Directed flow in heavy-ion collisions and softening of equation of state

Akira Ohnishi
in collaboration with
Yasushi Nara, Horst Stoecker

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QCD Phase Diagram

RHIC/LHC/Early Universe

T

QGP
AGS/SPS/NICA/FAIR/J-PARC

Hadron Matter

Neutron Star

CSC

μ_B

Nucleon Gas

Nuclear Matter

Quarkyonic?

Inhomo.?
Signals of QGP formation & QCD phase transition

- Signals of QGP formation at top RHIC & LHC energies
  - Jet quenching in AA collisions (not in dA)
  - Large elliptic flow (success of hydrodynamics)
  - Quark number scaling (coalescence of quarks)

- Next challenges
  - Puzzles: Early thermalization, Photon v2, Small QGP, ...
    → Complete understanding from initial to final states
  - Discovery of QCD phase transition

- Signals of QCD phase transition at BES energies?
  - Critical Point → Large fluctuation of conserved charges
  - First-order phase transition → Softening of EOS
    → Non-monotonic behavior of proton number moment ($\kappa\sigma^2$) and collective flow ($dv_1/dy$)
Net-Proton Number Cumulants & Directed Flow

STAR Collab. PRL 112(’14)032302

CPOD 2016: Nu Xu, Morita, Friman, Schaefer, Koch, Kitazawa, ...

STAR Collab., PRL 112(’14)162301.
Two ways to probe QCD phase transition

QGP → Hadrons
Final State Observables
Cumulants, …

Hadrons → QGP
Early Stage Observables
Caution: (Partial) Equilibration is necessary!

Randrup, Cleymans ('06,'09)
What is directed flow?

- $v_1$ or $\langle p_x \rangle$ as a function of $y$ is called directed flow.
- Created in the overlapping stage of two nuclei
  $\rightarrow$ Sensitive to the EOS in the early stage.
- Becomes smaller at higher energies.

How can we explain non-monotonic dependence of $dv_1/dy$?

$\rightarrow$ Softening or Geometry

$v_1 = \langle p_x/p \rangle = \langle \cos \varphi \rangle$
Does the “Wiggle” signal the QGP?

- Hydro predicts wiggle with QGP EOS.

- Baryon stopping + Positive space-momentum correlation leads wiggle (w/o QGP)

L. P. Csernai, D. Röhrich, PLB 45 (1999), 454.

Negative $dv_1/dy$ around $\sqrt{s_{NN}} \sim 10$ GeV

Yes in Hydrodynamics

- Black: Crossover, Red: 1st
  
  Y. B. Ivanov and A. A. Soldatov, PRC91 (2015)024915

No at around $\sqrt{s_{NN}} \sim 10$ GeV in transport models.

V. P. Konchakovski, W. Cassing, Y. B. Ivanov, V. D. Toneev, PRC90('14)014903
SPS(NA49) vs RHIC(STAR)

- **SPS (NA49),** $\sqrt{s_{NN}} = 8.9$ GeV
  - C. Alt et al. (NA49), PRC68 ('03) 034903
  - M. Isse, AO, N. Otuka, P. K. Sahu, Y. Nara, PRC72 ('05) 064908

- **RHIC (STAR),** 7.7-39 GeV
  - L. Adamczyk et al. (STAR), PRL 112(2014)162301

Mid-central: Green

Hadronic Transport w/ MF
Does Directed Flow Collapse Signal Phase Tr.? 

Negative $\frac{dv_1}{dy}$ at high-energy ($\sqrt{s_{NN}} > 20$ GeV) 

- Geometric origin (bowling pin mechanism), not related to FOPT
  R.Snellings, H.Sorge, S.Voloshin, F.Wang, N. Xu, PRL84,2803('00)

Negative $\frac{dv_1}{dy}$ at $\sqrt{s_{NN}} \sim 10$ GeV 

- Yes, in three-fluid simulations. → Thermalization?
  Y. B. Ivanov and A. A. Soldatov, PRC91('15)024915

- No, in transport models incl. hybrid.
  Exception: B.A.Li, C.M.Ko ('98) with FOPT EOS

We investigate the directed flow at BES energies in hadronic transport model
with / without mean field effects
with / without softening effects via attractive orbit.
Contents

- Introduction
  - Two ways to probe QCD phase transition
    - Collapse of Directed Flow at $\sqrt{s_{NN}} \sim 10$ GeV
- Hadronic Transport Model Approaches
  - Boltzmann equation with potential effects
  - Jet AA Microscopic transport model (JAM)
- Additional Softening Effects
  - Attractive Orbit Scattering
  - Transition Density and Pressure (conjecture)
- Summary
Hadronic Transport Approaches
Cascade / Cascade + Mean Field
Microscopic Transport Models

- **UrQMD 3.4** Frankfurt  public
  resonance model N*,D*, string pQCD, PYTHIA6.4

- **PHSD** Giessen (Cassing)  upon request
  D(1232),N(1440),N(1530), string, pQCD, FRITIOF7.02

- **GiBUU 1.6** Giessen (Mosel)  public
  resonance model N*,D*, string, pQCD, PYTHIA6.4

- **AMPT** public
  HIJING+ZPC+ART

- **JAM** (Y. Nara) public
  resonance model N*,D*, string, pQCD, PYTHIA6.1
### Transport Model

**Boltzmann equation with (optional) potential effects**

*E.g. Bertsch, Das Gupta, Phys. Rept. 160(88), 190*

\[
\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f - \nabla U \cdot \nabla_p f = I_{\text{coll}}
\]

\[
I_{\text{coll}}(r, p) = -\frac{1}{2} \int \frac{dp_2}{(2\pi)^3} d\Omega \ v_{12} \frac{d\sigma}{d\Omega} \ [f f_2 (1 - f_3)(1 - f_4)] - (12 \leftrightarrow 34)
\]

*(NN elastic scattering case)*

**Hadron-string transport model JAM**

- **Collision term** → Hadronic cascade with resonance and string excitation

- **Potential term** → Mean field effects in the framework of RQMD/S
  *Sorge, Stocker, Greiner, Ann. of Phys. 192 (1989), 266.
Relativistic QMD/Simplified (RQMD/S)

- RQMD is developed based on constraint Hamiltonian dynamics
  
  \[ H. \text{ Sorge, H. Stoecker, W. Greiner, Ann. Phys. 192 (1989), 266.} \]

- 8N dof → 2N constraints → 6N (phase space)

- Constraints = on-mass-shell constraints + time fixation

- RQMD/S uses simplified time-fixation
  

- Single particle energy (on-mass-shell constraint)
  
  \[ p_i^0 = \sqrt{p_i^2 + m_i^2 + 2 m_i V_i} \]

- EOM after solving constraints
  
  \[ \dot{r}_i = \frac{p_i}{p_i^0} + \sum_j \frac{m_j}{p_j^0} \frac{\partial V_j}{\partial p_i} \quad \dot{p}_i = -\sum_j \frac{m_j}{p_j^0} \frac{\partial V_j}{\partial r_i} \]

- Relative distances \((r_i - r_j)^2\) are replaced with those in the two-body c.m.
  
  → Potential becomes Lorentz scalar
Skryrne type density dependent + momentum dependent potential

\[ V = \sum_i V_i = \int d^3r \left[ \frac{\alpha}{2} \left( \frac{\rho}{\rho_0} \right)^2 + \frac{\beta}{\gamma + 1} \left( \frac{\rho}{\rho_0} \right)^{\gamma + 1} \right] + \sum_k \int d^3r d^3p d^3p' \frac{C_{ex}^{(k)}}{2\rho_0} \frac{f(r, p) f(r, p')}{1 + (p - p')^2/\mu_k^2} \]

<table>
<thead>
<tr>
<th>Type</th>
<th>( \alpha ) (MeV)</th>
<th>( \beta ) (MeV)</th>
<th>( \gamma )</th>
<th>( C_{ex}^{(1)} ) (MeV)</th>
<th>( C_{ex}^{(2)} ) (MeV)</th>
<th>( \mu_1 ) (fm(^{-1}))</th>
<th>( \mu_2 ) (fm(^{-1}))</th>
<th>( K ) (MeV)</th>
</tr>
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<td>MH1</td>
<td>-12.25</td>
<td>87.40</td>
<td>5/3</td>
<td>-383.14</td>
<td>337.41</td>
<td>2.02</td>
<td>1.0</td>
<td>371.92</td>
</tr>
<tr>
<td>MS1</td>
<td>-208.89</td>
<td>284.04</td>
<td>7/6</td>
<td>-383.14</td>
<td>337.41</td>
<td>2.02</td>
<td>1.0</td>
<td>272.6</td>
</tr>
</tbody>
</table>

Pot. Eff. on the $v_1$ is significant, but $dv_1/dy$ becomes negative only at $\sqrt{s_{NN}} > 20$ GeV.

Hadronic approach does not explain directed flow collapse at 10-20 GeV even with potential effects.

JAM/M: only formed baryons feel potential forces
JAM/Mq: pre-formed hadron feel potential with factor $2/3$ for diquark, and $1/3$ for quark
JAM/Mf: both formed and pre-formed hadrons feel potential forces.

Additional Softening Effects
Attractive orbit scattering simulates softening of EOS

P. Danielewicz, S. Pratt, PRC 53, 249 (1996)
H. Sorge, PRL 82, 2048 (1999).

\[ P = P_f + \frac{1}{3TV} \sum_{(i,j)} (q_i \cdot r_i + q_j \cdot r_j) \]

(Virial theorem)

Let us examine the EOS softening effects, which cannot be explained in hadronic mean field potential, by using attractive orbit scatterings!

Directed Flow with Attractive Orbits

Nara, AO, Stöcker ('16)

Softening!
Mean Field + Attractive Orbit

MF+Attractive Orbit make $dv_1/dy$ negative at $\sqrt{s_{\text{NN}}} \sim 10$ GeV
When is negative $v_1$ slope generated?

We need to make $v_1$ slope negative in the compressing stage.
Transport model results also show tilted-ellipsoid-like behavior, but it is not enough.
“Softening” should take place at $\sqrt{s_{NN}} = 11.5$ GeV $\rightarrow \rho/\rho_B \sim (6-10)$

- Attractive orbit
  $\rightarrow$ Larger interactions
  & Higher $T$ at later times
How much softening do we need?

**Virial theorem**

\[ \Delta P = \frac{1}{3} \langle \nu \rho^2 \sigma q \cdot \Delta r \rangle \]

Simple estimate:
\( \sigma = 30 \text{ mb}, \langle q \Delta R \rangle \sim -1 \)


**B. A. Li, C. M. Ko, PRC58 ('98) 1382**

A. Ohnishi @ CPOD 2016, May.31, 2016
How much softening do we need?

Virial theorem

\[ \Delta P = \frac{1}{3} \langle \nu \rho^2 \sigma_q \cdot \Delta r \rangle \]

Simple estimate: \( \sigma = 30 \text{ mb}, \langle q \Delta R \rangle \sim -1 \)

J. Steinheimer, J. Randrup, V. Koch, PRC89(‘14)034901.
How about $v_2$?

- Do we see softening effects in other observables, e.g. $v_2$?
- Yes, attractive orbits reduces proton $v_2$ by $\sim 0.2\%$. (but there is no qualitative change.)
Relation to Neutron Star Matter

We may need early transition (2-5 \( \rho_0 \)) to quark matter to solve the hyperon puzzle. Contradicting?

→ Temperature effects (T \( \sim \) 0 MeV & 100 MeV)
Isospin chem. pot. (Weaker transition with finite \( \delta \mu \))
Hyperon repulsion may push up the transition density.

AO, Ueda, Nakano, Ruggieri, Sumiyoshi, PLB704('11),284
H. Ueda, T. Z. Nakano, AO, M. Ruggieri, K. Sumiyoshi, PRD88('13),074006
Summary

- We may see QCD phase transition (1\textsuperscript{st} or 2\textsuperscript{nd}) signals at BES (or J-PARC) energies in baryon number cumulants and $v_1$ slope.

- Hadronic transport models cannot explain negative $v_1$ slope below $\sqrt{s_{NN}} = 20$ GeV.

  - Geometric (bowling pin) mechanism becomes manifest at higher energies (JAM, JAM-MF, HSD, PHSD, UrQMD, ...).

- Hadronic transport with EOS softening can describe negative $v_1$ slope below $\sqrt{s_{NN}} = 20$ GeV.


  - Attractive orbit scattering simulates EOS softening (virial theorem).

  - We need more studies to confirm its nature. First-order phase transition ? Crossover ? Forward-backward rapidities ? MF leading to softer EOS ?

- We need “re-hardening” at higher energies, e.g. $\sqrt{s_{NN}} = 27$ GeV.
Thank you!
Attractive orbits keeps matter to be high density for a longer time.

Steinheimer et al. ('13)

Gaussian smeared, CM (x, y, z=0), Collided/created hadrons, incl. those in formation time
Time-dependence of density

Anisotropy correction of the Fermi Sphere

\[ \rho_{iso} = \rho \left( \frac{P_{xy}^{\text{kin}}}{P_z^{\text{kin}}} \right)^{1/2} \]

- Std.
- Att.

7.7 GeV
11.5 GeV
Nuclear Liquid-Gas Phase Transition

Caloric curve → LG phase transition 
(Smoking gun)


AO, Randrup ('98)

T. Furuta, A. Ono ('09)
Non-monotonic behavior in $K^+/\pi^+$ ratio (Horn), m slope par. (Step or re-hardening), rapidity dist. width of $\pi$

Horn

Step

Dale

E.g. A. Rustamov (2012)

N. Otuka, P.K.Sahu, M. Isse, Y. Nara, AO, nucl-th/010205
**Hybrid Approaches**

Both Hybrid model (Frankfurt) and PHSD (Giessen) show higher balance E.

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**Figure Explanation**

- **Hybrid Model**
  - Graph (a): Plot showing the dependence of some variable on Energy with markers for different data sets.
  - Graph (b): Graph showing the same variable with markers for different energy levels.

**References**

- J. Steinheimer, J. Auvinen, H. Petersen, M. Bleicher, H. Stöcker, PRC89 (’14) 054913
- V. P. Konchakovski, W. Cassing, Yu. B. Ivanov, V. D. Toneev, PRC90(’14)014903
JAM results at AGS and SPS Energies

JAM w/ Mean-Field effects roughly explains $v_1$ and $v_2$ at AGS & SPS

$(1-158 \text{ A GeV} \to \sqrt{s_{NN}} = 2.5-20 \text{ GeV})$

$\sqrt{s_{NN}} = 8.9 \text{ GeV} \quad \sqrt{s_{NN}} = 17.3 \text{ GeV}$

M. Isse, AO, N. Otuka, P. K. Sahu, Y. Nara, PRC72('05)064908
Highest Density Matter at J-PARC?

Nara, Otuka, AO, Maruyama ('97)

Central 1 fm³ cube.

AO, JHF workshop (2002)
How do heavy-ion collisions look like?

Au+Au, 10.6 A GeV

Pb+Pb, 158 A GeV

JAMming on the Web  http://www.jcprg.org/jow/
J-PARC energy

Au+Au, 25 AGeV, b=5 fm (JOW)
BES (J-PARC) Energies?

**J-PARC Energies:** $\sqrt{s_{NN}} = 4-40$ GeV (or $\sqrt{s_{NN}} = 1.9-6.2$ GeV)

- $E(p)=30$ GeV $\rightarrow E(Au) \sim 12$ AGeV (full strip, $\sqrt{s_{NN}} = 5.1$ GeV for Au+Au)
- $E(p)=50$ GeV $\rightarrow E(Au) \sim 20$ AGeV ($\sqrt{s_{NN}} = 6.4$ GeV)
- $E(p)=30$ GeV (50 GeV) Collider $\rightarrow \sqrt{s_{NN}} = 26$ GeV (42 GeV)

Two Aspects of J-PARC energies

- Formation of highest baryon density matter
- Various non-monotonic behaviors $\rightarrow$ Onset of deconfinement

**Question**

Do these Non-mono. behaviors signal the onset of QCD phase transition and/or QCD critical point? or Do they show some properties of hadronic matter? $\rightarrow$ Let's examine in hadronic transport models!
How to treat mean-field for excited matter?

**Hadronic resonance dominant**

$NN \rightarrow N^* \Delta$

$\sqrt{s_{NN}} = 4.72 \text{ GeV}$

**constituent quark dominant due to string**

$NN \rightarrow \text{string} + \text{string}$

$\sqrt{s_{NN}} = 6.4 \text{ GeV}$

**Model 1 JAM/M: potential for all formed baryons**

**Model 2 JAM/Mq: potentials for quarks inside the pre-formed hadrons**

**Model 3: JAM/Mf: both formed and pre-formed baryons**
JAM hadronic cascade model: resonance and string excitation

Mean field by the framework of the Relativistic Quantum Molecular Dynamics

Nuclear cluster formation by phase space coalescence.

Statistical decay of nuclear fragment

Purpose: Effects of hadron mean field potential on the directed flow $v_1$
Relativistic QMD/Simplified (RQMD/S)

**RQMD** based on Constraint Hamiltonian Dynamcis


Single particle energy:

\[ p_i^0 = \sqrt{p_i^2 + m_i^2 + 2m_i V_i} \]

\[ \dot{r}_i = \frac{p_i}{p_i^0} + \sum_j \frac{m_j}{p_j^0} \frac{\partial V_j}{\partial p_i} \]

\[ \dot{p}_i = - \sum_j \frac{m_j}{p_j^0} \frac{\partial V_j}{\partial r_i} \]

Arguments of potential \( r_i - r_j \) and \( p_i - p_j \) are replaced by the distances in the two-body c.m.
Relativistic QMD/Simplified (RQMD/S)

- RQMD = Constraint Hamiltonian Dynamics
  
  \[ H_i \equiv p_i^2 - m_i^2 - 2m_i V_i \approx 0 \]

- Constraints: \( \varphi \approx 0 \) (Satisfied on the realized trajectory, by Dirac)
  
  \[ \chi_i \equiv \hat{a} \cdot (q_i - q_N) \approx 0 \quad (i = 1, \ldots, N-1) \]
  
  \[ \chi_N \equiv \hat{a} \cdot q_N - \tau \approx 0 \]

- Variables in Covariant Dynamics = 8N phase space: \((q_\mu, p_\mu)\)

- Variables in EOM = 6N phase space
  
  \( \rightarrow \) We need 2N constraints to get EOM

- On Mass-Shell Constraints

\[ \chi_i \equiv \hat{a} \cdot (q_i - q_N) \approx 0 \quad (i = 1, \ldots, N-1) \]

\[ \chi_N \equiv \hat{a} \cdot q_N - \tau \approx 0 \]

- Time-Fixation in RQMD/S

\[ \hat{a} = \text{Time-like unit vector in the Calculation Frame} \]

\[ (\text{Tomoyuki Maruyama et al., Prog. Theor. Phys. 96(1996), 263.}) \]
Hamiltonian is made of constraints

\[ H = \sum_i u_i \phi_i \ (\phi_i = H_i (i = 1 \sim N), \chi_{i-N} (i = N + 1 \sim 2N)) \]

Time Development

\[ \frac{d f}{d \tau} = \frac{\partial f}{\partial \tau} + \{f, H\}, \quad \{q_\mu, p_\nu\} = g_{\mu \nu} \]

Lagrange multipliers are determined to keep constraints

→ We can obtain the multipliers analytically in RQMD/S

\[ \frac{d \phi_i}{d \tau} \approx 0 \rightarrow \delta_{i,2N} + \sum_j u_j \{\phi_i, \phi_j\} \approx 0 \]

Equations of Motion

\[ H = \sum_i \left( p_i^2 - m_i^2 - 2m_i V_i \right) / 2p_i^0, \quad p_i^0 = E_i = \sqrt{\vec{p}_i^2 + m_i^2 + 2m_i V_i} \]

\[ \frac{d \vec{r}_i}{d \tau} \approx -\frac{\partial H}{\partial \vec{p}_i} = \vec{p}_i^0 + \sum_j \frac{m_j}{p_j^0} \frac{\partial V_j}{\partial \vec{r}_i}, \quad \frac{d \vec{p}_i}{d \tau} \approx \frac{\partial H}{\partial \vec{r}_i} = -\sum_j \frac{m_j}{p_j^0} \frac{\partial V_j}{\partial \vec{r}_i} \]

We can include MF in an almost covariant way in molecular dynamics
Particle “DISTANCE”

\[ r_{Tij}^2 \equiv r_\mu r^\mu - \left( r_\mu P_{ij}^\mu \right)^2 / P_{ij}^2 = \vec{r}^2 \quad (in \ CM) \]

\[ P_{ij} \equiv p_i + p_j , \quad r \equiv r_i - r_j \]

Particle “Momentum Difference”

\[ p_{Tij}^2 \equiv p_\mu p^\mu - \left( p_\mu P_{ij}^\mu \right)^2 / P_{ij}^2 = \vec{p}^2 \quad (in \ CM) \]

\[ p \equiv p_i - p_j \]

Lorentz Invariant, and Becomes Normal Distance in CM!