Purity is not eternal at the Planck scale

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MARIE CURIE ACTIONS

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40 Years of Hawking radiation

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Nature Vol. 248 March 1 1974

Black hole explosions?

QUANTUM gravitational effects are usually ignored in calculations of the formation and evolution of black holes. The justification for this is that the radius of curvature of space-time outside the event horizon is very large compared to the Planck length $(G\hbar/c^3)^{1/2} \approx 10^{-33}$ cm, the length scale on which quantum fluctuations of the metric are expected to be of order unity. This means that the energy density of particles created by the gravitational field is small compared to the space-time curvature. Even though quantum effects may be small locally, they may still, however, add up to produce a significant effect over the lifetime of the Universe $\approx 10^{17}$ s which is very long compared to the Planck time $\approx 10^{-43}$ s.

...

S. W. HAWKING

Department of Applied Mathematics and Theoretical Physics and Institute of Astronomy University of Cambridge

Received January 17, 1974.

[1] Bardeen, J. M., Carter, B., and Hawking, S. W., *Commun. math. Phys.*, **31**, 161–170 (1973).
[2] Hawking, S. W., *Mon. Not. R. astr. Soc.*, **152**, 75–78 (1971).
[3] Penrose, R., in *Relativity, Groups and Topology* (edit. by de Witt, C. M., and de Witt, B. S). Les Houches Summer School, 1963 (Gordon and Breach, New York, 1964).
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[5] Hawking, S. W., in *Black Holes* (edit. by de Witt, C. M., and de Witt, B. S), Les Houches Summer School, 1972 (Gordon and Breach, New York, 1973).
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The information paradox in a nutshell

The stage: free quantum field on a Schwarzschild black hole background

Essence of Hawking effect: vacuum state for a **free falling observer** $|0\rangle \neq$ vacuum state for **static observer** $|0\rangle_{out}$

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$$\rho = \mathrm{Tr}_{in}(|0\rangle\langle 0|)$$

however the **"full"** state $|0\rangle\langle 0|$ is *pure*.

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- The mixed state ρ cannot be a partial trace of a pure state since there's **no degrees of freedom to trace out** left!

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Commun. Math. Phys. 87, 395–415 (1982)

The Unpredictability of Quantum Gravity

S. W. Hawking
University of Cambridge, D.A.M.T.P., Cambridge CB3 9EW, England

"I present a number of axioms that the asymptotic Green functions should obey in any reasonable theory of quantum gravity. These axioms are the same as for ordinary quantum field theory in flat spacetime, except that one axiom, that of asymptotic completeness, is omitted. This allows pure quantum states to decay into mixed states."

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are preserved by time evolution they (re)-discovered the **Lindblad equation**

$$\dot{\rho} = -i[H, \rho] - \frac{1}{2} h_{\alpha\beta} \left(Q^\alpha Q^\beta \rho + \rho\, Q^\beta Q^\alpha - 2 Q^\alpha \rho\, Q^\beta \right)$$

$h_{\alpha\beta}$ is a hermitian matrix of constants and Q^α form a basis of hermitian matrices

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Motivated by Hawking's proposal that the quantum-mechanical density matrix ρ obeys an equation more general than the Schrödinger equation, we study the general properties of evolution equations for ρ. We argue that any more general equation for ρ violates either locality or energy-momentum conservation.

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Lindblad time evolution is still problematic since: *"[...] loss of purity is incompatible with the weakest possible form of Lorentz covariance."*

Symmetry deformation at the Planck scale

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MA: 1403.6457; Phys. Rev. D 90, 024016 (7 July 2014)

Outline

- **Topological particles and curved momentum space in 3d gravity**
- **Quantum double, deformed symmetries and Lindblad evolution**
- **4d case: de Sitter momentum space and κ-deformed symmetries**
- **Deformed Lindlblad evolution from κ-Poincaré**
- **Conclusions and outlook**

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Curved momentum space in *flatland*

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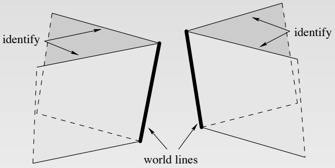

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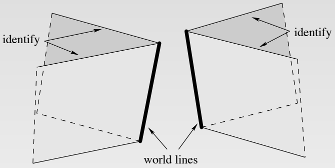

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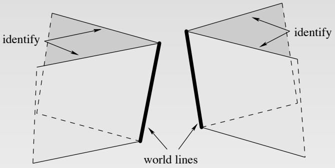

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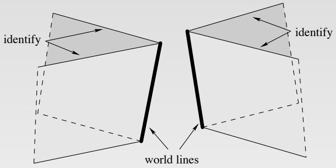

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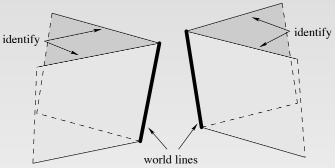

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Momenta of particles coupled to 3d gravity = elements of a non-abelian group!

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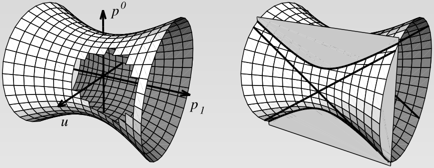

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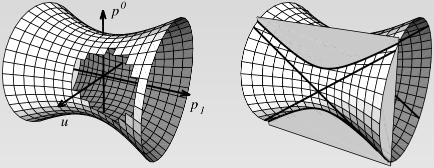

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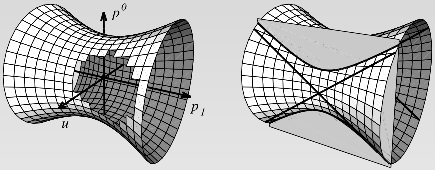

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AdS momentum space: embedding coordinates

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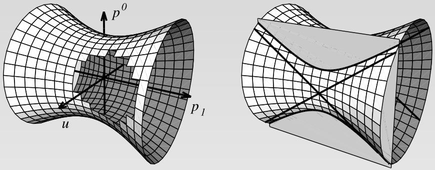

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- **mass-shell condition**: $p^2 = \kappa^2 \cos 4\pi G m = m_\kappa$
- **Lorentz transformation**: $h' = ghg^{-1}$, undeformed on p^μ e.g. boost in the 1-direction $g = e^{\frac{1}{2}\eta\gamma_2}$

$$\begin{cases} p'^0 = p^0 \cosh\eta - p^1 \sinh\eta \\ p'^1 = p^1 \cosh\eta - p^0 \sinh\eta \\ p'^2 = p^2 \end{cases}.$$

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ii) momentum coordinates obey a non abelian composition rule indeed

$$p_\mu \oplus q_\mu = v(q)\, p_\mu + u(p)\, q_\mu + \frac{1}{\kappa}\epsilon_{\mu\nu\sigma} p^\nu q^\sigma = p_\mu + q_\mu + \frac{1}{\kappa}\epsilon_{\mu\nu\sigma} p^\nu q^\sigma + \mathcal{O}(1/\kappa^2) \neq q_\mu \oplus p_\mu$$

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Plane waves = eigenfunctions of *translation generators* P_μ

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what does this mean physically?

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  consider e.g. a projector $\pi_k = |k\rangle\langle k|$ seen as the "outer product" of a ket and a bra namely $\pi_k = \pi(|k\rangle \otimes \langle k|) = |k\rangle\langle k|$

$$P_\mu(\pi_k) = \pi(P_\mu(|k\rangle \otimes \langle k|)) = \pi(P_\mu|k\rangle \otimes \langle k| - |k\rangle \otimes \langle k|P_\mu) = [P_\mu, \pi_k]$$

i.e. just the familiar **adjoint action**

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Key point: the action on operators will be deformed accordingly

Deformed translations and Lindblad evolution in three dimensions

The deformed translation generators of $\mathcal{D}(SL(2,\mathbb{R}))$ in the "cartesian" basis P_μ:

$$\Delta P_\mu = P_\mu \otimes \mathbb{1} + \mathbb{1} \otimes P_\mu + \frac{1}{\kappa}\,\epsilon_{\mu\nu\sigma} P^\nu \otimes P^\sigma \,, \;\; S(P_\mu) = -P_\mu \,.$$

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The adjoint action of undeformed translation generators $\Rightarrow$ **quantum adjoint action**

$$\mathrm{ad}_{P_\mu}(\rho) = (\mathrm{id} \otimes S)\Delta(P_\mu) \diamond \rho$$

with $(a \otimes b) \diamond c \equiv a\, c\, b$...

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which can be rewritten in Lindlblad form as

$$\boxed{\dot{\rho} = -i[P_0, \rho] - \frac{1}{2} h_{ij}\left(P^i P^j \rho + \rho\, P^j P^i - 2P^j \rho\, P^i\right)}$$

with "dissipation" matrix is given by

$$h = \frac{i}{\kappa}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$$

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BPS, Srednicki et al. restricted to *real* and *positive definite h*!

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  - *h positive definite* $\Longrightarrow$ **positivity** of ρ is preserved
  - in addition h real $\Longrightarrow$ entropy *increases* with evolution
  - energy is conserved if operators in *dissipation term* **commute** with P_0
- Srednicki tells us that requirement of energy conservation **not compatible** with Lorentz covariance
- Lindblad eq. above conserves energy and **is** Lorentz covariant...what's going on?

BPS, Srednicki et al. restricted to *real* and *positive definite h*!
In our case *h is not* **positive definite** *nor* **real**

Further work needed to establish properties of our Lindblad evolution...

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  - dual **Lie algebra** "non-commutative space-time" coordinates

$$[x_\mu, x_\nu] = -\frac{i}{\kappa}(x_\mu \delta^0_\nu - x_\nu \delta^0_\mu)\,.$$

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The non-abelian composition of momenta in "flat slicing" or **bicrossproduct** coordinates

$$\begin{aligned} p_0(k_0, \mathbf{k}) &= \kappa \sinh k_0/\kappa + \frac{\mathbf{k}^2}{2\kappa}\, e^{k_0/\kappa}, \\ p_i(k_0, \mathbf{k}) &= k_i\, e^{k_0/\kappa}, \\ p_4(k_0, \mathbf{k}) &= \kappa \cosh k_0/\kappa - \frac{\mathbf{k}^2}{2\kappa}\, e^{k_0/\kappa}. \end{aligned}$$

reads $\qquad k \oplus l = (k^0 + l^0;\ k^j\ e^{-\frac{l^0}{\kappa}} + l^j)$

κ-Poincaré and DSR

The non-abelian composition of momenta reflects a **non-Leibniz action** of *spatial* translation generators

$$K_i(e_{\mathbf{k}_1} \otimes e_{\mathbf{k}_2}) = K_i(e_{\mathbf{k}_1}) \otimes e_{\mathbf{k}_2} + \exp(-K_0/\kappa)(e_{\mathbf{k}_1}) \otimes K_i(e_{\mathbf{k}_2})$$

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in the limit $\kappa \longrightarrow \infty$ recover ordinary Poincaré algebra

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In analogy with the 3d case we consider **translation generators** P_μ associated to *embedding* coordinates p_μ on dS_4

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a straightforward calculation leads to a *non-symmetric* **Lindblad equation**

$$\boxed{\dot{\rho} = -i[P_0, \rho] + \frac{i}{\kappa} P_m \rho P_m - \frac{i}{\kappa} \rho \, \vec{P}^2}$$

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- however the quantum adjoint actions of N_i and P_0 satisfy

$$\mathrm{ad}_{\mathrm{ad}N_i(P_0)}(\cdot) = \mathrm{ad}_{N_i}(\mathrm{ad}_{P_0})(\cdot) - \mathrm{ad}_{P_0}(\mathrm{ad}_{N_i})(\cdot)$$

in this sense the κ-Lindblad equation follows a **deformed notion of covariance**

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  - *phenomenology* of κ-Lindblad evolution? (Ellis et al. "Search for Violations of Quantum Mechanics," Nucl. Phys. B **241**, 381 (1984) and following works)

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- and finally application to **black hole quantum evolution**...