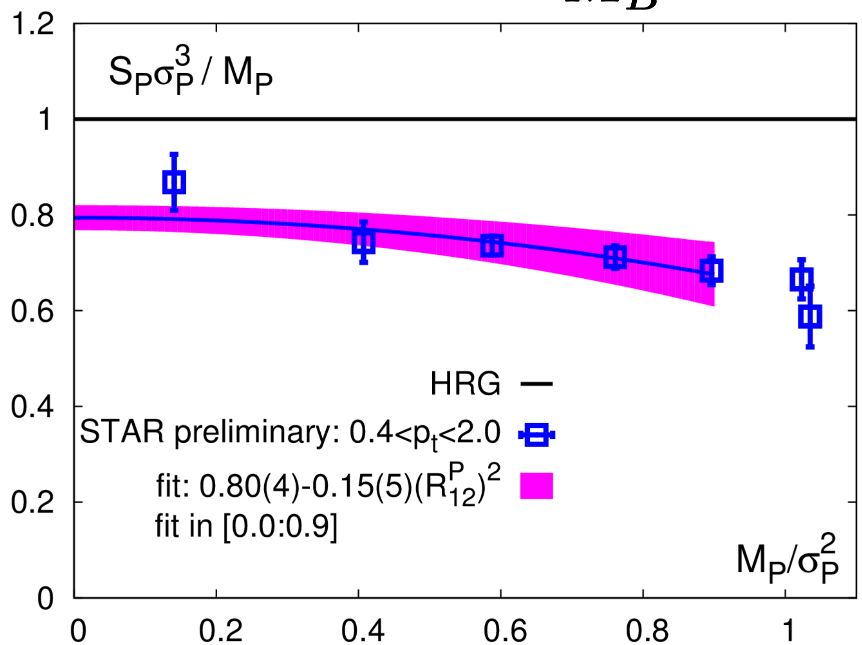
mean, variance and skewness

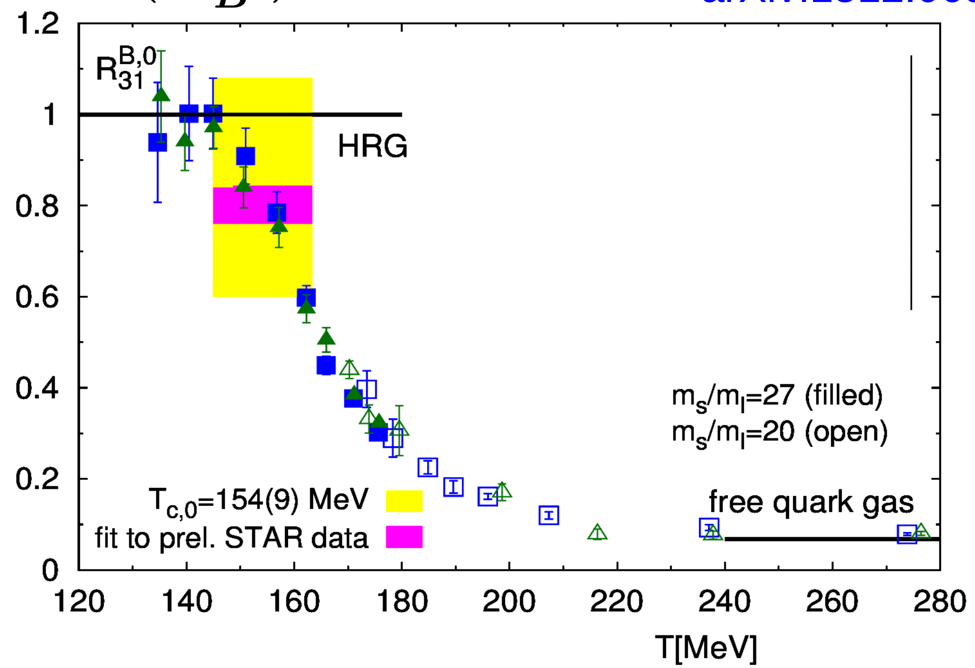
NLO Taylor expansion

$$S_B\sigma_B = \frac{\chi_4^B}{\chi_2^B}\frac{M_B}{\sigma_B^2} + \frac{1}{6}\left(\frac{\chi_6^B}{\chi_2^B} - \left(\frac{\chi_4^B}{\chi_2^B}\right)^2\right)\left(\frac{M_B}{\sigma_B^2}\right)^3 + \ldots \qquad \mu_Q = \mu_S = 0$$

$$\Leftrightarrow \quad R_{31}^B \equiv \frac{S_B\sigma_B^3}{M_B} = R_{31}^{B,0} + R_{31}^{B,2}\left(\frac{M_B}{\sigma_B^2}\right)^2$$

F. Karsch et al., arXiv:1512.06987





$$\text{e.g.,}\quad \mu_Q \neq \mu_S \neq 0: \quad R_{31}^{B,0} = \frac{\chi_4^B + \frac{\mu_S}{\mu_B}\chi_{31}^{BS} + \frac{\mu_Q}{\mu_B}\chi_{31}^{BQ}}{\chi_2^B + \frac{\mu_S}{\mu_B}\chi_{11}^{BS} + \frac{\mu_Q}{\mu_B}\chi_{11}^{BQ}}\ , \quad R_{31}^{B,2} = \ldots$$

Conserved charge fluctuations and freeze-out

mean, variance and skewness

NLO Taylor expansion

$$S_B\sigma_B = \frac{\chi_4^B}{\chi_2^B}\frac{M_B}{\sigma_B^2} + \frac{1}{6}\left(\frac{\chi_6^B}{\chi_2^B} - \left(\frac{\chi_4^B}{\chi_2^B}\right)^2\right)\left(\frac{M_B}{\sigma_B^2}\right)^3 + \dots$$

$$\Leftrightarrow \quad R_{31}^B \equiv \frac{S_B\sigma_B^3}{M_B} = R_{31}^{B,0} + R_{31}^{B,2}\left(\frac{M_B}{\sigma_B^2}\right)^2$$

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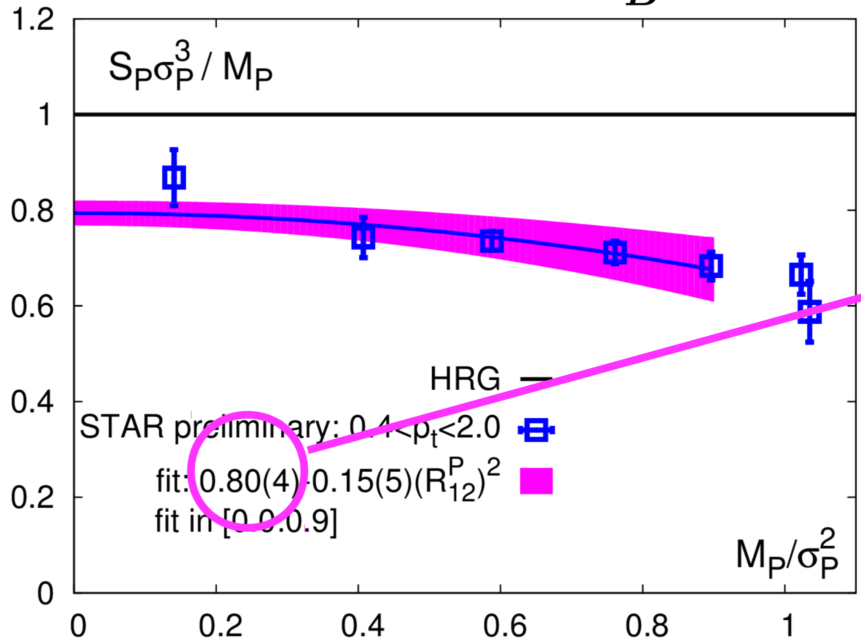
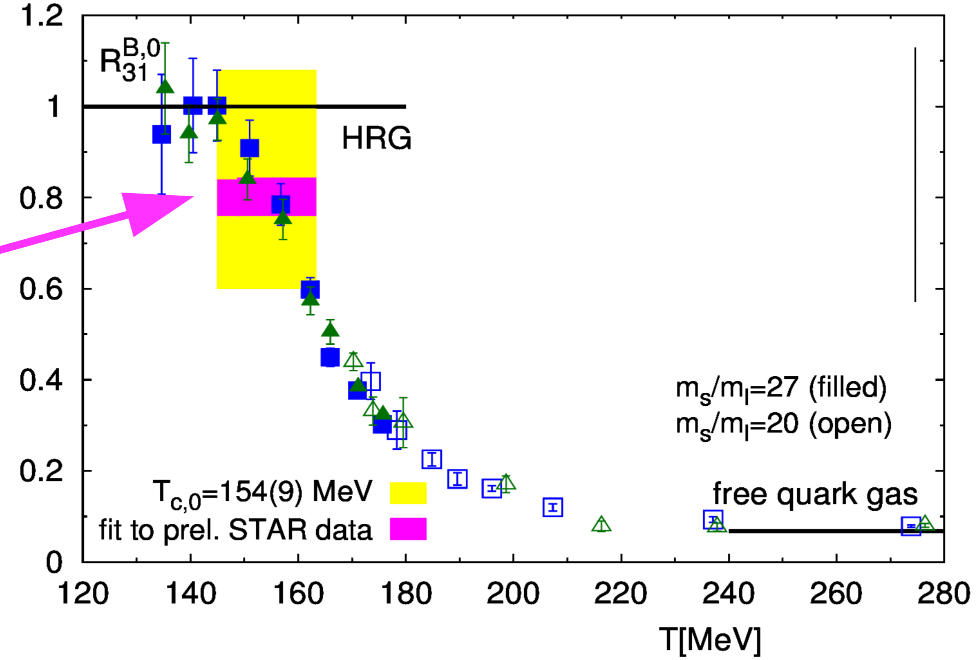




– intercept consistent with QCD result,

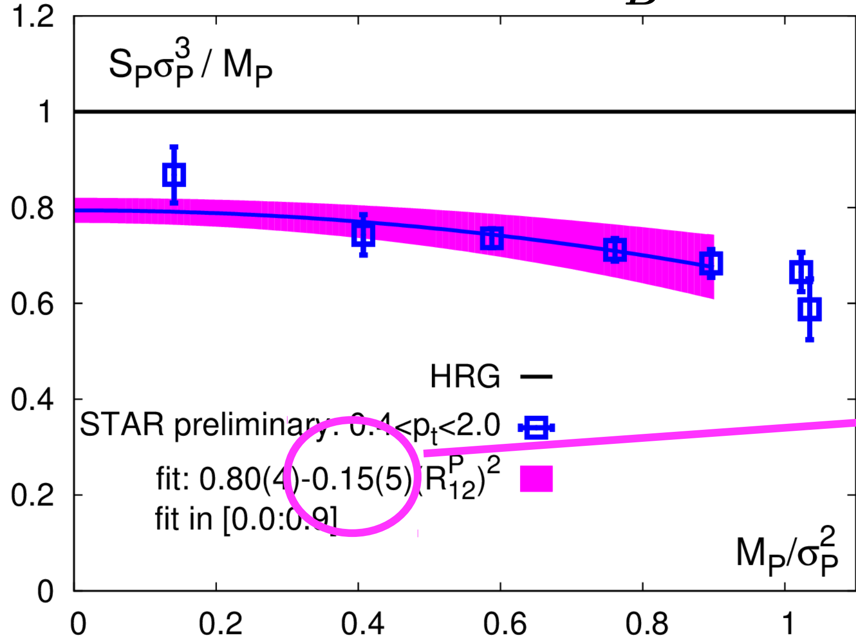
Conserved charge fluctuations and freeze-out
mean, variance and skewness

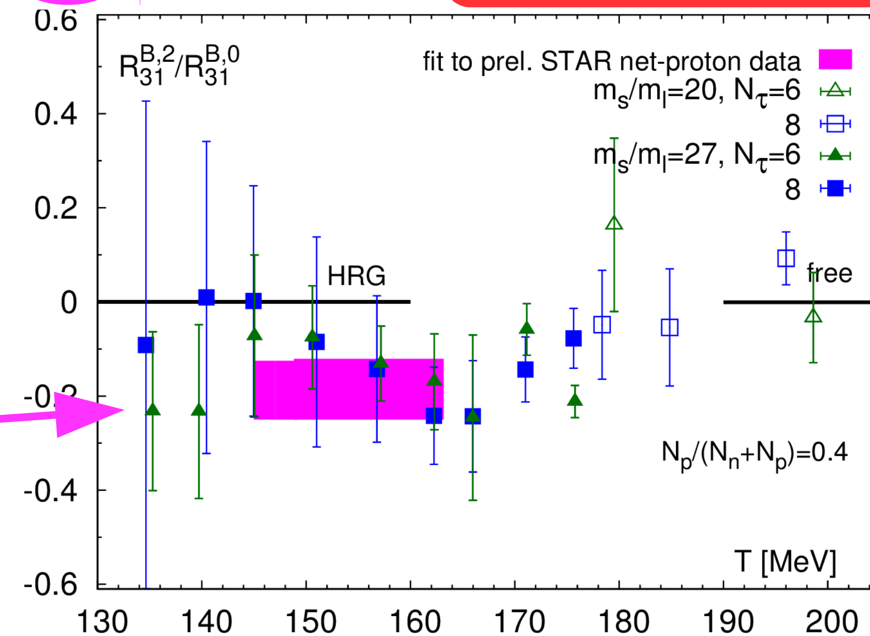
NLO Taylor expansion

$$S_B\sigma_B = \frac{\chi_4^B}{\chi_2^B}\frac{M_B}{\sigma_B^2} + \frac{1}{6}\left(\frac{\chi_6^B}{\chi_2^B} - \left(\frac{\chi_4^B}{\chi_2^B}\right)^2\right)\left(\frac{M_B}{\sigma_B^2}\right)^3 + \dots$$

$$R_{31}^B \equiv \frac{S_B\sigma_B^3}{M_B} = R_{31}^{B,0} + R_{31}^{B,2}\left(R_{12}^B\right)^2$$

lattice QCD calculation involves 6th order cumulants





– intercept consistent with QCD result,

– curvature consistent with QCD result (still noisy, coarse lattice)

Conserved charge fluctuations and freeze-out
mean, variance, skewness and kurtosis

in a NLO Taylor expansion $R_{31}^B \equiv S_B\sigma_B^3/M_B$, $R_{42}^B \equiv \kappa_B\sigma_B^2$ are closely related

F. Karsch et al., arXiv:1512.06987

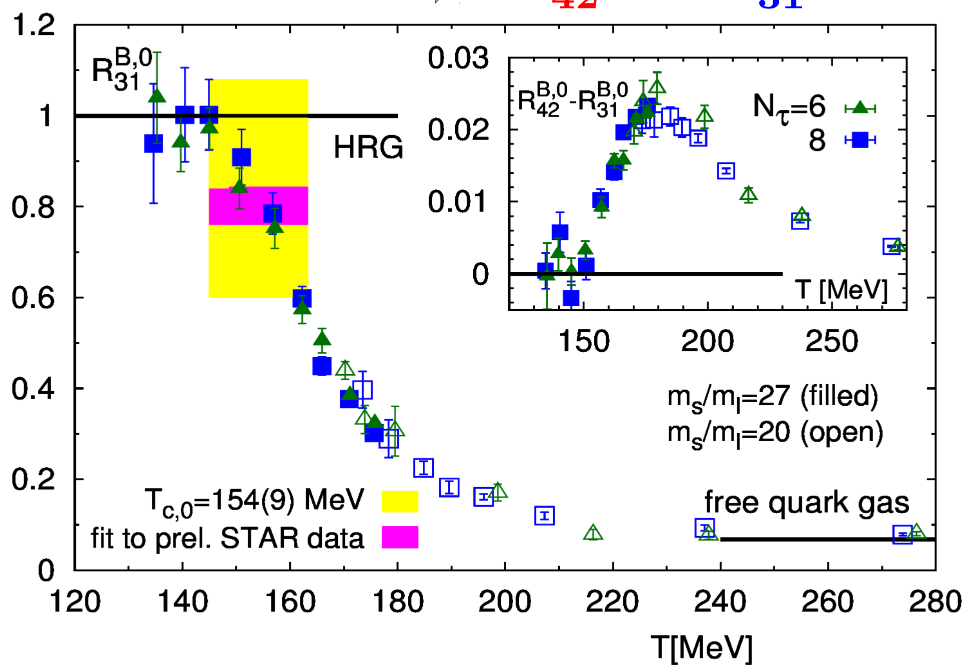
$$R_{31}^B = R_{31}^{B,0} + R_{31}^{B,2}\left(\frac{\mu_B}{T}\right)^2$$
$$R_{42}^B = R_{42}^{B,0} + R_{42}^{B,2}\left(\frac{\mu_B}{T}\right)^2$$

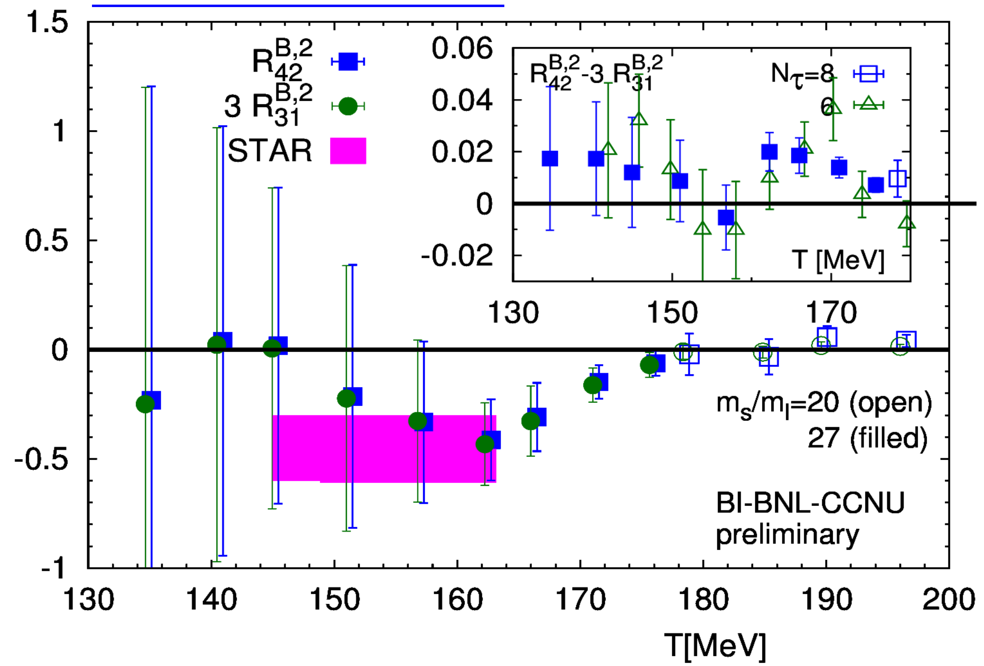
$\Rightarrow$ $R_{42}^{B,0} \simeq R_{31}^{B,0}$

$\mu_S = \mu_Q = 0:$

$$R_{42}^{B,2} = 3R_{31}^{B,2} = \frac{1}{2}\left(\frac{\chi_6^B}{\chi_2^B} - \left(\frac{\chi_4^B}{\chi_2^B}\right)^2\right)$$

$\mu_S \neq \mu_Q \neq 0:$





Conserved charge fluctuations and freeze-out
mean, variance, skewness and kurtosis

in a NLO Taylor expansion $R_{31}^B \equiv S_B \sigma_B^3 / M_B$, $R_{42}^B \equiv \kappa_B \sigma_B^2$ **are closely related**

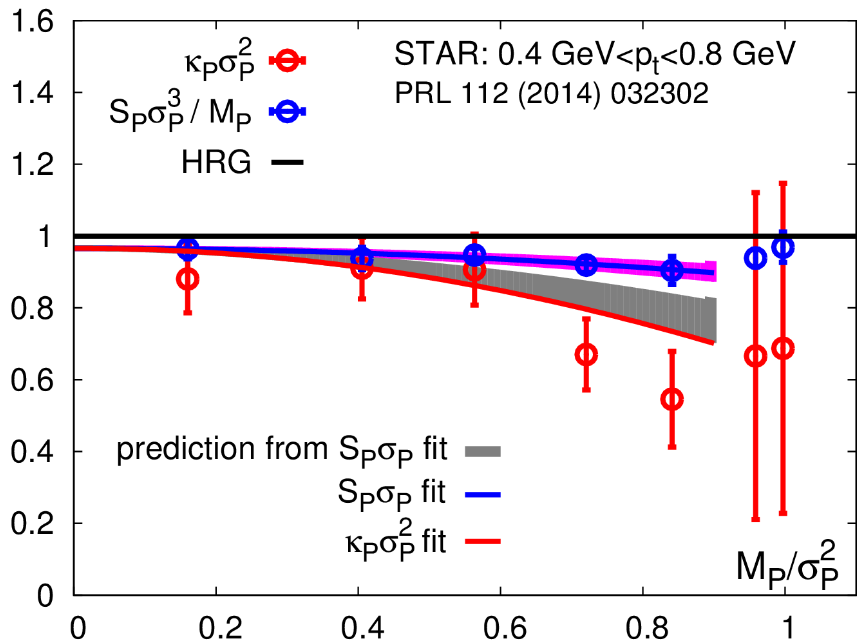
F. Karsch et al., arXiv:1512.06987

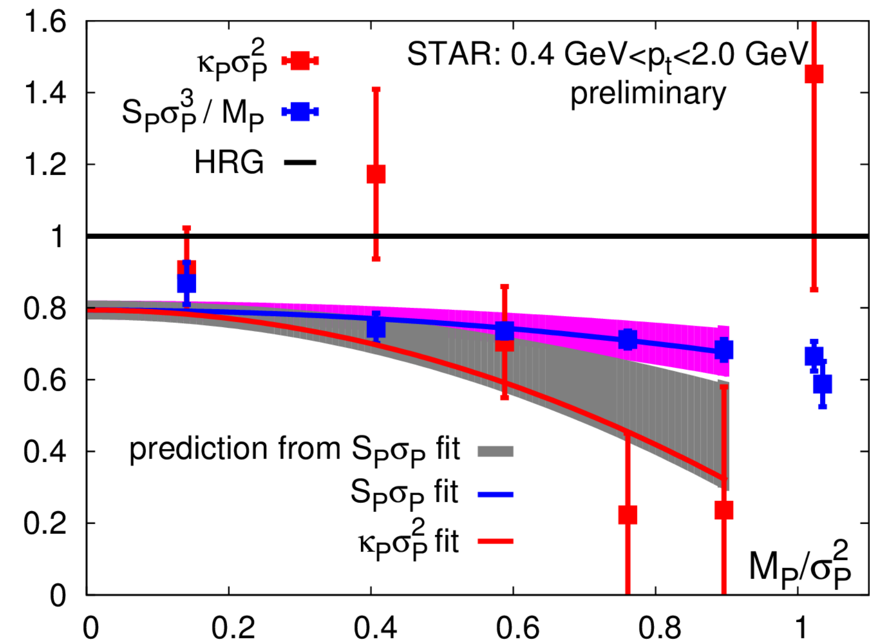
$$R_{31}^B = R_{31}^{B,0} + R_{31}^{B,2} \left(\frac{\mu_B}{T}\right)^2$$

$$R_{42}^B = R_{42}^{B,0} + R_{42}^{B,2} \left(\frac{\mu_B}{T}\right)^2$$

$\underline{\mu_S = \mu_Q = 0:}$

$$R_{42}^{B,2} = 3 R_{31}^{B,2} = \frac{1}{2} \left(\frac{\chi_6^B}{\chi_2^B} - \left(\frac{\chi_4^B}{\chi_2^B}\right)^2 \right)$$

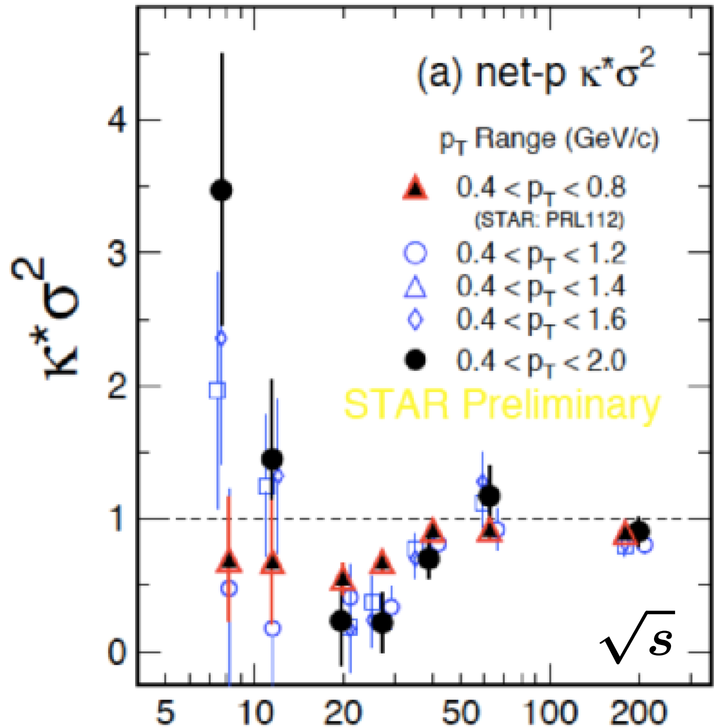




Constraining the location of the critical point

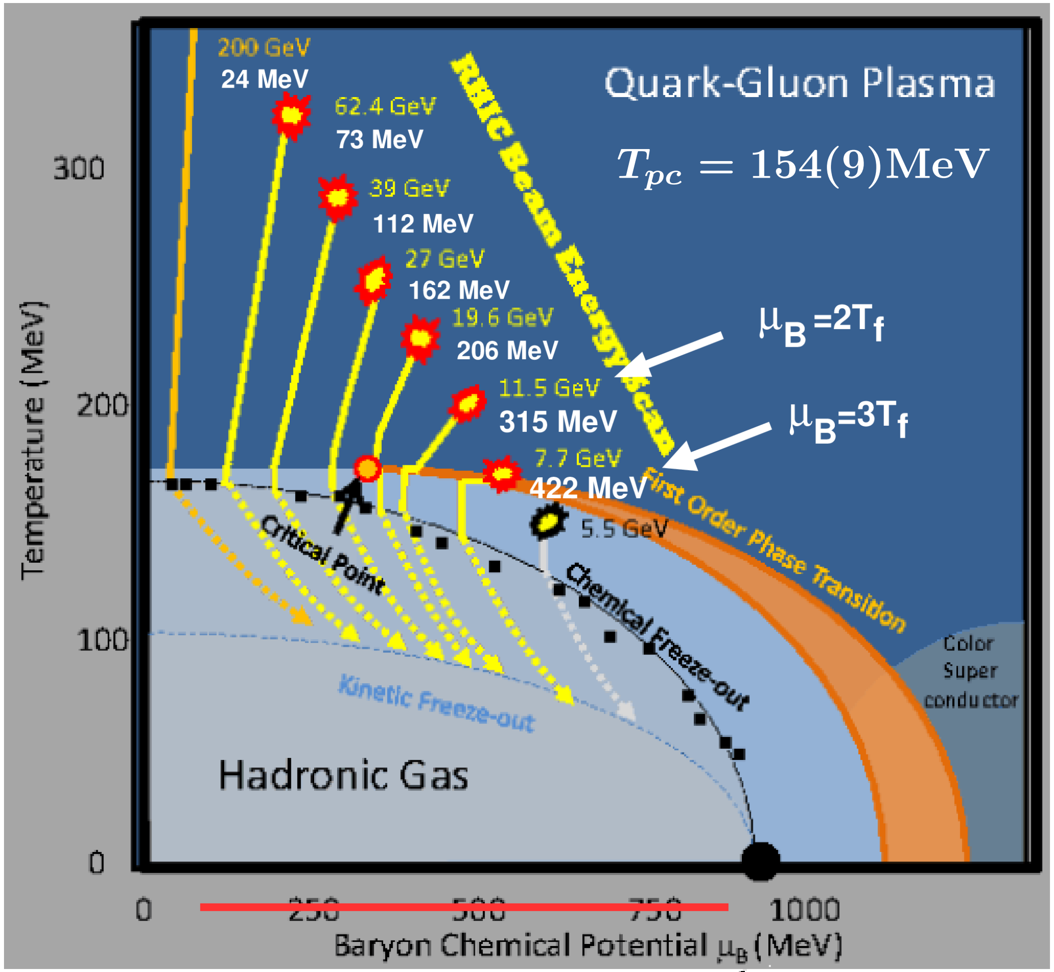
kurtosis*variance:

$$(\kappa\sigma^2)_X = \frac{\chi_4^X}{\chi_2^X}$$

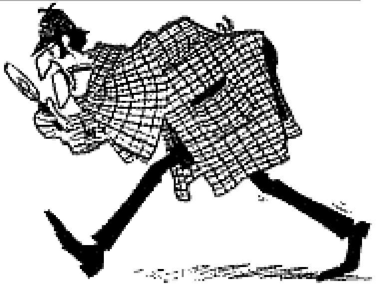


3 2 1.2 0.7 μ_B/T

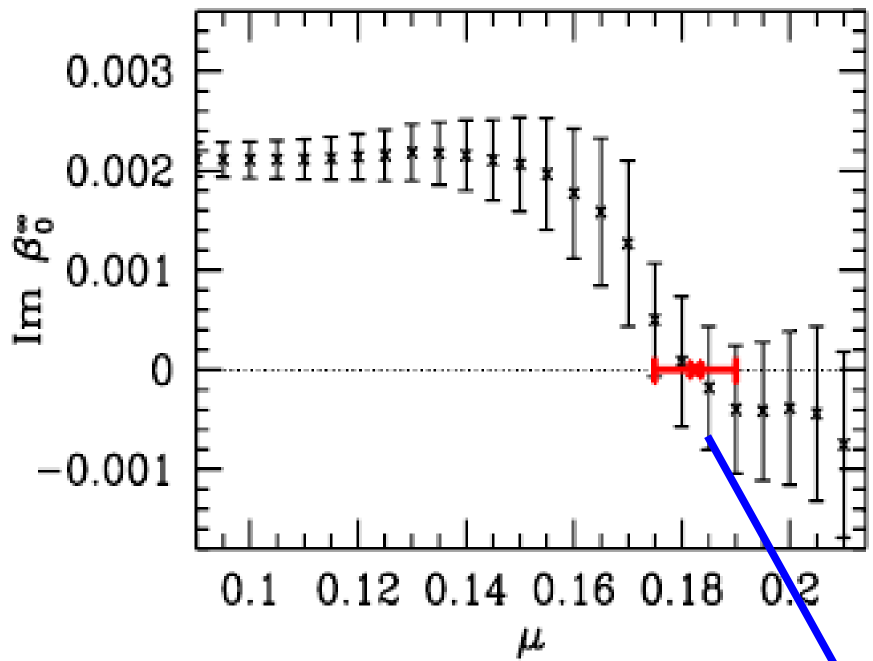
X. Luo (STAR Collaboration), PoS CPOD2014 (2014) 019



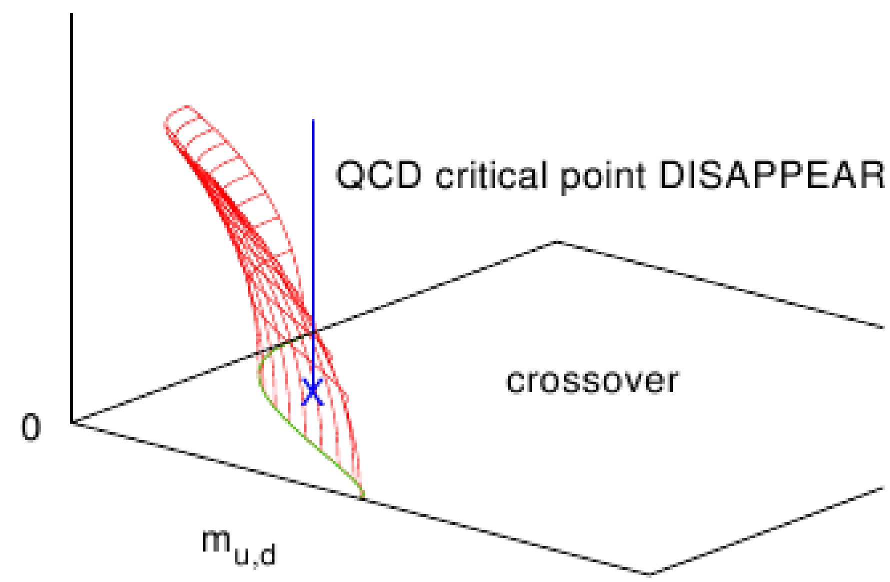
Where is the critical point?



LGT attempts to find the critical point



Z. Fodor, S. Katz. 2001, 2004



P. deForcrand, O. Philipsen, 2002

these calculations were possible because
(I) the lattices were coarse,
(II) the discretization schemes were crude

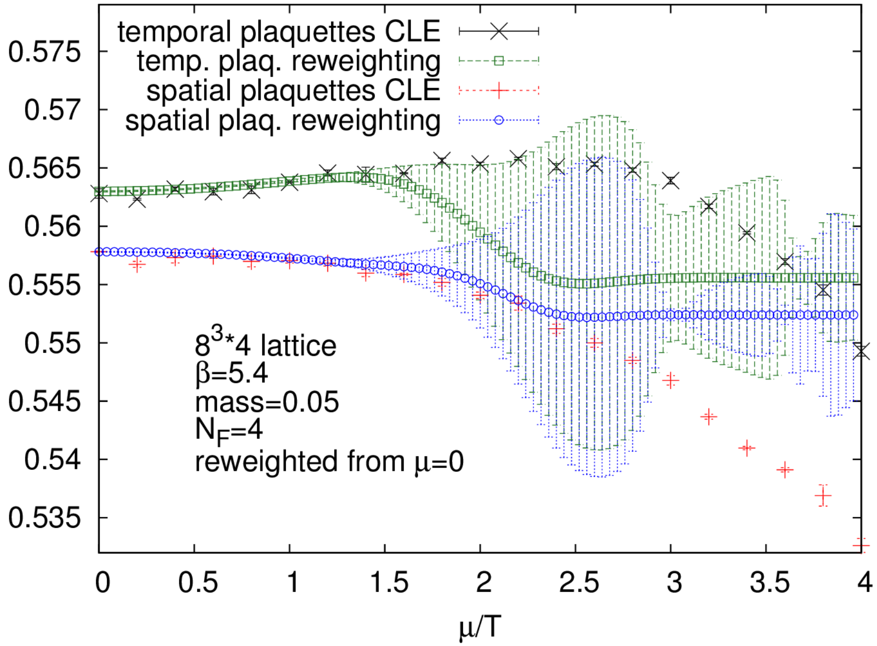
critical point or breakdown of the reweighting approach (loosing the overlap) ?

S. Ejiri, PRD69, 094506 (2004)

since 10 years no progress along this line

Complex Langevin vs. Reweighting

– the silent death of the Fodor/Katz critical point ? –



Z. Fodor, S. Katz. D Sexty, C. Torok, Phys. Rev. D 92 (2015) 094516

from Conclusion:

...reweighting from zero μ breaks down because of the overlap and sign problems around

$$\frac{\mu}{T} = 1 - 1.5$$

i.e. $$\frac{\mu_B}{T} = 3 - 4.5$$

this should be compared to the first Fodor/Katz critical point estimate on lattices with comparable parameters:

$$\frac{\mu_B^{crit}}{T} = 4.5(3)$$

Z. Fodor, S. Katz. JHEP 0203 (2002) 014

(calculations with physical quark masses eventually lead to a twice smaller estimate for the critical chemical potential)

Taylor expansion of the pressure and critical point

$$\frac{P}{T^4} = \sum_{n=0}^{\infty} \frac{1}{n!} \chi_n^B(T) \left(\frac{\mu_B}{T}\right)^n$$

for simplicity : $\mu_Q = \mu_S = 0$

estimator for the radius of convergence:

$$\left(\frac{\mu_B}{T}\right)^{\chi}_{crit,n} \equiv r_n^{\chi} = \sqrt{\left|\frac{n(n-1)\chi_n^B}{\chi_{n+2}^B}\right|}$$

– radius of convergence corresponds to a critical point only, iff

$$\chi_n > 0 \text{ for all } n \geq n_0$$

forces P/T^4 and $\chi_n^B(T, \mu_B)$ to be monotonically growing with μ_B/T

$\Updownarrow$

at T_{CP} : $\kappa_B \sigma_B^2 = \dfrac{\chi_4^B(T, \mu_B)}{\chi_2^B(T, \mu_B)} > 1$

if not:

– radius of convergence does not determine the critical point

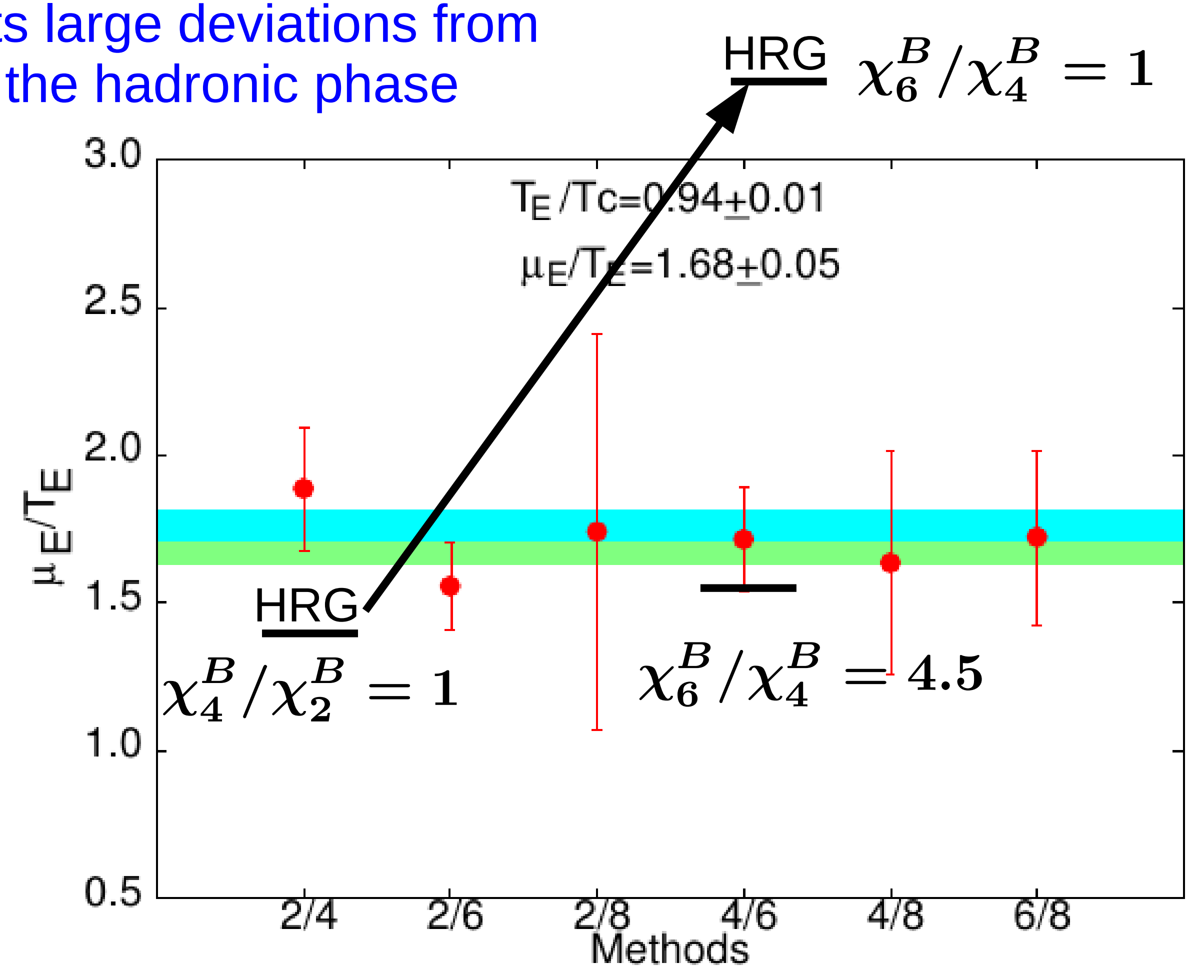
– Taylor expansion can not be used close to the critical point

Estimates of the radius of convergence

a challenging prediction from susceptibility series for standard staggered fermions:

$$\left(\frac{\mu_B}{T}\right)^{\chi}_{crit,n} \equiv r_n^{\chi} = \sqrt{\left|\frac{n(n-1)\chi_n^B}{\chi_{n+2}^B}\right|}$$

suggests large deviations from HRG in the hadronic phase



huge deviations from HRG in 6th order cumulants!

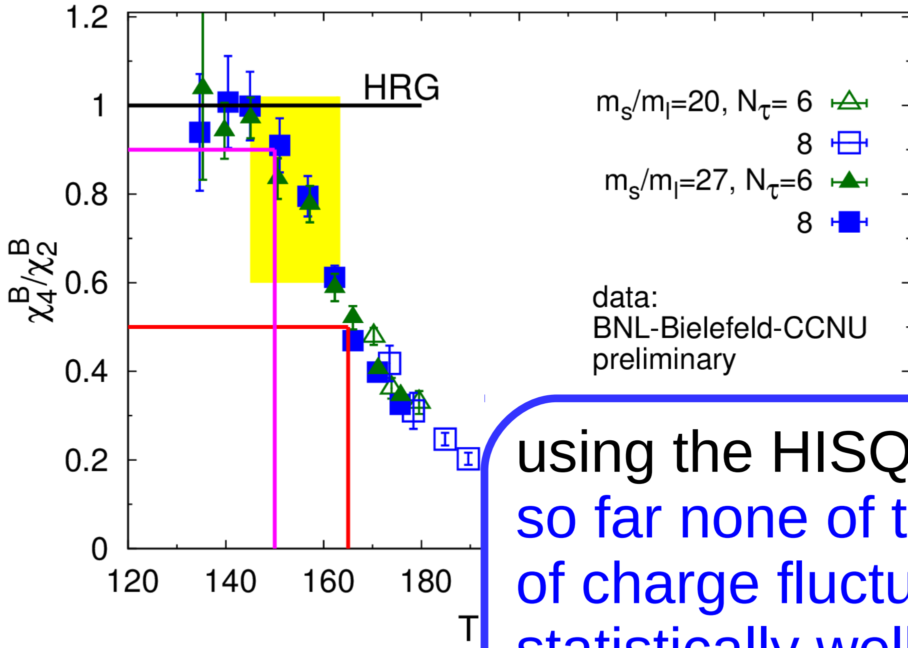
S. Datta et al., PoS Lattice2013 (2014) 202

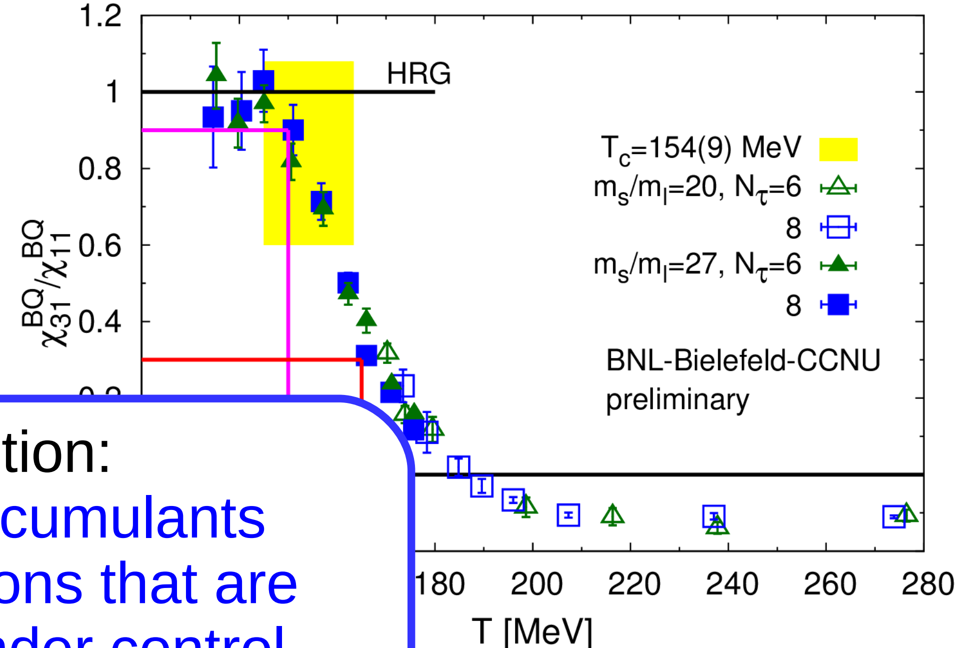
suggests a critical point for $\mu_B/T < 2$

at present, we cannot rule it out!

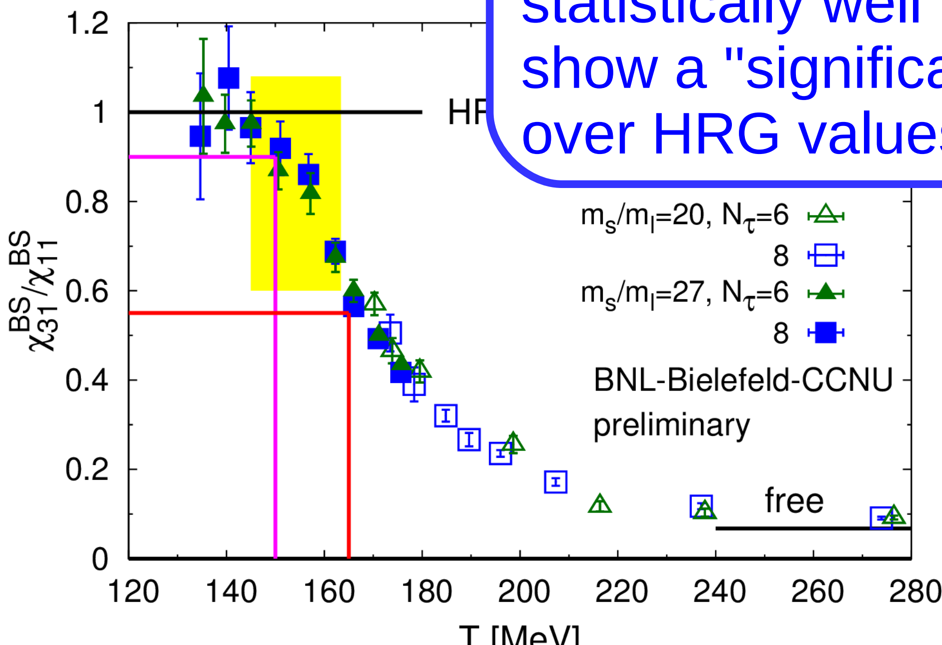
BNL-Bielefeld-CCNU

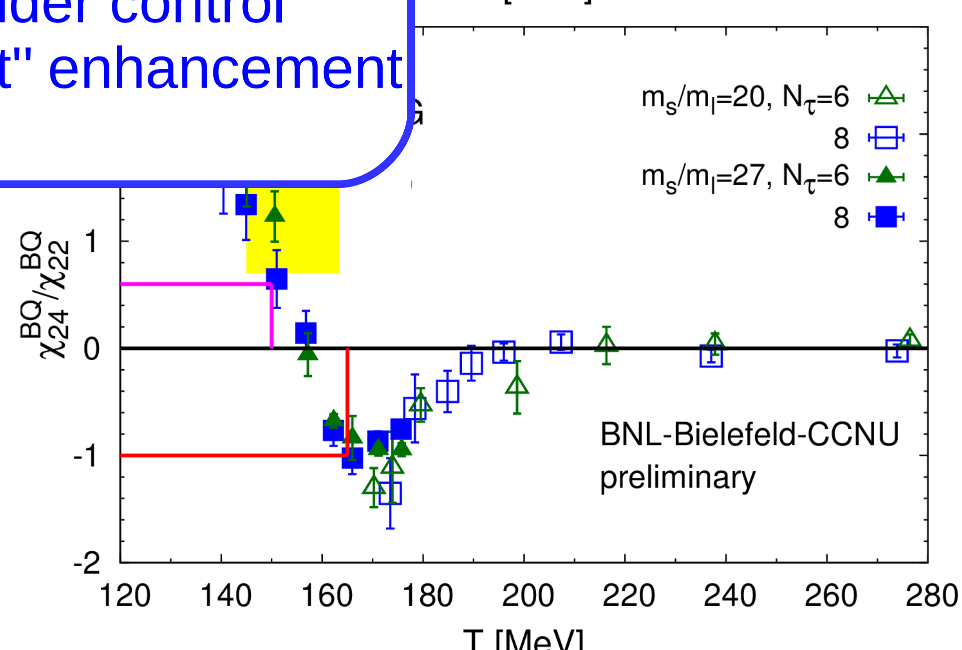
Some 4th and 6th order cumulants



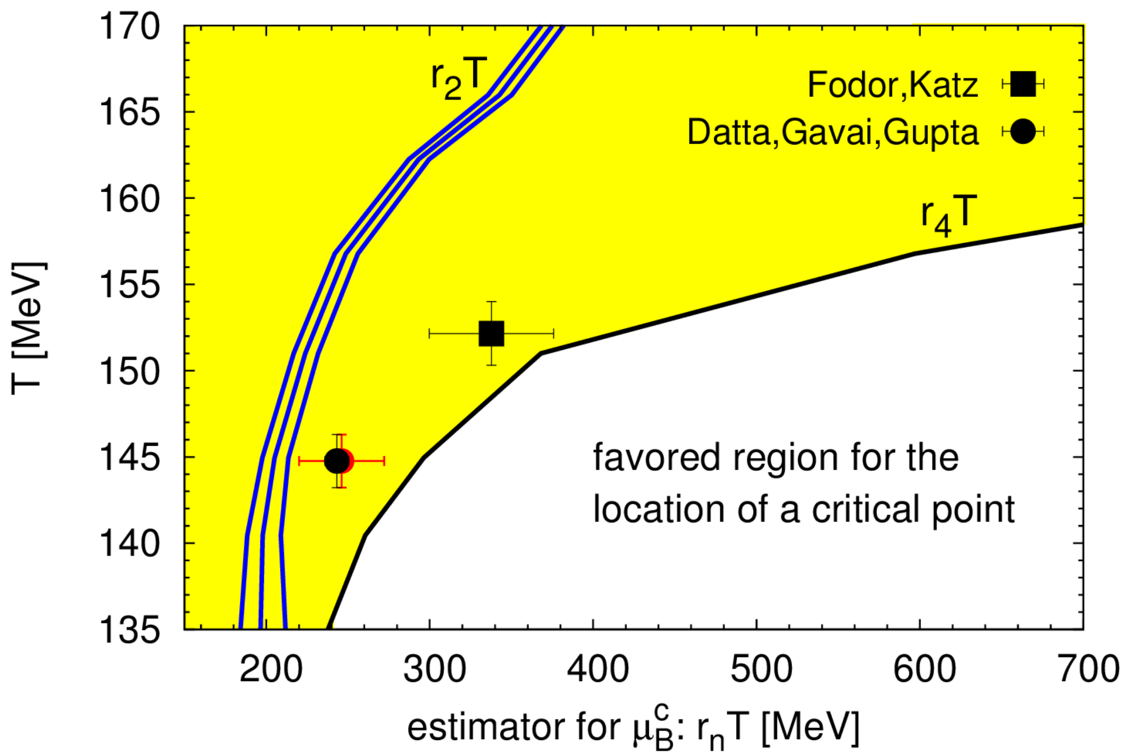


using the HISQ action:
so far none of the cumulants of charge fluctuations that are statistically well under control show a "significant" enhancement over HRG values





estimates/constraints on critical point location



05/30/16:
based on ongoing calculations of 6th order Taylor expansion coefficients performed by the Bielefeld-BNL-CCNU collaboration

Conclusions

– results on bulk thermodynamics coming from Taylor expansion of the QCD partition function are already now reliable in the range $0 \le \mu_B/T \le 2$

bulk QCD thermodynamics in the entire parameter range accessible to BES I and II may soon be accessible also through Taylor expansions

– attempts to understand freeze-out/hadronization in terms of HRG model based calculations at temperatures T > 160 MeV are difficult to conciliate with QCD;

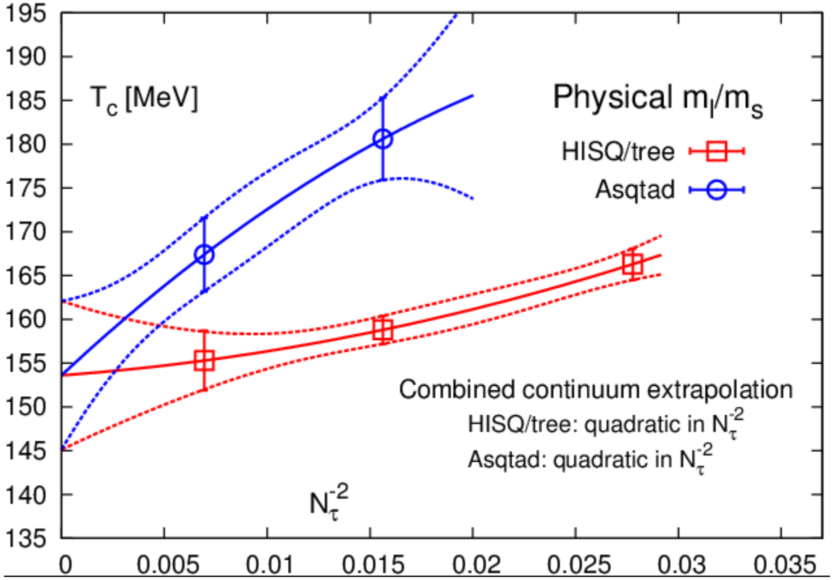
QCD thermodynamics is quite different from HRG thermodynamics at T > 160 MeV

– properties of cumulants measured in BES-I for $\sqrt{s_{NN}} \le 20\ \mathrm{GeV}$ clearly differ from HRG thermodynamics but are consistent with QCD thermodynamics close to the crossover transition temperature

$$S_B\sigma_B < M_B/\sigma_B^2 \ \ , \ \ \kappa_B\sigma_B^2 - S_B\sigma \sim \left(M_B/\sigma_B^2\right)^2$$

– with increasing statistical accuracy current LGT calculations seem to favor estimates for the location of the critical point (if it exists) at values of $\mu_B/T > 2$

Equation of state and transition temperature

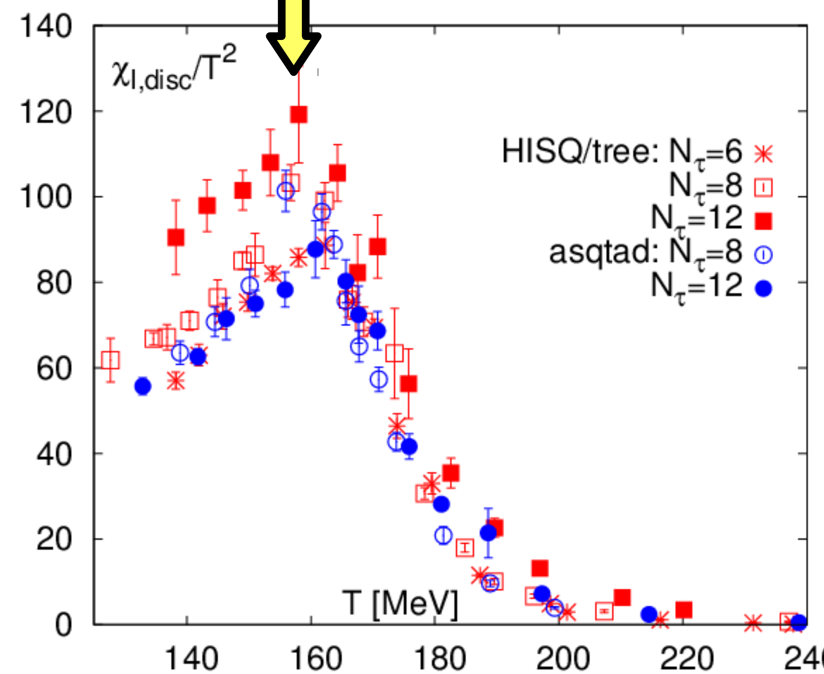


$$T_c = (154 \pm 9)\ \mathrm{MeV}$$

Critical temperature from location of peak in the fluctuation of the chiral condensate (order parameter):

Chiral susceptibility

$$\chi_l = \frac{T}{V}\frac{\partial^2 \ln Z}{\partial m_l^2} = \chi_{l,disc} + \chi_{l,con}$$



– well defined pseudo-critical temperature
– quark mass dependence of susceptibilities consistent with O(4) scaling

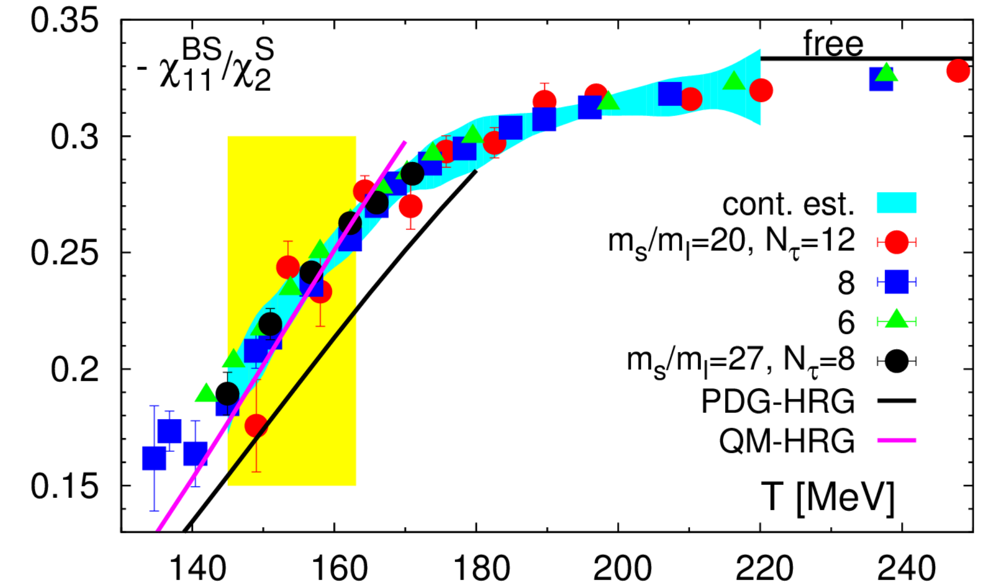
A. Bazavov et al. (hotQCD), Phys. Rev. D85, 054503 (2012), arXiv:1111.1710

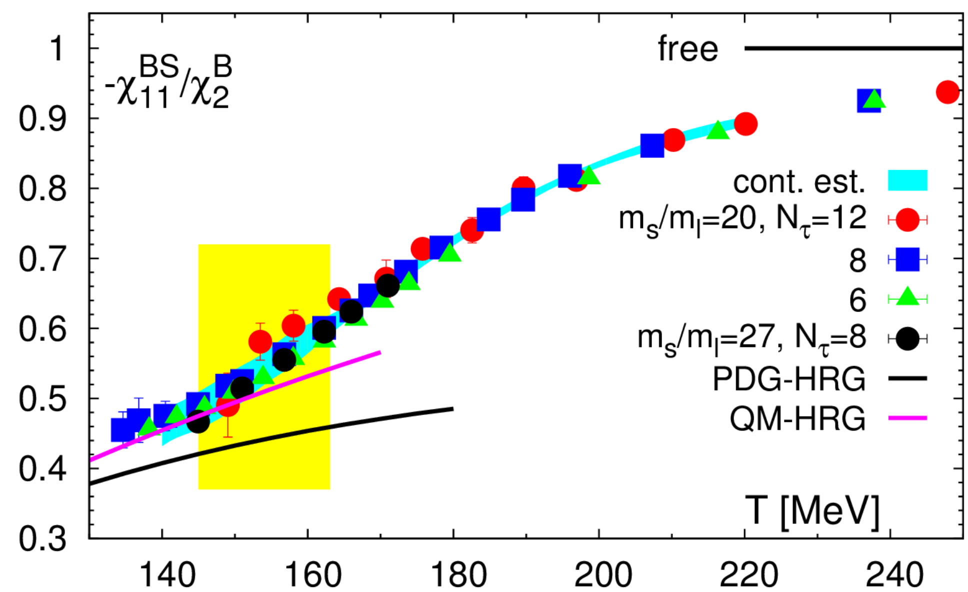
lattice: $N_\sigma^3 \cdot N_\tau$
temperature: $T = 1/N_\tau a$

consistent with Y. Aoki et al, JHEP 0906 (2009) 088

Strangeness-Baryon number Correlations
HRG vs. QCD

Koch ratios





A. Bazavov et al.,
Phys. Rev. Lett. 113, 072001 (2014), arXiv:1404.6511

continuum extrapolated results on strangeness-baryon correlations do NOT agree with a conventional hadron resonance gas, based on experimentally known resonances listed in the particle data tables

in the crossover region (and above):

PDG-HRG $\neq$ QCD

Conserved charge fluctuations and freeze-out

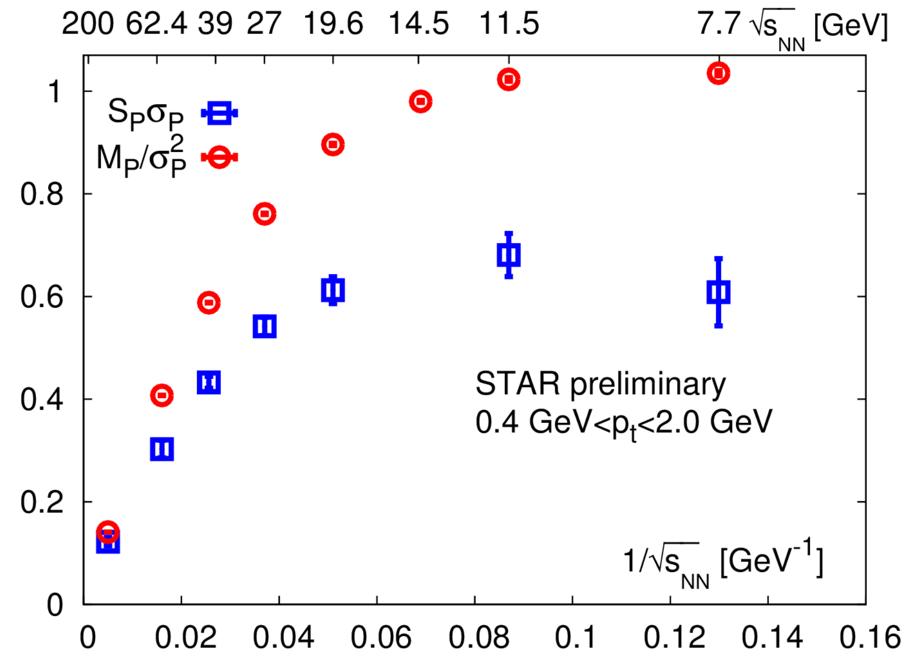
$$\frac{\Delta(T,\mu_B)}{T^4} = \frac{P(T,\mu_B)-P(T,0)}{T^4} = \frac{\chi_2^B}{2}\left(\frac{\mu_B}{T}\right)^2\left(1+\frac{1}{12}\frac{\chi_4^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^2\right)$$

$$\frac{M_B}{\sigma_B^2} = \frac{\mu_B}{T} + \mathcal{O}(\mu_B^3)$$

$$S_B\sigma_B = \frac{\mu_B}{T}\frac{\chi_4^B}{\chi_2^B} + \mathcal{O}(\mu_B^3)$$

no need for talking about a chemical potential

$$\boxed{S_B\sigma_B = \frac{M_B}{\sigma_B^2}\frac{\chi_4^B}{\chi_2^B} + \mathcal{O}(\mu_B^3)}$$



Conserved charge fluctuations and freeze-out
mean, variance, skewness and kurtosis

in a NLO Taylor expansion $R_{31}^B \equiv S_B\sigma_B/M_B$, $R_{42}^B \equiv \kappa_B\sigma_B^2$ } **are closely related**

$R_{31}^B = R_{31}^{B,}$ …

$R_{42}^B = R_{42}^{B,0}$ … $\left(\frac{\chi_6^B}{\chi_2^B} - \left(\frac{\chi_4^B}{\chi_2^B}\right)^2\right)$

need to understand systematics :

– non-equilibrium effects
(S. Mukherjee et al., arXiv:1506.00645)
– proton vs. baryon number distributions
(M. Kitazawa et al, arXiv:1205.3292, arXiv:1303.3338)
– acceptance and pt-cuts
(P. Garg et al, arXiv:1304.7133,
FK, K. Morita, K. Redlich, arXiv:1508.02614
A. Bzdak and V. Koch, arXiv:12064286)