Non-extensive critical effects in the nuclear mean field

J. Rozynek, Warsaw

Nonextensive NJ-L model of QCD matter revisited

Nonextensive distributions for a relativistic Fermi gas
Equation of State

Pressure $p_T(\rho)$ vs. Density $\rho$

$T_c$

$T=0$
Content:

- Motivation

- Example: NJL model of QCD

  → Nonextensive NJL model: q_NJL

- Results

- Summary
The critical phenomena in strongly interaction matter are generally investigated using the mean-field model and are characterized by well defined critical exponents.

However, such models provide only average properties of the corresponding order parameters and neglect altogether their possible fluctuations. Also the possible long range effect and correlations are neglected in the mean field approach.

One of the possible phenomenological ways to account for such effects is to use the nonextensive approach.

Here we investigate the critical behavior in the nonextensive version of the Nambu Jona-Lasinio model (NJL). It allows to account for such effects in a phenomenological way by means of a single parameter $q$, the nonextensivity parameter.

In particular, we show how the nonextensive statistics influences the region of the critical temperature and chemical potential in the NJL mean field approach.
The critical phenomena in strongly interaction matter are generally investigated using the mean-field model and are characterized by well defined critical exponents.

However, such models provide only average properties of the corresponding order parameters and neglect altogether their possible fluctuations. Also the possible long range effect and correlations are neglected in the mean field approach.

One of the possible phenomenological ways to account for such effects is to use the nonextensive approach.

(... digression on nonextensivity and the like ...)
Digression: .... nonextensive approach.... what does it mean?

The nonextensive statistical mechanics proposed by Tsallis [2] generalizes the usual BG statistical mechanics in that the entropy function (we use the convention that the Boltzmann constant is set equal to unity),

\[ S_{\text{BG}} = - \sum_{i=1}^{W} p_i \ln p_i \implies S_q = - \sum_{i=1}^{W} p_i^q \ln_q p_i, \quad (17) \]

\[ S_q \rightarrow S_{q=1} = S_{\text{BG}} \text{ for } q \rightarrow 1. \]

Here, \( q \) is the nonextensive parameter and \( \ln_q p = [p^{1-q} - 1]/(1 - q) \). The additivity for two independent subsystems A and B (i.e., such that \( p^{A\oplus B} = p^A \cdot p^B \)) is now lost and takes the form:

\[ S_{q}^{A\oplus B} = S_q^A + S_q^B + (1-q)S_q^A S_q^B, \quad (18) \]

they are called nonextensive\(^4\).

Digression: .... nonextensive approach.... what does it mean?

This phenomenon is ubiquitous in all branches of science and very well documented. It occurs always whenether:

(*) there are **long range correlations** in the system (or „system is small” – like our Universe with respect to the gravitational interactions)

(*) there are **memory effects** of any kind

(*) the **phase-space** in which system operates is **limited** or has **fractal structure**

(*) there are **intrinsic fluctuations** in the system under consideration

(*) .........................
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(*) ……

   Biró T S 2008 Europhys. Lett. 84 56003
Heat bath characterized by one parameter:
- temperature

N-particle system $\Rightarrow$
N-1 unobserved particles form "heat bath" which determines behaviour of 1 observed particle

Digression: .... nonextensive approach.... what does it mean?

But: In such ”thermodynamical” approach one has to remember assumptions of infinity and homogeneity made when proposing this approach - only then behaviour of the observed particle will be characterised by single parameter - the ”temperature” T

In reality: This is true only approximately and in most cases we deal with system which are neither infinite and nor homogeneous

In both cases: Fluctuations occur and new parameter(s) in addition to T is(are) necessary

Can one introduce it keeping simple structure of statistical model approach?

Yes, one can, by applying nonextensive statistical model.

Wilk G and Włodarczyk Z 2000 Phys. Rev. Lett. 84 1770
Wilk G and Włodarczyk Z 2001 Chaos Solitons Fractals 13 581
Digression: ...nonextensive approach... what does it mean?

$T$ varies $\iff$ fluctuations...

$T_0 = \langle T \rangle, q$

$q$ - measure of fluctuations of $T \Rightarrow q$-statistics (Tsallis)
Recently, q-statistics has been applied to the Walecka many-body field theory resulting (among others) in the enhancement of the scalar and vector meson fields in nuclear matter, in diminishing of the nucleon effective mass and in the hardening of the nuclear equation of state (only $q>1$ case was considered there).

Here we investigate the critical behavior in the nonextensive version of the Nambu Jona-Lasinio model (NJL). It allows to account for such effects in a phenomenological way by means of a single parameter $q$, the nonextensivity parameter. The NJL model we modify is that presented in


In particular, we show how the nonextensive statistics influences the region of the critical temperature and chemical potential in the NJL mean field approach.

Abstract. In the present work we apply non extensive statistics to obtain equations of state suitable to describe stellar matter and verify its effects on microscopic and macroscopic quantities. Two snapshots of the star evolution are considered and the direct Urca process is investigated with two different parameter sets. q-values are chosen as 1.05 and 1.14. The equations of state are only slightly modified, but the effects are enough to produce stars with slightly higher maximum masses. The onsets of the constituents are more strongly affected and the internal stellar temperature decreases with the increase of the q-value, with consequences on the strangeness and cooling rates of the stars.
In the extensive approach to dense, hot matter the particle and antiparticle occupation numbers, \( n_i \) and \( \bar{n}_i \), can be obtained from the Jayne’s extremalization of the entropic measure

\[
S = \sum_i \left[ n_i \ln n_i + (1 - n_i) \ln(1 - n_i) \right] + [n_i \rightarrow \bar{n}_i],
\]

(1)

under the constraints imposed by the total number of particles, \( N \), and the total energy of the system, \( E \) (\( \epsilon_i \) is the energy of the \( i \)-th energy level) [16],

\[
\sum_i (n_i - \bar{n}_i) = N \quad \text{and} \quad \sum_i (n_i + \bar{n}_i) \epsilon_i = E.
\]

(2)

As a result we get the Fermi-Dirac distributions,

\[
n_i = \frac{1}{\exp(x_i) + 1}, \quad \bar{n}_i = \frac{1}{\exp(\bar{x}_i) + 1},
\]

(3)

which depend on the dimensionless quantities

\[
x = \beta(\epsilon - \mu) \quad \text{and} \quad \bar{x} = \beta(\epsilon + \mu).
\]

(4)

where \( \beta = 1/T \), \( \epsilon = \sqrt{p^2 + m^2} \), \( m \) is fermion mass and \( \mu \) its chemical potential. For \( \epsilon = 0 \), the distributions \( n(x) \) and \( \bar{n}(\bar{x}) = n(\bar{x}) \) satisfy following relation:

\[
n(x) + n(\bar{x}) = 1.
\]

(5)
Entrophy Extremalization gives:

\[ S_q(a) = \sum_i \left[ n_{q_i}^q \ln_q n_{q_i} + (1-n_{q_i})^q \ln_q (1-n_{q_i}) \right] + \]

where \( \ln_q x = \frac{x^{1-q} - 1}{1-q} \).

\[ q=1 \quad \rightarrow \quad \ln(n_i) \]

\[ n_{q_i} = \frac{1}{e_q(x_{q_i}) + 1}, \quad \tilde{n}_{q_i} = \frac{1}{e_q(\bar{x}_{q_i}) + 1}, \]

where \( x_{q_i} = \beta(E_{q_i} - \mu), \quad \bar{x}_{q_i} = \beta(E_{q_i} + \mu), \)

and \( e_q(x) = [1 + (q-1)x]^{\frac{1}{q-1}}. \)

with conditions

\[ \sum_i (n_{q_i}^q - \tilde{n}_{q_i}^q) = \hat{N}, \quad \sum_i (n_{q_i}^q + \tilde{n}_{q_i}^q) E_{q_i} = \hat{E}. \]

(follow Lagrange multipliers)
Our **NJL** Lagrangian has the usual form:

\[
\mathcal{L} = \bar{q} (i \partial \cdot \gamma - \hat{m}) q + \frac{g_s}{2} \sum_{a=0}^{8} \left[ (\bar{q} \lambda^a q)^2 + (\bar{q} (i \gamma_5) \lambda^a q)^2 \right] \\
+ g_D \left[ \det [\bar{q}(1 + \gamma_5)q] + \det [\bar{q}(1 - \gamma_5)q] \right].
\]

We have 2 coupled **equations** for quark **mass** and **condensate**:

\[
M_i = m_i - 2g_s \langle \bar{q}_i q_i \rangle - 2g_D \langle \bar{q}_j q_j \rangle \langle \bar{q}_k q_k \rangle,
\]

\[
\langle \bar{q}_i q_i \rangle = -\frac{N_c}{\pi^2} \sum_{i=u,d,s} \left[ \int \frac{p^2 M_i}{E_i} (1 - n_i - \bar{n}_i) \right] dp.
\]
NJL - Finite Densities in the Grand Canonical Ensemble \{V,T,\mu_i\}

\[ \Omega(T, V, \mu_i) = E - TS - \sum_{i=u,d,s} \mu_i N_i. \]

\[ E = -\frac{N_c}{\pi^2} V \sum_{i=u,d,s} \left[ \int p^2 dp \frac{p^2 + m_i M_i}{E_i} (1 - n_i - \bar{n}_i) \right] - \]

\[ - g_S V \sum_{i=u,d,s} (\langle \bar{q}_i q_i \rangle)^2 - 2 g_D V \langle \bar{u} u \rangle \langle \bar{d} d \rangle \langle \bar{s} s \rangle, \]

\[ S = -\frac{N_c}{\pi^2} V \sum_{i=u,d,s} \int p^2 dp \cdot \tilde{S}, \]

where \[ \tilde{S} = [n_i \ln n_i + (1 - n_i) \ln(1 - n_i)] + [n_i \rightarrow 1 - \bar{n}_i], \]

\[ N_i = \frac{N_c}{\pi^2} V \int p^2 dp (n_i - \bar{n}_i). \]
\( \Omega_q(T, V, \mu_i) = E_q - TS_q - \sum_{i=u,d,s} \mu_i N_{qi}. \) \( (42) \)

The pressure and the energy density are defined as, respectively,

\[ P_q(\mu, T) = -\frac{1}{V} [\Omega_q(\mu, T) - \Omega_q(0, 0)], \] \( (43) \)

\[ \varepsilon_q(\mu, T) = \frac{1}{V} [E_q(\mu, T) - E_q(0, 0)], \] \( (44) \)

where \( \Omega_q(0, 0) = E_q(0, 0) \) denotes the vacuum energy.

The first derivatives give, respectively, \( q \)-entropy and \( q \)-density,

\[ S_q = \sum_{i=u,d,s} \frac{\partial \Omega_q}{\partial T} \bigg|_\mu \quad \text{and} \quad \rho_q = \sum_{i=u,d,s} \frac{\partial \Omega_q}{\partial \mu} \bigg|_T. \] \( (45) \)

The second derivatives result in nonextensive versions of the heat capacity, \( C_\mu \), and the barionic susceptibility \( \chi_B \),

\[ C_\mu = \frac{\partial S_q}{\partial T} \bigg|_\mu \quad \text{and} \quad \chi_B = \frac{\partial \rho_q}{\partial \mu} \bigg|_T. \] \( (46) \)
Because
\[ dE_q = \frac{\partial E_q}{\partial S_q} dS_q + \frac{\partial E_q}{\partial N_q} dN_q = \frac{\partial E_q}{\partial T} dT + \frac{\partial E_q}{\partial \mu} d\mu, \quad (47) \]

one can define temperature the $T$ and the heat capacity $C_\mu$ as, respectively,
\[ \frac{1}{T} = \frac{\partial S_q}{\partial E_q} \quad \text{and} \quad \frac{1}{T^2 C_\mu} = -\frac{\partial^2 S_q}{\partial E_q^2}. \quad (48) \]

The corresponding nonextensive energy, $E_q$, entropy, $S_q$, and number density, $N_q$, are now given by
\[ E_q = -\frac{N_c}{\pi^2} V \sum_{i=u,d,s} \left[ \int p^2 dp \frac{p^2 + m_i M_{qi}}{E_{qi}} (1 - n_{qi}^q - \bar{n}_{qi}^q) \right] - \]
\[ - g_S V \sum_{i=u,d,s} \left( \langle \bar{q}_i q_i \rangle_q \right)^2 - 2 g_D V \langle \bar{u}u \rangle_q \langle \bar{d}d \rangle_q \langle \bar{s}s \rangle_q; \quad (49) \]
\[ S_q = -\frac{N_c}{\pi^2} V \sum_{i=u,d,s} \int p^2 dp \cdot \tilde{S}_{qi}^{(R)} \quad (50) \]
\[ N_{qi} = \frac{N_c}{\pi^2} V \int p^2 dp \left( n_{qi}^q - \bar{n}_{qi}^q \right). \quad (51) \]
\[ d\varepsilon_q = T\, ds_q + \mu\, d\rho_q, \quad dP_q = s_q\, dT + \rho_q\, d\mu, \] (57)

where the energy density \( \varepsilon_q = E_q/V \), the entropy density \( s_q = S_q/V \) and the density \( \rho_q = N_q/V \). The relations to be checked are

\[ T = \frac{\partial \varepsilon_q}{\partial s_q} \bigg|_{\rho_q}, \quad \mu = \frac{\partial \varepsilon_q}{\partial \rho_q} \bigg|_{s_q}, \quad \rho_q = \frac{\partial P_q}{\partial \mu} \bigg|_T, \quad s_q = \frac{\partial P_q}{\partial T} \bigg|_{\mu}. \] (58)

The last two are easy to check numerically. In the first two one has first to convert derivatives in \( s \) and \( \rho \) to derivatives in \( T \) and \( \mu \) and to calculate

\[ T = \frac{\partial \varepsilon_q}{\partial s_q} \bigg|_{\rho_q} = \frac{\partial \varepsilon_q}{\partial T} + \frac{\partial \varepsilon_q}{\partial \mu} \frac{d\mu}{dT} \frac{\partial \mu}{\partial T}, \quad \text{where} \quad \frac{d\mu}{dT} = -\frac{\partial \rho_q}{\partial T}, \] (59)

\[ \mu = \frac{\partial \varepsilon_q}{\partial \rho_q} \bigg|_{s_q} = \frac{\partial \varepsilon_q}{\partial T} + \frac{\partial \varepsilon_q}{\partial \mu} \frac{d\mu}{dT} \frac{\partial \mu}{\partial T}, \quad \text{where} \quad \frac{d\mu}{dT} = -\frac{\partial s_q}{\partial T}. \] (60)
We concentrate on such features of the $q$-NJL model:

(1) Chiral symmetry restoration in the $q$-NJL („Results-chiral”)

(2) Spinodial decomposition in the $q$-NJL („Results-spinodial”)

(3) Critical effects in the $q$-NJL („Results-critical effects”)

As our goal was to demonstrate the sensitivity to the nonextensive effects represented by $|q-1| \neq 0$, we do not reproduce here the whole wealth of results provided in


but concentrate on the most representative results.
Figure 1. (a) Quark condensates and (b) effective quark masses as functions of the temperature for different values of the nonextensive parameter $q$ ($q = 1$ corresponds to Boltzmann–Gibbs statistics).

Figure 2. Masses of $\pi$ and $\sigma$ mesons as functions of the temperature for different values of the nonextensive parameter $q$ ($q = 1$ corresponds to BG statistics).
They were calculated assuming zero chemical potentials and solving numerically the $q$-version of gap equation

$$M_i = m_i - 2g_s \langle \bar{q}_i q_i \rangle - 2g_D \langle \bar{q}_j q_j \rangle \langle \bar{q}_k q_k \rangle,$$

with $\langle \bar{q}_i q_i \rangle \rightarrow \langle \bar{q}_i q_i \rangle_q$ given by:

$$\langle \bar{q}_i q_i \rangle_q = -\frac{N_c}{\pi^2} \sum_{i=u,d,s} \left[ \int \frac{p^2 M_i}{E_i} (1 - n_{q_i}^q - \bar{n}_{q_i}^q) \right] dp.$$
(*) Notice the difference between $q<1$ and $q>1$ cases.

(*) The effects caused by nonextensivity are practically invisible for heavier quarks.

Figure 1. (a) Quark condensates and (b) effective quark masses as functions of the temperature for different values of the nonextensive parameter $q$ ($q = 1$ corresponds to Boltzmann–Gibbs statistics).
Figure 2. Masses of $\pi$ and $\sigma$ mesons as functions of the temperature for different values of the nonextensive parameter $q$ ($q = 1$ corresponds to BG statistics).
(*) Temperature for which $\sigma$ mass reaches minimum is smaller for $q<1$ and larger for $q>1$ (by amount $\sim |q-1|$).

(*) Final values of masses is larger for $q<1$ and smaller for $q>1$ (by amount $\sim |q-1|$).

(*) Fluctuations ($q>1$) dilute the region where the chiral phase transition takes place.

(*) Correlations ($q<1$) only shift the condensates, quark masses and meson masses towards smaller temperatures.

(*) They refer to quarks, not hadrons as in $q$-Walecka model (where only $q>1$, i.e., fluctuations were considered with similar effect).
Figure 5. The pressure calculated for different values of the nonextensivity parameter $q$ for temperatures $T = 30$ MeV (a) and $T = 50$ MeV (b) as a function of the compression $\rho/\rho_0$. The curves for $q$ for which the temperature considered is the critical temperature are also shown, they correspond to $q = 1.19$ for $T = 30$ MeV and $q = 1.063$ for $T = 50$ MeV.
Fig. 3. (Color online) Entropy as a function of the chemical potential (left panel) and temperature (right panel) calculated for different values of $q$ corresponding to different realizations of the $q$-NJL model and compared with the BG case of $q = 1$.

Fig. 4. (Color online) Compression $\rho/\rho_0$ as a function of the chemical potential (left panel) and the pressure (right panel) for different values of $q$ corresponding to different realizations of the $q$-NJL model and compared with the BG case of $q = 1$. 
Fig. 6. (Color online) The pressure at critical temperature $T_{cr}$ as a function of compression $\rho/\rho_0$ calculated for different values of $q$ corresponding to different realizations of the $q$-NJL model and compared with the BG case of $q = 1$ (the area marked on the upper panel is shown in detail in the lower panel). The dots indicate the positions of the inflection points for which the first derivative of pressure by compression vanishes.

Fig. 7. (Color online) Compression $\rho/\rho_0$ as a function of chemical potential (left panel) and temperature (right panel) calculated in the vicinity of the phase transition for different values of $q$ corresponding to different realizations of the $q$-NJL model and compared with the BG case of $q = 1$. 
Fig. 8. (Color online) Left panel: Heat capacity as a function of temperature calculated in the vicinity of the phase transition point for different values of $q$ corresponding to different realizations of the $q$-NJL model and compared with the BG case of $q = 1$. The respective values of chemical potential used are indicated. Right panel: Susceptibility as a function of chemical potential calculated in the vicinity of the phase transition point, and for different values of $q$ corresponding to different realizations of the $q$-NJL model and compared with the BG case of $q = 1$. The respective values of the temperatures are indicated.

Fig. 9. (Color online) Phase diagram in the $q$-NJL model in the $T - \mu$ plane for different values of $q$ corresponding to different realizations of the $q$-NJL model and compared with the BG case of $q = 1$. The left panel shows a general view where, for the scale used, all curves essentially coincide. The right panel shows an enlarged region near the critical point (CEP).
The pressure of hot QCD up to $g^6 \ln(1/g)$

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Center for Theoretical Physics, MIT, Cambridge, MA 02139, USA

Abstract

The free energy density, or pressure, of QCD has at high temperatures an expansion in the coupling constant $g$, known so far up to order $g^5$. We compute here the last contribution which can be determined perturbatively, $g^6 \ln(1/g)$, by summing together results for the 4-loop vacuum energy densities of two different three-dimensional effective field theories. We also demonstrate that the inclusion of the new perturbative $g^6 \ln(1/g)$ terms, once they are summed together with the so far unknown perturbative and non-perturbative $g^6$ terms, could potentially extend the applicability of the coupling constant series down to surprisingly low temperatures.
Lattice Calculations
Bazavov et al. PRD 90 (2009)
Thermodynamics with effective fugacities $z(t)$ which capture all interaction

\[ f_{eq}^{g/q} = \frac{1}{z_{g/q}^{-1} \exp(\beta p) + 1} \]
Summary

We have investigated two possible scenarios corresponding to $q<1$ and $g>1$ which correspond to different physical interpretations (correlation or fluctuations – 2 different environments).

For $q<1$ we observe decreasing of pressure which reaches negative value for a broad range of temperatures.

**Lower Entropy - correlated particles in equilibrium.**

The $q>1$ we observe the increasing of critical barion chemical potential and therefore above the critical line we have quark gas and not mixed phase.

For a given density we obtain bigger pressure,

**Higher Entropy out of equilibrium - decay into hadrons**

- stiffer EOS like in Walecka model
Nonextensive example – Dynamical Bag Model

Finite volume effect in compressed medium

Nucleon inside saturated NM

Compressed inside Neutron Star or in H I collision

Nucleon properties inside compressed nuclear matter

Equation of State for Nuclear Matter:

- Monopole excitation of nuclei: „Breathing Mode“

Exotics: Strange matter, Δ matter, Dibaryons,…

Compressibility of nuclear matter
Vibrations of giant monopole resonances

Electric L=0

\( E_0, T=0 \)

Electric L=0

\( E_0, T=1 \)
(In) Compressibility from GMR

\[ K_A = K_{\text{vol}} + K_{\text{surf}} A^{-1/3} + K_{\text{curv}} A^{-2/3} \]

\[ + (K_{\text{sym}} + K_{ss} A^{-1/3}) \left( \frac{N-Z}{A} \right)^2 \]

\[ + K_{\text{Coul}} \frac{Z^2}{A^{4/3}} + \cdots , \]

where \( K_A \) is defined by

\[ K_A = \frac{m}{\hbar^2} E_{\text{GMR}}^2 \langle r^2 \rangle . \]

K\( \sim \)200MeV !!
Monopole giant resonances and nuclear compressibility in relativistic mean field theory

Vretener at al.

Table 2: Constrained GCM energies of isoscalar monopole states. The values of $K_{nm}$ and the excitation energies are in MeV.

<table>
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<th>$K_{nm}$</th>
<th>$^{16}\text{O}$</th>
<th>$^{40}\text{Ca}$</th>
<th>$^{48}\text{Ca}$</th>
<th>$^{90}\text{Zr}$</th>
<th>$^{208}\text{Pb}$</th>
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<td>1</td>
<td>NL-1</td>
<td>211.7</td>
<td>20.2</td>
<td>16.6</td>
<td>15.9</td>
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<td>18.9</td>
<td>16.9</td>
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<td>25.0</td>
<td>22.0</td>
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<td>27.1</td>
<td>24.4</td>
<td>23.0</td>
<td>21.9</td>
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Table 3: Constrained GCM energies of isovector monopole states. The values of $a_{\text{sym}}$ and the excitation energies are in MeV.

<table>
<thead>
<tr>
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<th>$a_{\text{sym}}$</th>
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<th>$^{90}\text{Zr}$</th>
<th>$^{208}\text{Pb}$</th>
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<td>NL-3</td>
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<td>27.4</td>
<td>18.0</td>
</tr>
<tr>
<td>NL-SH</td>
<td>36.1</td>
<td>28.5</td>
<td>27.9</td>
<td>18.4</td>
</tr>
<tr>
<td>NL-2</td>
<td>43.9</td>
<td>30.3</td>
<td>28.8</td>
<td>16.9</td>
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<tr>
<td>exp. [21]</td>
<td>31.1±2.2</td>
<td>28.5±2.6</td>
<td>26.0±3.0</td>
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</tr>
</tbody>
</table>
New Generation of Experiments:

1. $\alpha p \rightarrow \alpha X$ (Saclay)

$\Gamma \approx 190$ MeV

$\Gamma \approx 400$ MeV

- scalar-isoscalar probe $\alpha$
- however:
- interfering background from projectile excitation

Hirenzaki et al., PRC 53, 277 (1996)

Morsch et al., PRL 69, 1336 (1992) and PRC 61, 024002 (1999)
**Equation of State**

Compressibility of the nucleon:
- Monopole excitation of the nucleon: „Breathing Mode“

**Compressibility of nucleon**

Quark sum rules \( K_N^{-1} \Rightarrow M E_x^2 <r_N^2> \) (Morsch PRL (1992) and later works)
So far excluded volume effects were calculated for a constant nucleon mass and in constant radius.

To improve it let us introduce enthalpy in a Bag Model of nucleon.

- **Enthalpy** is a measure of the total energy of a thermodynamic system. It includes the system's internal energy and thermodynamic potential (a state function), as well as its volume $\Omega$ and pressure $p_H$ (the energy required to "make room for it" by displacing its environment, which is an extensive quantity).

\[
H_A = E_A + p_H \Omega_A \quad \text{Nuclear Enthalpy} \quad (1)
\]

\[
H_N = M_{pr} + p_H \Omega_N \quad \text{Nucleon Enthalpy} \quad (2)
\]

Our version of Hugenholtz-Van Hove relation for finite nucleons in NM

\[
H_A^T/A = \varepsilon_A - (\partial M_A/\partial \Omega_A)_A/\rho = \varepsilon_A + p_H/\rho = E_F
\]
Enthalpy vs Hugenholz - van Hove relation with chemical potential

\[ \mu \equiv (\partial M_A / \partial A)_\Omega_A \equiv (\partial H_A / \partial A)_{p_H} = \varepsilon_A + \frac{p_H}{\varrho} = H_A / A \]

\[ E_F \equiv P^0_N (P_F) = (\partial M_A / \partial A)_{\Omega_A} = \varepsilon_A + p_H / \varrho = \mu \]

In the NM in equilibrium \( p_H = 0 \) therefore \( H_A = E_A \). Dividing \( H_A \) by \( A \) we obtain the following relation between single particle enthalpy \( h_A \) and \( \varepsilon_A = E_A / A \),

\[ h_A = \varepsilon_A + p_H / \varrho. \]  

(1a)

Please note that the same equation fulfills a Fermi energy \( E_F \equiv P^0_N (p_F) = \varepsilon_A + p_H / \varrho \) of nucleon with a Fermi momentum \( p_F \); well-known as the HvH [9] relation, also proven in the self-consistent RMF approach [10]. It turns out that definitions of the Fermi energy or a single particle enthalpy have the same energy balance.

We will argue that the enthalpy rather than the rest energy should be used in the momentum distribution sum rules (MSR) in NM and in a nucleon.
Bag Model in Compress Medium

\[ E_{Bag}^0(R) = \frac{3\omega_0 - Z_0}{R} + \frac{4\pi}{3} B(\varphi_0) R^3 \sim \frac{1}{R}, \]

\[ p_H = p_B = \frac{3\omega_0 - Z_0}{4\pi R^4} - B(\varphi) \rightarrow \left( B(\varphi) + p_H \right) R^4 = const \]

\[ R = \left[ \frac{3\omega_0 - Z_0}{4\pi (B(\varphi) + p_H)} \right]^{1/4}, \]

\[ M_{pr} = E_{Bag} = 4\pi R^3 \left[ \frac{4}{3} (B + p_H) - \frac{p_H}{3} \right] = E_{Bag}^0 \frac{R_0}{R} - p_H \Omega_N. \]

\[ H_N = E_{Bag}^0 \frac{R_0}{R} \sim \frac{1}{R}. \]
Two Scenarios affecting nuclear compressibility $K^{-1}$

- **Constant Mass**
  - $= \text{Increasing Enthalpy}$
  - $1/R$

- **Constant Volume**
  - $= \text{Constant Enthalpy}$

Energy transfer to bags

No energy transfer to bags
Thermodynamical Consistency

\[ H_A^T/A = \varepsilon_A - (\partial M_A/\partial \Omega_A) \frac{1}{\rho} = \varepsilon_A + \rho H/\rho = E_F. \]

\[ M_{pr}(\varrho) = M_N - \rho H(\varrho) \Omega_N \]

\[ \rho H(\varrho) = \varrho^2 \varepsilon'_A(\varrho)/(1 - \varrho \Omega_N). \]

\[ \delta \rho_H/\delta \rho = E_F = \mu. \]
Nuclear compressibility and two scenarios

A - Constant Nucleon Mass

\[ K_N^{-1} \big|_{MN,R\to R_0} = -3\Omega_N^2 \frac{\partial [M_N (R_0 / R - 1) / \Omega_N]}{\partial \Omega_N} = M_N \simeq 940 \text{ MeV} \]

B - Constant Nuclear Radius

\[ K_N^{-1} \big|_{\Omega_N \to \infty} \]

Semi-experimental Value

Quark sum rules \( K_N^{-1} \Rightarrow M E_x^2 \langle r_N^2 \rangle \) (Morsch PRL (1992) and later works)

\[ K_N^{-1} = (1.4 \pm 0.3) \text{ GeV} \]

7 GeV alfa-p

Same For scenario A (with energy transfer to bag) and B (no transfer):

\[ K_A^{-1} - K_N^{-1} = 9 \Omega_N^2 \frac{\partial^2 p_H \Omega_N}{\partial \rho^2} = 9 \Omega_N^2 \frac{\partial^2}{\partial \rho^2} \left[ \frac{r \rho}{1-r} \frac{\partial \varepsilon}{\partial \rho} \right] = \frac{9 r \rho^2}{1-r} \left[ f(\rho) \frac{d \varepsilon}{d \rho} + \left( \frac{6-5 r}{1-r} \right) \frac{d^2 \varepsilon}{d \rho^2} + \ldots \right] \]

\[ K_N^{-1} \big|_{p_H=0} \approx \left[ \frac{(1 - \rho \Omega_N)^2}{1 + 4 \rho \Omega_N (1 - \rho \Omega_N)} \right] K_A^{-1} \simeq \frac{1}{2} K_N^{-1} \big|_{p_H=0} + K_N \]
Dynamical Bag

Bag constant in function of nuclear pressure

\[ B = B(\rho_0)(R_0/R)^4 - p. \]

Nucleon radius in compressed NM for a constant nucleon mass

\[ M_N R_0/R = M_N + 4/3\pi R^3 p. \]
Dynamical Bag - continue

Nucleon Mass for different nucleon radii in compressed NM

\[ M_{pr}(\rho) = M_N - p_H(\rho)\Omega_N, \]
\[ p_H(\rho) = \rho^2\varepsilon_A(\rho)/(1 - \rho\Omega_N). \]

Nuclear compressibility for different constant nucleon radii in compressed NM

\[ K^{-1} = 9\partial p_H/\partial \rho \]

\[ K^{-1} = 235 \text{ MeVfm}^{-3} \]
Energy transfer to nucleon

Energy Transfer [MeV]

$R_0 = 0.7 \text{ fm}$

Energy Density [MeV/fm$^3$]

$R_0 = 0.7 \text{ fm}$

- nucleon - $M_N$ const
- nucleon - $R$ const
- nuclear
Equation of State
Energy transfer

\[ \varepsilon_{cr}^B = \frac{M_N}{\varepsilon_A} \varepsilon_{cp} = \frac{3M_N}{4\pi \varepsilon_B(\varepsilon_{cr}) R^3(\varepsilon_{cr})}. \]

Order of phase transition?

No energy transfer

\[ \varepsilon_{cr}^A = \frac{(M_N - p\Omega_N)}{\varepsilon_A} \varepsilon_{cp} = \frac{M_N}{\varepsilon_A(\varepsilon_{cr})} \varepsilon_{cp} - p/\varepsilon_A. \]
Finite Nucleon Volumes - Conclusions

A. Lower compressibility

B. Constant nucleon mass requires increasing enthalpy energy transfer to bag

STIFFER EOS

C. Constant nucleon volume give the constant enthalpy with decreasing nucleon mass

SOFTER EOS

Similar to Relativistic Mean Field nonlinear corrections

\[ U(\sigma) = \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{3} b_2 \sigma^3 + \frac{1}{4} b_3 \sigma^4 \]

which provide proper compressibility of Nuclear Matter and were introduced by Boguta & Bodmer – (new fit?)
Finite volume effect in compressed medium

Nucleon inside saturated NM

Compressed inside Neutron Star or in HI collision

Where? pressure
Neutron Stars

Average density $5 \times 10^{14} \text{g/cm}^3 \sim 2$ nuclear density

Masa~2M - New Limit of Tolman-Oppenheimer-Volkoff equation
- Large Mass ($\sim 2 \, M_\odot$) and radius ($R \geq 12 \, \text{km}$) $\Rightarrow$ stiff quark matter EoS;  
  Note: DU problem of DBHF removed by deconfinement! and: CFL core Hybrids unstable!

- Flow in Heavy-Ion Collisions $\Rightarrow$ not too stiff EoS!  
  Note: Quark matter removes violation by DBHF at high densities

Astrophysical observations and "Data"

Average density $5 \times 10^{14}$ g/cm$^3$ ~ 2 nuclear density

A new quark-hadron hybrid equation of state for astrophysics

I. High-mass twin compact stars

Sanjin Benić$^{1,2}$ *, David Blaschke$^{3,4}$ **, David E. Alvarez-Castillo$^{4,5}$ ***, Tobias Fischer$^{3}$ †, and Stefan Typel$^6$

Volume effects with constant mass and radius !!!
“Neutron” Star Composition in 2005
“Neutron” Star Composition in 2005

Proton superconductivity and the masses of neutron stars
Negreiros, Schramm, Weber
The nuclear equation of state

\[ E/A - m_N \] (MeV) as a function of the density \( \rho \) (fm\(^{-3}\))

**T=0**

- **hard**
  - \( K=380 \text{ MeV} \)
  - 380 w/PT
  - 210

- **soft**
  - -16MeV

**Pressure as a function of density (astrophysics)**
Przybliżenie nukleonów przez obiekty punktowe w obliczeniach struktury jądrowej jest chyba najpoważniejszym niekontrolowanym przybliżeniem - (P. Ring)

Dotyczy ono np. takiej wielkości jak ścisłość materii jądrowej. Wiemy że same nukleony mają swoją „twardość” więc wydaje się oczywiste że obliczenia ścisłości jądrowej zależeć będą od ścisłości zawartych w nich nukleonów. Taki rachunek został przeprowadzony i prowadzi do istotnej modyfikacji jądrowego równanie stanu.

Dotychczasowe rachunki albo wogóle nie uwzględniają rozmiarów nukleonu albo ustalają jego rozmiary i masę dla całego przebiegu gęstości i ciśnienia.

Dlatego głównym motywem wyboru takiego tematu pracy jest uwzględnienie rozmiarów nukleonu, zarówno dla gęstości równowagi klasycznej materii jądrowej której składnikami są hadrony o zmieniających się rozmiarach, jak i w obszarze krytycznym gdzie następuje przejście fazowe z materii hadronowej do (silnie skorelowanej?) materii kwarkowej.
Jaka jest masa nukleonu w ośrodku materii jądrowej? Czy może zależeć od modelu?

Wiemy z opisu reakcji jądrowych że masa nukleonu się nie zmienia w materii jądrowej dla gęstości równowagi. Nie jest to już tak oczywiste w relatywistycznej teorii średniego pola RMF gdzie są nukleony poruszają się w silnych polach skalarnym i wektorowym.

Dlatego opis efektu EMC na tarczy związanego nukleonu odpowiedział no to pytanie.
Efekt EMC +DY - przedstawione dopasowania głęboko nieelastycznych procesów uzyskano dla niezmienionej masy nukleonu w ośrodku.

\[ M_x = M_N + \frac{1 - f(x)}{2} < V_N > \]

Shifting pion mass

Only 1% of nuclear pions


The relativistic nuclear dynamics of nucleons in the nucleus is described by the Light Cone (LC), momentum distribution function $f_N(y)$ (Jaffe) where $y = AP_N^+/P_A^+$, a fraction of longitudinal momentum of A nucleons in the nucleus is Lorentz invariant. Let us now focus our attention on the sum rule for longitudinal momenta $P_N^+ = P_N^0 + P_N^Z$. Do they sum in the rest frame to the nuclear energy $E_A$, or rather to nuclear enthalpy $H_A$? To answer this question we can examine the distribution

$$f_N(y) = \int \frac{d^4 P_N}{(2\pi)^4} \delta \left(y - \frac{A P_N^+}{P_A^+}\right) Tr[\gamma^+ S(P_N, P_A)].$$

(4)
Finally with a good normalization of $S_N$ we have:

\[
f_N(y) = \frac{4}{\varrho} \int_0^{P_F} \frac{S_N(P_N) d^3P_N}{(2\pi)^3} \left(1 + \frac{P_N^3}{E_N^*}\right) \delta(y - AP_N^+/P_A^0) = (3/4)\left[\frac{P_A^0}{(AP_F)}\right]^3 \left[(AP_F/P_A^0)^2 - (y - AEF/P_A^0)^2\right]. \quad (5)
\]

\[P_A^0 = \varepsilon_A = AE_A\]

and Momentum Sum Rule

\[
\frac{1}{A} \int dy y f_N(y) = \frac{E_F}{P_A^0} \equiv \frac{\partial}{\partial A} \left(\frac{E_A}{\Omega_A}\right) = \frac{\varepsilon_A + pH/\varrho}{P_A^0}. \quad (6)
\]

\[
\int dy y f_N(y) = \frac{E_F}{h_A} = 1. \quad \text{Enthalpy/A}
\]
So far excluded volume effects were calculated for a constant nucleon mass and in constant radius.

To improve it let us introduce enthalpy in a Bag Model of nucleon.

- **Enthalpy** is a measure of the total energy of a thermodynamic system. It includes the system's internal energy and thermodynamic potential (a state function), as well as its volume $\Omega$ and pressure $p_H$ (the energy required to "make room for it" by displacing its environment, which is an extensive quantity).

$$H_A = E_A + p_H \Omega_A$$  \hspace{1cm} \text{Nuclear Enthalpy} \hspace{1cm} (1)$$

$$H_N = M_{pr} + p_H \Omega_N$$  \hspace{1cm} \text{Nucleon Enthalpy} \hspace{1cm} (2)$$

Our version of Hugenholtz-Van Hove relation for finite nucleons in NM

$$H_A^T/A = \varepsilon_A - (\frac{\partial M_A}{\partial \Omega_A})_A/q = \varepsilon_A + p_H/q = E_F$$
Wnioski i co dalej.

1. Efekty związane ze zmianą objętości i masy nukleonów w materii jądrowej odkrywają niezmiernie ważną rolę w równaniu stanu i w związku z tym są istotne w każdym obszarze zastosowań: od ścisłości dla gęstości równo wagi do obliczeń modelowych mas gwiazd neutronowych i eksperymentów ciężkojonowych.

2. Rachunki są kontynuowane dla gorącej materii jądrowej z uwzględnieniem hiperonów i rezonansów barionowych.


4. Kwarki konstytuentne?
Finally with a good normalization of $S_N$ we have:

$$f_N(y) = \frac{4}{9} \int_0^{P_F} \frac{S_N(P_N) d^3P_N}{(2\pi)^3} \left(1 + \frac{P^3_N}{E^*_N}\right) \delta(y - AP_N^+/P_A^0) =$$

$$= (3/4)[P_A^0/(AP_F)]^3\left[(AP_F/P_A^0)^2 - (y - AE_F/P_A^0)^2\right]. \quad (5)$$

$$P_A^0 = E_A = A\varepsilon_A$$

and Momentum Sum Rule

$$\frac{1}{A} \int dy y f_N(y) = \frac{E_F}{P_A^0} \equiv \frac{\partial}{\partial A} \left(\frac{E_A}{\Omega_A}\right) = \frac{\varepsilon_A + pH/\rho}{P_A^0}. \quad (6)$$

$$\int dy y f_N(y) = \frac{E_F}{h_A} = 1.$$
A model for parton distribution

Prumordial quark transverse momentum distribution $p_{\text{rest}}^+ = H(R)$

$$k^+ = x p^+$$

Kinematical conditions for Monte Carlo technique

$$m_i^2 < j^2 < W^2 \quad \text{and} \quad r^2 > \sum_i m_i^2$$

Neglecting transverse quark momenta

$$f_i(x) = N'({\tilde{\sigma}}_i) \exp \left(-\frac{x^2}{4\tilde{\sigma}_i^2}\right) \text{erf} \left(\frac{1-x}{2\tilde{\sigma}_i}\right)$$

$$\tilde{\sigma} = \frac{1}{d_n m_n}$$
2. The valence quark and gluon distributions obtained from the model applied to (a) the proton, (b) the pion, (c) the strange meson $K^+$, the charm meson $D^0$. The Gaussian widths used are 135 MeV for gluons and 150 MeV for $q$ and $\bar{q}$, except $\sigma_u = 180$ MeV in (a)
Fig. 2. The valence quark and gluon distributions obtained from the model applied to (a) the proton, (b) the pion, (c) the strange meson $K^+$, (d) the charm meson $D^0$. The Gaussian widths used are 135 MeV for gluons and 150 MeV for $q$ and $\bar{q}$, except $\sigma_g = 180$ MeV in (a).
Fig. 3. The DIS structure function $F_2$ versus $Q^2$ in bins of $x$. Fixed target NMC data [7] compared to the model starting from only valence quarks and gluons (dashed) and including also a sea quark component (full). (The small break in the curves at $Q^2 \sim m_c^2$ is due to the charm threshold.)
Nuclear Models - equilibrium

\[ M_x = M_N + \frac{(1 - f(x))}{2} < V_N > \]

Shifting pion mass

Only 1% of nuclear pions


Fig. 1. (Color online) Schematic view of $S_\sigma = \tilde{S}_{q}^{(R=a,b)}$ defined by Eqs. (54) and (55) (left panel) and $S_{q}^{(c)} = \tilde{S}_{q}^{(R=c)}$ defined by Eq. (56) (right panel), all presented as a function of the scaled variable $x/\mu$. The respective values of $q$ used are shown. All these results are compared with the extensive case of $q = 1$ (blue curves). Calculations were performed assuming $M_{q1} = 0$, $T = 60$ MeV and $\mu = 322$ MeV. The meaning of $q$ and $\tilde{q}$ on the right panel corresponds to definition of $\tilde{q}$ presented in Eq. (36).

Fig. 2. (Color online) The same as in the left panel of Fig. 1 but for different masses $M_\sigma$. 
Toy Model

(Neglecting transverse quark momenta)

\[ f_i(x) = N'(\tilde{\sigma}_i) \exp \left( - \frac{x^2}{4\tilde{\sigma}_i^2} \right) \text{erf} \left( \frac{1 - x}{2\tilde{\sigma}_i} \right) \]

\[ \tilde{\sigma} = \frac{1}{d_h m_h} \]

In our case \( d_h m_h \Rightarrow R^*H(R) \) is const.

But the \( x=k^+/H(R) \) depends on nucleon radius
Two Scenarios
for NN repulsion with qq attraction

• Constant Mass
  = Increasing Enthalpy
  \[ \frac{1}{R} \]

• Constant Volume
  = Constant Enthalpy
Large Mass ($\sim 2 M_\odot$) and radius ($R \geq 12 \text{ km}$) $\Rightarrow$ stiff quark matter EoS;

Note: DU problem of DBHF removed by deconfinement! and: CFL core Hybrids unstable!

Flow in Heavy-Ion Collisions $\Rightarrow$ not too stiff EoS!

Note: Quark matter removes violation by DBHF at high densities
Astrophysical "data"

A new quark-hadron hybrid equation of state for astrophysics

I. High-mass twin compact stars

Sanjin Benić¹,² *, David Blaschke³,⁴ **, David E. Alvarez-Castro⁴,⁵ ***, Tobias Fischer³ †, and Stefan Typeł⁶
The E0 strength distribution in $^{90}$Zr, $^{116}$Sn, $^{144}$Sm and $^{208}$Pb have been measured with inelastic scattering of 240 MeV α particles at small angles. The E0 strengths in $^{116}$Sn, $^{144}$Sm and $^{208}$Pb were found to be concentrated in symmetric peaks and centroid of the strength distributions were located at $E_x = 16.00 \pm 0.7$, $15.31 \pm 1.1$, and $14.24 \pm 1.1$ MeV respectively. In $^{90}$Zr the E0 distribution was found to have a high energy tail extending up to $E_x = 25$ MeV. The resulting centroid of the E0 strength for $^{90}$Zr is $E_x = 17.89 \pm 0.20$ MeV. These results and the previously reported result for $^{40}$Ca lead to $K_{nm} = 231 \pm 5$ MeV by comparing to microscopic
Nuclear convolution Model