Residual Symmetry and Scaling Ansatz in Neutrino Mass Matrix: Maximal CP violation

Rome Samanta, Probir Roy and Ambar Ghosal

Saha Institute of Nuclear Physics, India

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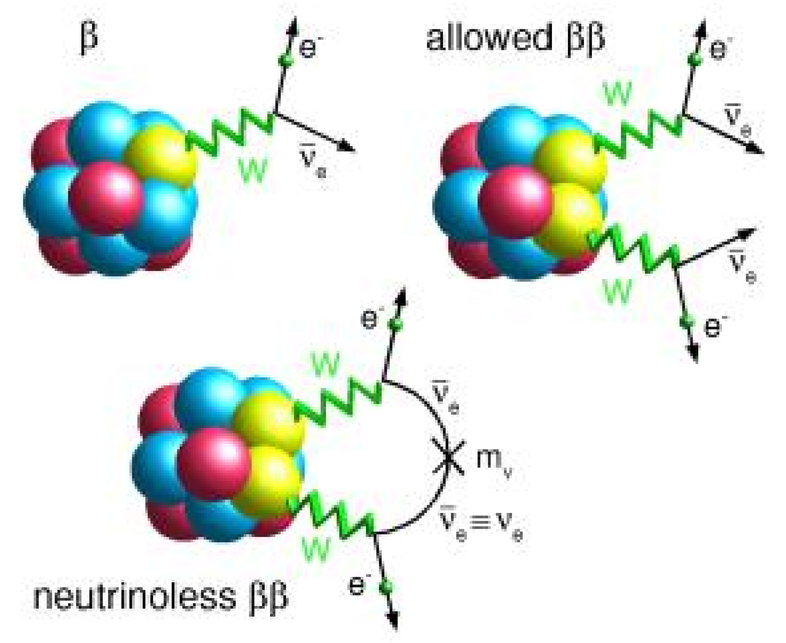

Abstract

The residual symmetry approach, pioneered by Lam and given a complex extension for invariance under $\mu\tau$ interchange by Grimus and Lavoura, is a powerful tool to uncover the flavor structure of the 3×3 neutrino Majorana mass matrix M_ν. We utilize this to propose a complex extension of the real scaling ansatz for M_ν which was introduced some years ago. Unlike the latter, our proposal allows a nonzero mass for each of the three light neutrinos as well as a non vanishing θ_{13}. A major falsifiable result of this scheme is that leptonic Dirac CP violation must be maximal while atmospheric neutrino mixing need not to be exactly maximal. Moreover, each of the two Majorana phases, to be probed by the search for $0\nu\beta\beta$ decay, has to be zero or π. There are other interesting consequences such as the allowed occurrence of a normal hierarchy.

Introduction

If $G^TM_\nu G = M_\nu$ is a horizontal symmetry and $U^TM_\nu U = M_d$ is the diagonalization condition, then another unitary matrix $V = Ud$ also diagonalizes M_ν, provided d is diagonal matrix with $d_{jj} = \pm 1$. The matrix G and d are connected with the relation

$$U^\dagger G U = d. \tag{1}$$

Now as the entries of d are ± 1, it defines a Z_2 symmetry, and hence, G is also representation of z_2 symmetry. Although there are eight structures of d it can easily be proved that among them only two are independent, and hence, there is always two independent G matrices or in other words there is always a hidden $Z_2\times Z_2$ symmetry in the mass matrix. Motivated by this idea, we investigate the scaling neutrino mass matrix interpreting the ansatz as a $Z_2\times Z_2$ symmetry and further extend the symmetry to find relevant phenomenology.

Main Objectives

- Scaling ansatz in neutrino mass matrix is a good approximate symmetry in neutrino mass matrix.
- C.S. Lam argued in a novel way that if the Majorana mass term of the Lagrangian enjoys a horizontal symmetry, it has to be $Z_2\times Z_2$ symmetry.
- Scaling ansatz can also be interpreted as a $Z_2\times Z_2$ symmetry.
- As in the leading order, scaling mass matrix leads to vanishing value of θ_{13}, the residual $Z_2\times Z_2$ symmetry is not a good symmetry, and should be modified somehow.
- Best way to generate nonzero θ_{13} is to take the theory under the lamppost of another symmetry with the previous one, without breaking the symmetry which are well studied in literature.

Materials and Methods

Initial idea: We first consider the scaling neutrino mass matrix originally proposed by Mohapatra and Rodejohann.

$$M_\nu = \begin{pmatrix} X & -Yk & Y \\ -Yk & Zk^2 & -Zk \\ Y & -Zk & Z \end{pmatrix}. \tag{2}$$

The corresponding mixing matrix is

$$U = \begin{pmatrix} c_{12} & s_{12}e^{i\frac{\alpha}{2}} & 0 \\ -\frac{ks_{12}}{\sqrt{1+k^2}} & \frac{kc_{12}e^{i\frac{\alpha}{2}}}{\sqrt{1+k^2}} & \frac{e^{i\frac{\beta}{2}}}{\sqrt{1+k^2}} \\ \frac{s_{12}}{\sqrt{1+k^2}} & -\frac{c_{12}e^{i\frac{\alpha}{2}}}{\sqrt{1+k^2}} & \frac{ke^{i\frac{\beta}{2}}}{\sqrt{1+k^2}} \end{pmatrix}. \tag{3}$$

Thus G_2 and G_3 can be calculated as

$$G_2^k = \begin{pmatrix} -\cos 2\theta_{12} & \frac{k\sin\theta_{12}}{\sqrt{1+k^2}} & -\frac{\sin\theta_{12}}{\sqrt{1+k^2}} \\ \frac{k\sin\theta_{12}}{\sqrt{1+k^2}} & \frac{k^2\cos 2\theta_{12}-1}{1+k^2} & \frac{-k(\cos 2\theta_{12}+1)}{1+k^2} \\ -\frac{\sin\theta_{12}}{\sqrt{1+k^2}} & \frac{-k(\cos 2\theta_{12}+1)}{1+k^2} & \frac{\cos 2\theta_{12}-k^2}{1+k^2} \end{pmatrix}, G_3^{scaling} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & \frac{1-k^2}{1+k^2} & \frac{2k}{1+k^2} \\ 0 & \frac{2k}{1+k^2} & \frac{k^2-1}{1+k^2} \end{pmatrix} \tag{4}$$

As previously mentioned, these G_is can not be the only symmetry as it leads to vanishing value of θ_{13}. In the next section we consider the idea of CP transformation along with one of the root symmetry $G_3^{scaling}$.

Idea for viable neutrino mass matrix

Taking

$$G_3^{scaling}M_\nu G_3^{scaling} = M_\nu^* \tag{5}$$

instead of

$$G_3^{scaling}M_\nu G_3^{scaling} = M_\nu \tag{6}$$

Which implies

$$G_3U^* = U\tilde{d} \tag{7}$$

with $\tilde{d}$ a a diagonal matrix. Once again, $\tilde{d}_{lm} = \pm\delta_{lm}$ if neutrino mass m_1, m_2, m_3 are all nondegenerate. Now the LHS of the Eqn.(7)

$$\begin{pmatrix} -(U_{e1}^{CES})^* & -(U_{e2}^{CES})^* & -(U_{e3}^{CES})^* \\ \frac{1-k^2}{1+k^2}(U_{\mu 1}^{CES})^* + \frac{2k}{1+k^2}(U_{\tau 1}^{CES})^* & \frac{1-k^2}{1+k^2}(U_{\mu 2}^{CES})^* + \frac{2k}{1+k^2}(U_{\tau 2}^{CES})^* & \frac{1-k^2}{1+k^2}(U_{\mu 3}^{CES})^* + \frac{2k}{1+k^2}(U_{\tau 3}^{CES})^* \\ \frac{2k}{1+k^2}(U_{\mu 1}^{CES})^* - \frac{1-k^2}{1+k^2}(U_{\tau 1}^{CES})^* & \frac{2k}{1+k^2}(U_{\mu 2}^{CES})^* - \frac{1-k^2}{1+k^2}(U_{\tau 2}^{CES})^* & \frac{2k}{1+k^2}(U_{\mu 3}^{CES})^* - \frac{1-k^2}{1+k^2}(U_{\tau 3}^{CES})^* \end{pmatrix} \tag{8}$$

Now $\tilde{d}$ has eight structures. However, as we already know the parametrization of U_{PMNS}, only the following structures are allowed

$$\tilde{d}_a \equiv diag.(-1,1,1), \tag{9}$$
$$\tilde{d}_b \equiv diag.(-1,1,-1), \tag{10}$$
$$\tilde{d}_c \equiv diag.(-1,-1,1), \tag{11}$$
$$\tilde{d}_d \equiv diag.(-1,-1,-1). \tag{12}$$

These structures of $\tilde{d}$ and Eqn (7) leads to the following equations:

Elements of U^{CES}	Constraint conditions
$\mu 1$	$2kU_{\mu 1}^{CES} = (1-k^2)U_{\tau 1}^{CES} - (1+k^2)(U_{\tau 1})^*$
$\tau 1$	$2kU_{\tau 1}^{CES} = -(1-k^2)U_{\mu 1}^{CES} - (1+k^2)(U_{\mu 1})^*$
$\mu 2$	$2kU_{\mu 2}^{CES} = (1-k^2)U_{\tau 2}^{CES} + \eta(1+k^2)(U_{\tau 2})^*$
$\tau 2$	$2kU_{\tau 2}^{CES} = -(1-k^2)U_{\mu 2}^{CES} + \eta(1+k^2)(U_{\mu 2})^*$
$\mu 3$	$2kU_{\mu 3}^{CES} = (1-k^2)U_{\tau 3}^{CES} + \xi(1+k^2)(U_{\tau 3})^*$
$\tau 3$	$2kU_{\tau 3}^{CES} = -(1-k^2)U_{\mu 3}^{CES} + \eta(1+k^2)(U_{\mu 3})^*$

The equations presented above implies the following predictions in the mixing sector:

$\tilde{d}$	α	β	$\cos\delta$
$\tilde{d}_a = diag(-1,+1,+1)$	π	0	0
$\tilde{d}_b = diag(-1,+1,-1)$	π	π	0
$\tilde{d}_c = diag(-1,-1,+1)$	0	0	0
$\tilde{d}_d = diag(-1,-1,-1)$	0	π	0

Along with $\tan\theta_{23} = k^{-1}$. Which means atmospheric mixing angle need **not to be strictly maximal.**

Mass matrix and the relevant phenomenology

The most general matrix invariant under Eqn. (7) can be written as

$$M_\nu^{CES} = \begin{pmatrix} x_1 & -y_1k + i\frac{y_2}{k} & y_1 + iy_2 \\ -y_1k + i\frac{y_2}{k} & z_1 - w_1\frac{k^2-1}{k} - iz_2 & w_1 - i\frac{k^2-1}{2k}z_2 \\ y_1 + iy_2 & w_1 - i\frac{k^2-1}{2k}z_2 & z_1 + iz_2 \end{pmatrix}. \tag{13}$$

We use 3σ ranges of the oscillation data and the upper bound (0.23 eV) on sum of the light neutrino masses to constrain the mass matrix.

Quantity	3σ ranges
$\lvert\Delta m_{31}^2\rvert$ (N)	2.31$< \Delta m_{31}^2(10^3 eV^{-2}) < 2.74$
$\lvert\Delta m_{31}^2\rvert$ (I)	2.21$< \Delta m_{31}^2(10^3 eV^{-2}) < 2.64$
Δm_{21}^2	7.21$< \Delta m_{21}^2(10^5 eV^{-2}) < 8.20$
θ_{12}	$31.3^o < \theta_{12} < 37.46^o$
θ_{23}	$36.86^o < \theta_{23} < 55.55^o$
θ_{13}	$7.49^o < \theta_{13} < 10.46^o$

Predictions

- Both types of hierarchy are present in the model.
- For normal hierarchy the lightest mass m_1 ranges from $10^{-}4$ eV- 0.07 eV and for inverted hierarchy the lightest mass m_3 ranges from 10^{-4} eV - 0.068 eV.
- For both the hierarchies, $|m_{ee}|$ can reach upto 0.14 eV. which will be probed by $GERDA$ phase II data.

Forthcoming Research

We are building explicit model for with Type I seesaw mechanism to obtain the general structure of M_ν. After the model we shall emphasize on leptogenesis.

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