Properties of strongly coupled quantum fluids

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1 Research project objectives

Strongly coupled quantum systems have been studied with increasing attention in recent years [1, 2, 3]. They can be found in nature in various forms, ranging from compact objects of astronomical size such as neutron stars to the tiniest droplet of Quark-Gluon Plasma (QGP) which exists only for extremely small fractions of a second in a heavy-ion collision experiment. Also, they can differ in temperature by orders of magnitude, ranging from micro Kelvin in ultracold atomic Fermi and Bose gases to QGP temperatures being almost a million times larger than in the centre of our sun. State-of-the-art and future technologies involve strongly coupled quantum systems in form of high-temperature superconductors or spintronic devices for information processing and storage.

A major goal is to better understand phenomena encountered in these many-body systems. With quite some surprise, it was found that strongly coupled quantum systems share interesting features despite their apparant differences and albeit being described by different microscopic theories. For example, very similar almost inviscous fluid dynamical flow was observed in ultracold Fermi gases [4, 5] and for the QGP [6]. This led to the conclusion that both systems constitute nearly perfect fluids with a shear viscosity to entropy density ratio as small as the KSS-limit [7] conjectured from the duality between gravity and quantum field theory.

Fermionic alkali atoms can form strongly coupled quantum fluids at extremely low temperatures when magnetically tuned into a Fano-Feshbach resonant scattering state by means of an external magnetic field. In this state, the scattering length is large compared to the interparticle spacing and the cross section is bound only by unitarity. In the unitary limit, the system is scale and conformally invariant and the properties of the matter are universal functions of temperature and density [8, 9, 10]. The unique possibility to manipulate the interaction strength, and to trap and cool a Fermi gas allowed experimentalists to study for the first time the theoretically predicted crossover between a Bardeen-Cooper-Schrieffer (BCS) type superfluid and a Bose-Einstein condensate (BEC) of strongly bound molecules [1]. This was one of the major scientific breakthroughs in the last decades.

The QGP is a particular color-deconfined phase, which QCD matter forms at high temperatures and densities [11]. At the very first stages in the cosmic evolution, our universe was filled with QGP for the blink of a moment. Here on earth, this Big Bang matter is transciently recreated in high-energy heavy-ion collision experiments [12]. Properties of the QGP can, however, only indirectly be inferred from experimental data as with decreasing temperature the expanding matter hadronizes and only color-confined hadrons arrive in the detectors. Of particular interest is the study of the QCD phase diagram. With low collision energy experiments, one aims at finding signatures of an anticipated critical point or an adjacent first-order phase transition region at large densities.

The aim of this project is to study the properties and the dynamics of both ultracold Fermi gases and the QGP. In particular the transport coefficients in these fluids, notably shear and bulk viscosities, give insight into the basic structure of the matter and the fundamental interactions among its constituents.

Their knowledge is crucial for approaches that aim at a dynamical description of the evolution of these systems. Fluctuations in conserved quantities, such as baryon number or energy, play also an important role. They enable us to study the thermodynamic properties of the matter and to explore its phase diagram. Moreover, they are important for a consistent description of dissipative processes taking place in these systems. My goal is to better understand the thermodynamic and transport properties of the two different strongly coupled quantum fluids, to look for similarities and differences, thus, helping to span a bridge between different active research fields in physics.

2 Work plan

The topics related to ultracold Fermi gases and the QGP, which will be addressed in this project, are outlined in this section. A list of milestones and an estimate for the time schedule are given.

2.1 Ultracold Fermi gases

First-principle calculations of the properties of ultracold Fermi gases are provided by functional methods such as T-matrix calculations [13], Monte Carlo techniques [14, 15, 16] or Functional Renormalization Group (FRG) approaches [17]. Insight into the behavior of ultracold Fermi gases in the dilute gas limit can be obtained via kinetic theory. Moreover, the collective behavior of the system near unitarity and at low temperatures is correctly described by fluid dynamics [18]. The topics to be addressed are:

- *Determination of the temperature and density dependence of the shear viscosity in unitary Fermi gases*: The shear viscosity of a unitary Fermi gas can be measured by studying the expansion dynamics of a gas cloud, which is initially confined in an asymmetric optical trap and then released from the trap: pressure gradients accelerate the cloud stronger in the shorter direction, while the shear viscosity works against this differential acceleration. Analyzing expansion data with fluid dynamical simulations allows one to extract the shear viscosity. In contrast to the centre, the dilute corona of the gas cloud does, however, not behave fluid dynamically. To deal with this situation, we developed a new framework called anisotropic fluid dynamics for non-relativistic fluids [19]. A similar development took place for relativistic fluids in order to describe the dynamics of a heavy-ion collision more realistically [20, 21, 22]. We applied the framework [19] to analyze the high-temperature expansion data [5]. In this way, we verified expectations from kinetic theory for the temperature dependence of the shear viscosity without making uncontrolled assumptions about the corona [23]. As a next step, I will apply this method to the expansion data at temperatures just above the superfluid transition [24, 25] in order to constrain the temperature and density dependence of the shear viscosity in this region. This will require to employ the measured equation of state [10].

- *Study of the properties in the superfluid phase*: At very low temperature, the unitary Fermi gas undergoes a second-order phase transition into a superfluid phase [3, 10]. A realistic treatment of this phase in the centre of the gas cloud requires the development of a new framework, which combines the evolution of an inviscous and a viscous normal fluid [26]. By extending the existing anisotropic fluid dynamics code [19] to a two-fluid scenario, I will be able to achieve this goal. Similar scenarios have been proposed to describe heavy-ion collisions at high densities, cf. [27]. The developed two-fluid model will allow me to study transport phenomena such as second sound [28] dynamically.

- *Study of spin diffusion in dynamical simulations*: The above mentioned idea of a two-fluid framework will, in principle, allow me to analyze experimental data on the transport of spin in different spatial dimensions [29] and to determine the spin diffusion coefficient in unitary Fermi gases. At low temperature, Luttinger-Ward theory predicts a universal quantum limit for spin diffusion [30]. Within an extended anisotropic two-fluid framework, which embeds spin diffusion, I will be able to test this expectation for the first time. Moreover, I will study a possible connection between momentum diffusion, as measured by the shear viscosity, and spin diffusion. A similar connection between shear viscosity and heavy-quark diffusion in the QGP has been pointed out in [31]. The analysis of spin diffusion will help understanding the mechanisms of charge transport in matter. This is of fundamental importance in spintronics, the transfer of information and its storage.

- *Impact of medium effects on the spin transport away from the unitary limit*: In the unitary limit, when interactions are strongest, the bulk viscosity vanishes [9]. Away from unitarity, one can study the transport coefficients of ultracold Fermi gases in the dilute regime by applying kinetic theory. In this regime, the bulk viscosity was found to be non-zero [32]. In [33], we studied the shear viscosity. We found that with decreasing temperature the shear viscosity is more and more minimized on the BEC-side of the crossover in qualitative agreement with previously made experimental observations [34]. In kinetic theory, this anomalous behavior of the shear viscosity can be understood as a medium effect: Pauli-blocking is found to be more efficient in reducing the scattering cross section on the BCS-side of the crossover. I will extend my previous calculation [33] to study spin diffusion away from the unitary limit. This will require to take medium effects on the scattering of particles with opposite spin into account. It will be interesting to see if spin diffusion exhibits a similar unexpected behavior. A corresponding experiment has not yet been performed. I will propose such a measurement and make a prediction for its results.

- *Study of thermal fluctuations in dynamical simulations of ultracold Fermi gases*: Thermal fluctuations play an important role for the dynamical description of fluids undergoing phase transitions or developing instabilities [35]. The nonlinearities in the fluid dynamical equations can amplify the effect of microscopic fluctuations causing macroscopic phenomena. In the case of non-relativistic fluids, as relevant in molecular biology or chemistry, it was shown that a proper numerical treatment of thermal fluctuations in line with the fluctuation-dissipation theorem is feasible [36], in particular, in one-dimensional systems. I will study the impact of fluctuations on the behavior of ultracold Fermi gases in order to understand how one has to embed fluctuations in a dynamical simulation. This will require further conceptual development of the fluid dynamical framework as well as extending current algorithms to include a correct treatment of thermal fluctuations. The gained knowledge will be important for a later numerical study of fluid dynamical fluctuations in the QGP (see section 2.2 below).

2.2 Quark-Gluon Plasma

First-principle calculations of the thermodynamic and transport properties of hot and dense QCD matter are provided by lattice gauge theory simulations and functional methods such as FRG approaches. While nowadays final results on equilibrium quantities are available from lattice QCD calculations [37, 38], the evaluation of higher-order fluctuation observables [39, 40, 41] and, in particular, of dynamical quantities such as transport coefficients is a highly non-trivial task on the lattice [42]. Presently, only for Yang-Mills theory preliminary lattice results for the shear and bulk viscosities are available [43, 44] supported by recent progress reported from an FRG approach [45]. In such a situation, phenomenological models can help giving a qualitative insight into the behavior of the transport coefficients or fluctuation observables in dense QCD matter. Fluid dynamics is again the method of choice to study the dynamical evolution of the fireball created in a heavy-ion collision. The topics to be addressed are:

- *Study of multiplicity fluctuations*: In the thermodynamic limit, fluctuations in the conserved charges of QCD diverge at a critical point. In heavy-ion collisions, the situation is more complicated: neutrons are difficult to measure such that proton-number has to serve as a proxy for baryon-number. Also, both size and lifetime of the created fireball are small and dynamical effects play a dominant role. Previously, we analyzed measured multiplicity fluctuations with a thermal model to determine chemical freeze-out conditions [46] and to quantify the impact of resonance regeneration and decay on the fluctuations [47]. In continuation of this work, I will study the role of critical and of volume fluctuations. I will determine if and to what extent the recently reported data on net-proton fluctuations [48] are driven by the possible presence of a critical point. This will make the inclusion of local charge conservation [49] and dynamical effects [50] into the model necessary. Furthermore, I will study the influence of the hadronization on fluctuation observables by using a coalescence model [51].

- *Determination of QGP transport coefficients within a phenomenological model*: Applying effective kinetic theory based on a quasiparticle picture, I calculated previously the shear and bulk viscosities in Yang-Mills theory [52]. The shear viscosity over entropy density ratio was found to be small near the confinement transition, comparable with the KSS-limit [7] and in agreement with the lattice results [43, 44]. A recent study using FRG methods confirmed the validity of such a picture impressively [45]. I will extend my work and calculate the temperature and density dependence of the viscosities and the heat conductivity in the QGP. I will also investigate possible relations among the various transport coefficients, cf. [31, 53, 54].

- *Study of thermal fluctuations in dynamical simulations of relativistic heavy-ion collisions*: I plan to apply the knowledge gained in connection with ultracold Fermi gases (see section 2.1 above) to study the impact of fluid dynamical fluctuations on the dynamics of the QGP. This is essential for the search of the critical point as the dynamics of critical fluctuations is tightly connected with the evolution of the baryon density [55]. Ultimately, I want to study the impact of critical fluctuations on particle multiplicity distributions by means of a dynamical simulation that is capable of describing the evolution of fluid dynamical quantities and their fluctuations simultaneously. This will require firm knowledge of the behavior of the transport coefficients and the equation of state near the critical point [56, 57].

2.3 Milestones

According to the objectives described in sections 2.1 and 2.2, the following milestones can be identified with an estimated time schedule:

1. **Feb. – May 2017**
   Application of anisotropic fluid dynamics and determination of the temperature and density dependence of the shear viscosity of unitary Fermi gases in the normal phase through comparison with low-temperature expansion data.

2. **Jun. – Sept 2017**
   Analysis of net-proton fluctuation data within a phenomenological model and quantification of possible critical fluctuation contributions.

3. **Oct. 2017 – Jan. 2018**
   Calculation of the viscosity coefficients and the heat conductivity in the QGP within effective kinetic theory.

4. **Apr. 2017 – Feb. 2018**
   Numerical study of the influence of fluid dynamical fluctuations on the dynamics of ultracold Fermi gases and the QGP. Analysis of the impact of critical fluctuations on multiplicity distributions in a dynamical framework for heavy-ion collisions.

5. **Mar. 2018 – Jan. 2019**
   Calculation of the spin diffusion coefficient in ultracold Fermi gases away from the unitary limit within kinetic theory taking in-medium effects into account. Numerical study of spin diffusion at unitarity and inclusion of the superfluid phase in dynamical simulations.

The research related to milestones 4 and 5 will be carried out over a longer time period as both topics are highly involved. I plan to combine the work on milestones 2 and 4 with a study visit at the École des Mines de Nantes, France, and work on milestones 4 and 5 with a study visit at the University of Heidelberg, Germany. My international research contacts are Prof. Nahrgang in Nantes, and Prof. Pawlowski, Dr. Enss and Dr. Floerchinger in Heidelberg.

3 Research methodology

In this section, I discuss in more detail the methods, which will be used to achieve the goals summarized in section 2.3. In order to determine the temperature and density dependence of the shear viscosity in ultracold Fermi gases in the normal phase, I will apply the existing anisotropic fluid dynamics code [19]. While at high temperatures the ideal gas equation of state could be used [23], at lower temperatures the measured equation of state [10] will have to be employed. I will make an ansatz for the functional form of the shear viscosity based on a Virial expansion, which contains the kinetic theory result tested in [23] as leading-order contribution and, in addition, density dependent corrections. By comparing fluid dynamical simulations with the expansion data [24, 25], I will be able to extract the correct analytic form and the prefactors of the Virial expansion. The study of the properties of the superfluid phase can start from this work. As a superfluid can be described as a mixture of an inviscous with a viscous normal fluid [26], all that remains to be done is to extend the code to a two-fluid scenario.

For the dynamical study of spin diffusion, first, the anisotropic spin diffusion equations will be derived from kinetic theory in analogy to anisotropic fluid dynamics [19]: a suitable ansatz for the distribution function is made and moments of the Boltzmann equation are taken. This ensures that also the dilute regions of the gas clouds are taken into account properly. Finally, the spin diffusion equations will be embedded in the existing anisotropic fluid dynamics code. Experimentally, spin diffusion can be studied, for example, by preparing two separate clouds of opposite spin and letting them interact. This situation can be simulated within a two-fluid scenario.

To study fluid dynamical fluctuations in ultracold Fermi gases, I will extend the existing isotropic Navier-Stokes fluid dynamics code used in [58]. I will implement stochastic source terms, which will be correlated in line with the fluctuation-dissipation theorem, into the fluid dynamical equations. The stability of the simulations will be tested extensively, and maybe coarse-graining techniques will have to be introduced in order to deal with the potential extreme gradients in the fluid dynamical quantities from one fluid cell to another. Ultimately, simulation results for spatial and temporal correlations between fluid dynamical quantities will be contrasted with the existing numerical results [36]. The gained experience will be used to study fluid dynamical fluctuations in the QGP. For this purpose, I will extend the relativistic, viscous second-order fluid dynamics code of I. Karpenko [59] analogously. The implementation of fluctuations through stochastic source terms in the fluid dynamical equations respects the back-reaction of fluctuations on the fluid dynamical quantities. This will be a self-consistent improvement of a previous work [60] which studied the evolution of fluid dynamical quantities and their fluctuations independently.

Again, numerical stability will be tested extensively, and an dependence on the coarse-graining procedure be investigated. Finally, simulation results for correlations will be confronted with theoretical expectations [61] before quantitative studies in comparison with heavy-ion collision data can be made.

The kinetic theory calculations of the spin diffusion coefficient in ultracold Fermi gases away from unitarity, and of the transport coefficients in the QGP, will significantly profit from my previous calculations of comparable quantities in [33, 52, 53]. I will take medium effects into account, which modify the quasiparticle dispersion relations and scattering cross sections, and solve the collision terms either with the Chapman-Enskog method or in relaxation time approximation. To analyze measured multiplicity fluctuations, I will employ the phenomenological hadron resonance gas model used in [46, 47] and include non-thermal effects such as critical fluctuations following the idea elaborated in [62].

4 Significance of the project

Understanding the equilibrium properties of strongly coupled quantum systems, and the mechanisms that lead to their transport behavior, is very important. Such systems are realized in nature in various forms and they play an immense role in current technological developments such as quantum computing, spintronics or high-temperature superconductivity. Prominent examples being actively studied in the laboratories are ultracold Fermi gases and the QGP. Here, one wants to reveal the thermodynamic properties of both fluids and quantify their transport coefficients.

The physics questions to be addressed in this project aim exactly at achieving these goals. I propose cross-discipline studies of spin (or baryon number) and momentum transport in ultracold Fermi gases and in the QGP, to determine the temperature and density dependence of the corresponding transport coefficients with numerical simulations and in kinetic theory, and to search for the critical point as the landmark in the QCD phase diagram by means of simulations which include the effects of fluid dynamical fluctuations self-consistently.

These research objectives are both timely and prestigious. Key experimental results in cold atom physics, for example obtained by the group of J. E. Thomas at North Carolina State University, are regularly published in Nature or Science. Together with Prof. Schäfer from North Carolina State University, I am currently part of a Department of Energy supported research project, in which we collaborate with Prof. Thomas. Recently, the DFG (Deutsche Forschungsgemeinschaft) approved funding the long-term Collaborative Research Centre ISOQUANT at the University of Heidelberg and joint institutes with a total budget of about 10.5 million Euros. ISOQUANT will investigate the physics of ”Isolated Quantum Systems and Universality under Extreme Conditions” and make Heidelberg a centre of interdisciplinary research on strongly coupled quantum systems in the coming years. My proposed project is, furthermore, tightly connected with the heavy-ion collision program at LHC, the beam energy scan program at RHIC, and the future experiments at FAIR, NICA and J-PARC, which will probe very dense QCD matter in search for phase transition signatures. A goal of my project is to develop the tools that allow me to analyze, in particular, data from these future experiments and the upcoming RHIC beam energy scan phase II.

Scientific expertise, which will help me achieving my goals, is abundant at the host institution in Wrocław. The theoretical nuclear physics group at the University of Wrocław is recognized worldwide having made leading contributions in high-energy nuclear physics. Group members, notably Prof. Redlich and Prof. Sasaki, are experts in thermal field theory, effective thermal models, fluctuations, the equation of state, and the QCD phase diagram. Prof. Blaschke is a leading expert in kinetic theory and dense nuclear matter. Furthermore, I will have the opportunity to profit from an exchange of ideas with Prof. Florkowski and Dr. Ryblewski at the INP in Kraków, who are experts in the application of anisotropic fluid dynamics for relativistic fluids.

The quality of local and national expertise will be very useful for the successful completion of the planned project goals. The necessary tools will be developed from existing tools straightforwardly, and experience with the necessary calculational techniques has already been gained. The project provides ample opportunities for graduate research contributions. I plan to get two graduate students involved in the work to be part of an exciting and growing research field.

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