TOWARDS HARMONIC SUPERFIELD FORMULATION OF $\mathcal{N} = 4$ SYM THEORY WITH GAUGED CENTRAL CHARGE

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Aims

Review part

- Brief review of superfield models
- Brief review of Sohnius, Stelle, West approach to $\mathcal{N} = 4$ SYM theory

Basic part

- Developing the superfield formulation for $\mathcal{N} = 4$ SYM theory with central charge in terms of $\mathcal{N} = 4$ superfields
  Component formulation of $\mathcal{N} = 4$ SYM theory with central charge has been given by Sohnius, Stelle and West, 1980.
- Formulation of $USp(4)$ central charge harmonic superspace and construction of superfield action

Based on I.L.B, N.G. Pletnev, Nucl.Phys. B877, 936, 2013

Plan

Review part

- What is a supersymmetry
- What are the superspace and superfields
- Component and superfield formulation of supersymmetric field models
- Example: $\mathcal{N} = 1$ supersymmetric field models
- Advantages of superfield formulation in quantum field theory
- Conventional $\mathcal{N} = 4$ SYM theory
- Sohnius, Stelle, West (SSW) formulation

Plan

Basic part

- $USp(4), \mathcal{N} = 4$ superspace
- Gauge theory in $USp(4), \mathcal{N} = 4$ superspace
- $\mathcal{N} = 4$ central charge harmonic superspace
- Gauge theory in $\mathcal{N} = 4$ central charge harmonic superspace
- Superfield action and its component form
- Summary

- Supersymmetry in physics means a hypothetical symmetry of Nature relating the bosons and fermions. In its essence, the supersymmetry is an extension of special relativity symmetry. One can say, the supersymmetry is a special relativity symmetry extended by the symmetry between bosons and fermions
- From mathematical point of view the relativistic symmetry is expressed in terms of Poincare group with the generators P_m and J_{mn} satisfying the known commutation relations

$$\begin{aligned} [P_r, P_s] &= 0, \\ [J_{rs}, P_m] &= i(\eta_{rm} P_s - \eta_{sm} P_r), \\ [J_{mn}, J_{rs}] &= i(\eta_{mr} J_{ns} - \eta_{ms} J_{nr} + \eta_{ns} J_{mr} - \eta_{nr} J_{ms}) \end{aligned} \tag{1}$$

η_{mn} is the Minkowski metric with the signature $(-, +, +, +)$.

- Extension of special relativity means extension of the Poincare group by the generators (supercharges) $Q^i{}_\alpha$, $\bar{Q}_{i\dot{\alpha}}$, $i = 1, 2, ..., \mathcal{N}$
- The relations among the supercharges (Poincare superalgebra) are given in terms of anticommutators

$$\begin{aligned}
\{Q^i{}_\alpha, Q^j{}_\beta\} &= \epsilon_{\alpha\beta} Z^{ij} \\
\{\bar{Q}_{i\dot{\alpha}}, \bar{Q}_{j\dot{\beta}}\} &= \epsilon_{\dot{\alpha}\dot{\beta}} \bar{Z}_{ij} \\
\{Q^i{}_\alpha, \bar{Q}_{j\dot{\alpha}}\} &= 2\delta^i{}_j \sigma^m{}_{\alpha\dot{\alpha}} P_m
\end{aligned} \tag{2}$$

  Z^{ij}, $\bar{Z}_{ij}$ are the central charges; $\epsilon_{\alpha\beta}$, $\epsilon_{\dot{\alpha}\dot{\beta}}$ are the invariant tensors of the $SL(2|C)$ group, $\sigma_m = (\sigma_0, \sigma_i)$.
- Supersymmetric field model means a field model invariant under the above superalgebra. Since the supercharges are the spin $s = \frac{1}{2}$ spinors under the Lorentz groups one can expect that any supersymmetric field model must contain both bosonic and fermionic fields

Superspace and superfields

- Minkowski space coordinates x^m have the same tensor structure as the generators of space-time translations P^m. Analogously ones introduce the additional spinor coordinates $\theta^{i\alpha}$ and $\bar{\theta}^{i\dot{\alpha}}$ associated with the supercharges ${Q^i}_\alpha$ and $\bar{Q}_{i\dot{\alpha}}$. The additional coordinates have the fermionic structure as well as the supercharges and anticommute among themselves.
- Manifold parametrized by the commuting (bosonic) coordinates x^m and the anticommuting (fermionic) coordinates $\theta^{i\alpha}$, $\bar{\theta}^{i\dot{\alpha}}$ is called superspace
- Function defined on superspace is called superfield
- Since the fermionic coordinates are anticommuting, any superfield is no more then polinomial in these coordinates. The coefficients of such a polinomial are the standard bosonic and fermionic fields on Minkowski space. All these coefficients are called the component fields of the superfield.
- Ones define the supersymmetry transformations (supertranslations): $\theta^{i\alpha} \to \theta^{i\alpha} + \epsilon^{i\alpha}$, $\bar{\theta}^{i\dot{\alpha}} \to \bar{\theta}^{i\dot{\alpha}} + \bar{\epsilon}^{i\dot{\alpha}}$, $x^m \to x^m + \delta x^m$. The $\epsilon^{i\alpha}$ and $\bar{\epsilon}^{i\dot{\alpha}}$ are the anticommuting transformation parameters, δx^m are expressed in special form through the fermionic coordinates and the anticommuting parameters. The supertranslations define the supersymmetry transformations of the component fields.

Terminology

- Let $F(x, \theta, \bar{\theta})$ is a some superfield and (b, f) is a set of its bosonic and fermionic component fields.
- Supersymmetry transformations have the following general structure

$$b' = b + \delta b, \delta b \sim f$$

$$f' = f + \delta f, \delta f \sim b.$$

- If a supersymmetric field model is formulated in terms of component fields (b, f) ones get a component formulation.

$$\delta S[b, f] = 0$$

- If the same field model is formulated in terms of superfield $F(x, \theta, \bar{\theta})$ ones get a superfield formulation.

Example: $\mathcal{N}=1$ supersymmetric field models

- $\mathcal{N}=1$ superspace: coordinates $(x^m, \theta^\alpha, \bar\theta^{\dot\alpha})$.
- Chiral scalar superfield $\Phi(x,\theta,\bar\theta) = e^{i(\theta\sigma^m\bar\theta)\partial_m}\Phi(x,\theta)$. Component content $\Phi(x,\theta) = A(x) + \theta^\alpha\psi_\alpha(x) + F(x)\theta^2$
- Antichiral scalar superfield $\bar\Phi(x,\theta,\bar\theta) = e^{-i(\theta\sigma^m\bar\theta)\partial_m}\bar\Phi(x,\bar\theta)$. Component content $\bar A(x,\bar\theta) = \bar A(x) + \bar\theta_{\dot\alpha}\bar\psi^{\dot\alpha}(x) + \bar F(x)\bar\theta^2$
- Superfield model (Wess-Zumino model)

$$S[\Phi,\bar\Phi] = \int d^4x d^2\theta d^2\bar\theta \bar\Phi(x,\theta,\bar\theta)\Phi(x,\theta,\bar\theta) + (\int d^4x d^2\theta V(\Phi) + c.c.)$$

$V(\Phi) = \frac{m}{2}\Phi^2 + \frac{\lambda}{3!}\Phi^3$.
Manifest supersymmetry.

- Component form of Wess-Zumino model

$$S = \int d^4x(-\partial^m\bar A\partial_m A - \frac{i}{2}\psi^\alpha\sigma^m{}_{\alpha\dot\alpha}\partial_m\bar\psi^{\dot\alpha} + \bar F F + F(mA + \frac{\lambda}{2}A^2) +$$

$$+\bar F(m\bar A + \frac{\lambda}{2}\bar A^2) - \frac{1}{4}(m+\lambda A)\psi^\alpha\psi_\alpha - \frac{1}{4}(m+\lambda\bar A)\bar\psi_{\dot\alpha}\bar\psi^{\dot\alpha})$$

Non-manifest supersymmetry.

- Real scalar superfield $V(x,\theta,\bar\theta)$. Component content
$V(x,\theta,\bar\theta) = A(x) + \theta^\alpha\psi_\alpha(x) + \bar\theta_{\dot\alpha}\bar\psi^{\dot\alpha}(x) + \theta^2 F(x) + \bar\theta^2\bar F(x) +$
$(\theta\sigma^m\bar\theta)A_m(x) + \bar\theta^2\theta^\alpha\lambda_\alpha(x) + \theta^2\bar\theta_{\dot\alpha}\bar\lambda^{\dot\alpha}(x) + \theta^2\bar\theta^2 D(x)$
- Superfield model ($\mathcal{N}=1$ supersymmetric Yang-Mills theory)

$$S_{SYM}[V] = \frac{1}{2g^2}\int d^4x d^2\theta tr(W^\alpha W_\alpha)$$

Superfield V takes the values in Lie algebra of gauge group,
$W_\alpha = -\frac{1}{8}\bar D^2(e^{-2V}D_\alpha e^{2V})$, $D^2 = D^\alpha D_\alpha$, $D_\alpha = \partial_\alpha + i(\sigma^m)_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\partial_m$.
Manifest supersymmetry.

- Component form of supersymmetric Yang-Mills theory

$$S_{SYM} = \frac{1}{g^2}\int d^4x tr(-\frac{1}{4}G^{mn}G_{mn} - i\lambda^\alpha\sigma^m_{\alpha\dot\alpha}\nabla_m\bar\lambda^{\dot\alpha} + 2D^2)$$

Non-manifest supersymmetry.

Advantages of superfield formulation

- The SUSY models can be formulated in terms of bosonic and fermionic component fields. Supersymmetry is hidden. It is not very convenient in quantum field theory: very many Feynman diagrams, miraculous cancelations.
- Superfield formulation: manifest SUSY, comparatively small number of supergraphs, origin of miraculous cancelations. Problem: how to formulate the of $\mathcal{N}$-extended SUSY models in terms of unconstrained $\mathcal{N}$-extended superfields. General solution for arbitrary $\mathcal{N}$ is unknown.
- Why do we want to get a formulation in terms of unconstrained superfields? In quantum field theory we should calculate the variational derivatives of the actions with respect of fields. But it is possible only if the fields are the unconstrained functional arguments of the actions.
- Conventional $\mathcal{N} = 4$ SYM theory can be formulated in component form, terms of $\mathcal{N} = 1$ superfields, in terms of $\mathcal{N} = 2$ harmonic superfields and in terms of $\mathcal{N} = 3$ harmonic superfields. Formulation in terms of unconstrained $\mathcal{N} = 4$ superfields is unknown

Conventional $\mathcal{N}=4$ SYM theory
Properties:

- Theory was obtained in component form by dimensional reduction from d=10, $\mathcal{N}=1$ SYM to d=4
  L. Brink, J.H. Schwarz, J. Scherk, 1977
- Field content: 1 vector, 6 real scalars, 4 Majorana spinors
- Global R-symmetry $SU(4)$
- Most symmetric field theory (with maximal spin 1)
- Conformal quantum field theory
- Finite quantum field theory
- Relations with D=3 branes
- Basic theory for AdS/CFT correspondence
- Formulation in terms of unconstrained off-shell $\mathcal{N}=4$ superfields is unknown.
  Attempt of superfield formulation: M.F. Sohnius, 1978.
  Result: superfield constraints are so strong that they are equivalent to on-shell conditions.

Basic property: non-zero central charge

Central charge breaks $SU(4)$ R-symmetry group of conventional $\mathcal{N}=4$ SYM theory to some of its subgroup, e.g. to $USp(4)$ group.
$\mathcal{N}=4$ SYM theory with central charge.
Properties:

- Theory was constructed in component form
  M.F. Sohnius, K. Stelle, P.C. West, 1980
- Field content: 1 vector A_m, 4 $USp(4)$-Majorana spinior fields λ_i, 5 antisymmetric, Ω-traceless scalar fields ϕ_{ij} (Ω is invariant metric of $USp(4)$ -group). Besides, there are 1 pseudovector V_m and 5 antisymmetric, Ω-traceless scalars H_{ij}
- Action

$$S = \mathrm{tr}\int d^4x(-\frac{1}{4}F_{mn}F^{mn} - \frac{1}{2}V_mV^m + \frac{1}{2}D_m\phi_{ij}D^m\phi^{ij} + \frac{1}{2}H_{ij}H^{ij} \quad (3)$$

$$-\frac{i}{4}\bar{\lambda}^i \not{D}\lambda_i - \bar{\lambda}^i[\lambda^j,\phi_{ij}] + \frac{1}{4}[\phi_{ij},\phi_{kl}][\phi^{ij},\phi^{kl}])$$

Sohnius, Stelle and West formulation

- Action is invariant under central charge transformations
- Action is $\mathcal{N}=4$ supersymmetric if the following the additional constraint is imposed

$$D^m V_m + \frac{1}{2}\{\bar{\lambda}^i, \gamma^5 \lambda_i\} - i[\phi_{ij}, H^{ij}] = 0$$

- The constraint can be introduced into action with help of Lagrange multiplier A_5
- Solutions to equations of motion for V_m and H_{ij} are

$$V_m = -D_m A_5, \quad H_{ij} = i[A_5, \phi_{ij}]$$

- If to substitute these solutions into action, one gets the conventional $SU(4)$, $\mathcal{N}=4$ SYM theory.
  Scalar field A_5 is unified with 5 scalar fields ϕ_{ij} and one obtains 6 scalar fields of the conventional $SU(4)$, $\mathcal{N}=4$ SYM theory.

Aim: superfield formulation of this theory

Abelian theory.

- Constraint

$$\partial_m V^m = 0$$

- Solution in terms of second rank antisymmetric field

$$V_m = \epsilon_{mnrs} G^{nrs}$$

$$G_{mnr} = \partial_m B_{nr} + \partial_r B_{mn} + \partial_n B_{rm}$$

- Substitution of this solution to action yields $\mathcal{N} = 4$ Abelian SYM theory where one of scalars is replaced by antisymmetric tensor B_{mn}

Attempts of superfield consideration of $\mathcal{N}=4$ SYM theory with central charge:

- B. Milewsli, 1983; Constraint in $\mathcal{N}=1$ superfied form
- J. Saito, 2005; Superfield strengths in $\mathcal{N}=4$ central charge superspace, superfield constraints and their solution in terms of a single superstrength
- I.L.B, O. Lechtenfeld, I.B. Samsonov, 2008; $USp(4), \mathcal{N}=4$ harmonic superspace, $\mathcal{N}=4$ SYM theory with central charge, low energy effective action in terms of $\mathcal{N}=4$ superfields (F^4 term)
- I.L.B, N.G. Pletnev, 2013; $\mathcal{N}=4$ harmonic superspace, superfield formulation of $\mathcal{N}=4$ SYM theory with central charge in terms of constrained $\mathcal{N}=4$ harmonic superfields

$\mathcal{N}$=4 central charge superspace

- Coordinates $Z^M = \{x^m, z, \theta_i^\alpha, \bar\theta^i_{\dot\alpha}\}$ $(i,j=1,2,3,4)$
- Supercovariant derivatives $D_M = (\partial_m, \partial_z, D^i_\alpha, \bar D_i^{\dot\alpha})$

$$D^i_\alpha = \frac{\partial}{\partial\theta_i^\alpha} + i\bar\theta^{\dot\alpha i}\partial_{\alpha\dot\alpha} - i\theta^i_\alpha\partial_z, \quad \bar D_{\dot\alpha i} = -\frac{\partial}{\partial\bar\theta^{\dot\alpha i}} - i\theta_i^\alpha\partial_{\alpha\dot\alpha} - i\bar\theta_{\dot\alpha i}\partial_z$$

- Algebra

$$\{D^i_\alpha, \bar D_{\dot\alpha j}\} = -2i\delta^i_j\partial_{\alpha\dot\alpha}, \{D^i_\alpha, D^j_\beta\} = -2i\varepsilon_{\alpha\beta}\Omega^{ij}\partial_z,$$
$$\{\bar D_{\dot\alpha i}, \bar D_{\dot\beta j}\} = 2i\varepsilon_{\dot\alpha\dot\beta}\Omega_{ij}\partial_z$$

- Invariant metric on $USp(4)$ group

$$\Omega_{ij} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$$

- Rising and lowering the indices $(\Omega_{ij}\Omega^{jk} = \delta_i^k)$

$$\psi^{\alpha i} = \varepsilon^{\alpha\beta}\Omega^{ij}\psi_{\beta j}, \quad \bar\psi^{\dot\alpha i} = \varepsilon^{\dot\alpha\dot\beta}\Omega^{ij}\bar\psi_{\dot\beta j}$$

Gauge theory in $USp(4), \mathcal{N} = 4$ superspace

- Gauge covariant derivatives $\nabla_M = D_M + i\Gamma_M$
- Superconnections $\Gamma_M(x, z, \theta, \bar{\theta})$
- Gauge transformations $\nabla'_M = e^{i\tau} \nabla_M e^{-i\tau}$
- Algebra of covariant spinor derivatives. Definition of superstrength W_{ij}

$$\{\nabla_{\alpha i}, \nabla_{\beta j}\} = 2i\varepsilon_{\alpha\beta}\Omega_{ij}\nabla_z + 2i\varepsilon_{\alpha\beta}W_{ij}$$

$$\{\bar{\nabla}_{\dot{\alpha} i}, \bar{\nabla}_{\dot{\beta} j}\} = 2i\varepsilon_{\dot{\alpha}\dot{\beta}}\Omega_{ij}\nabla_z - 2i\varepsilon_{\dot{\alpha}\dot{\beta}}W_{ij}$$

$$\{\nabla_{\alpha i}, \bar{\nabla}_{\dot{\alpha} j}\} = -2i\Omega_{ij}\nabla_{\alpha\dot{\alpha}},$$

- Definitions of superstrengths $G_{\alpha i}, \bar{G}_{\dot{\alpha} i}, F_{\alpha i m}, \bar{F}_{\dot{\alpha} i m}, \mathcal{V}_m, F_{mn}$

$$[\nabla_{\alpha i}, \nabla_z] = iG_{\alpha i} \quad [\bar{\nabla}_{\dot{\alpha} i}, \nabla_z] = -i\bar{G}_{\dot{\alpha} i}$$

$$[\nabla_{\alpha i}, \nabla_m] = iF_{\alpha i m} \quad [\bar{\nabla}_{\dot{\alpha} i}, \nabla_m] = -i\bar{F}_{\dot{\alpha} i m}$$

$$[\nabla_m, \nabla_z] = i\mathcal{V}_m, \quad [\nabla_m, \nabla_n] = iF_{mn}$$

- Superstrengths satisfy the reality conditions

Bianchi identities $\longrightarrow$ Constraints on superstrengths

J. Saito, 2005

All superstrengths are expressed in terms of real superfield W_{ij} and its gauge covariant spinor derivatives

Solutions to constraints (I)

$$F_{\alpha im} = -\sigma^m_{\alpha\dot\alpha}\bar G^{\dot\alpha}_i, \qquad \bar F_{\dot\alpha im} = G^\alpha_i \sigma^m_{\alpha\dot\alpha}$$

$$\nabla_{\alpha k} W_{ij} = i\Omega_{ij} G_{\alpha k} + 2i\Omega_{k[i} G_{\alpha j]}, \quad \bar\nabla_{\dot\alpha k} W_{ij} = i\Omega_{ij}\bar G_{\dot\alpha k} + 2i\Omega_{k[i}\bar G_{\dot\alpha j]}$$

$$5iG_{\hat\alpha i} = \nabla^k_{\hat\alpha} W_{ki}, \qquad \nabla_z W_{ij} \equiv \mathcal{H}_{ij}$$

$$\nabla_{\alpha i} G_{\beta j} = -\varepsilon_{\alpha\beta}\mathcal{H}_{ij} - \frac{1}{2}\Omega_{ij} F_{\alpha\beta} + \frac{1}{2}\varepsilon_{\alpha\beta}[W_{ik}, W_j{}^k]$$

$$\bar\nabla_{\dot\alpha i}\bar G_{\dot\beta j} = -\varepsilon_{\dot\alpha\dot\beta}\mathcal{H}_{ij} + \frac{1}{2}\Omega_{ij}\bar F_{\dot\alpha\dot\beta} - \frac{1}{2}\varepsilon_{\dot\alpha\dot\beta}[W_{ik}, W_j{}^k]$$

$$\bar\nabla_{\dot\alpha i} G_{\alpha j} = i\Omega_{ij}\mathcal{V}_{\alpha\dot\alpha} - \nabla_{\alpha\dot\alpha} W_{ij}, \qquad \nabla_{\alpha i}\bar G_{\dot\alpha j} = i\Omega_{ij}\mathcal{V}_{\alpha\dot\alpha} + \nabla_{\alpha\dot\alpha} W_{ij}$$

Solutions to constraints (II)

$$\nabla_z \bar{G}_{\dot\alpha i} = \nabla_{\alpha\dot\alpha} G^\alpha_i + [W_{ik}, \bar{G}^k_{\dot\alpha}], \qquad \nabla_z G^\alpha_i = \nabla^{\dot\alpha\alpha} \bar{G}_{\dot\alpha i} - [W_{ik}, G^{\alpha k}]$$

$$\nabla_{\alpha i} \mathcal{V}_m = \sigma^m_{\alpha\dot\alpha}[W_{ik}, \bar{G}^{\dot\alpha k}] + i(\sigma_{mn})^\beta_\alpha \nabla_n G_{\beta i}$$

$$\bar\nabla_{\dot\alpha i} \mathcal{V}_m = -\sigma^m_{\alpha\dot\alpha}[W_{ik}, G^{\alpha k}] + i\nabla_n \bar{G}_{\dot\beta i}(\bar\sigma_{mn})^{\dot\beta}_{\dot\alpha}$$

$$\nabla_{\alpha i}\mathcal{H}_{jk} = -i\Omega_{jk}\nabla_{\alpha\dot\alpha}\bar{G}^{\dot\alpha}_i - 2i\Omega_{i[j}\nabla_{\alpha\dot\alpha}\bar{G}^{\dot\alpha}_{k]}$$

$$-i[W_{jk}, G_{\alpha i}] - i\Omega_{jk}[W_{il}, G^l_\alpha] - 2i\Omega_{i[j}[W_{k]l}, G^l_\alpha]$$

$$\bar\nabla_{\dot\alpha i}\mathcal{H}_{jk} = i\Omega_{jk}\nabla_{\alpha\dot\alpha}G^\alpha_i + 2i\Omega_{i[j}\nabla_{\alpha\dot\alpha}G^\alpha_{k]} +$$

$$i[W_{jk}, \bar{G}_{\dot\alpha i}] + i\Omega_{jk}[W_{ip}, \bar{G}^p_{\dot\alpha}] + 2i\Omega_{i[j}[W_{k]p}, \bar{G}^p_{\dot\alpha}]$$

$$\nabla_{\dot\alpha i}F_{\alpha\beta} = 2i\nabla_{(\alpha\dot\alpha}G_{\beta)i}, \qquad \nabla_{\gamma i}F_{\alpha\beta} = -2i\varepsilon_{\gamma(\alpha}\nabla_{\beta)\dot\alpha}\bar{G}^{\dot\alpha}_i$$

$$\nabla_{\alpha i}F_{\dot\alpha\dot\beta} = 2i\nabla_{\alpha(\dot\alpha}\bar{G}_{\dot\beta)i}, \qquad \bar\nabla_{\dot\gamma i}F_{\dot\alpha\dot\beta} = -2i\varepsilon_{\dot\gamma(\dot\alpha}\nabla_{\alpha\dot\beta)}G^\alpha_i$$

Solutions to constraints (III)

$$\nabla_m \mathcal{V}_m = -\frac{i}{4}[W_{ik}, \mathcal{H}^{ik}] + \frac{1}{2}\{G_{\alpha k}, G^{\alpha k}\} + \frac{1}{2}\{\bar{G}_{\dot{\alpha} k}, \bar{G}^{\dot{\alpha} k}\}$$

$$\nabla_z \mathcal{V}_m = -\sigma^m_{\alpha\dot{\alpha}}\{G^{\alpha i}, \bar{G}^{\dot{\alpha}}_i\} - \frac{i}{4}[W_{ik}, \nabla_m W^{ik}] - \nabla_a F_{am}$$

$$\nabla_z \mathcal{H}_{jk} = \Box W_{jk} - \frac{i}{2}\Omega_{jk}\{G_{\alpha i}, G^{\alpha i}\} - 2i\{G_{\alpha j}, G^{\alpha}_k\} +$$

$$\frac{i}{2}\Omega_{jk}\{\bar{G}_{\dot{\alpha} i}, \bar{G}^{\dot{\alpha} i}\} + 2i\{\bar{G}_{\dot{\alpha} j}, \bar{G}^{\dot{\alpha}}_k\} + \frac{1}{8}\Omega_{jk}[W_{il}[W^i{}_p, W^{lp}]]$$

Result: all superfields $G_{\alpha i}, \bar{G}_{\dot{\alpha} i}, F_{\alpha i m}, \bar{F}_{\dot{\alpha} i m}, \mathcal{V}_m, F_{mn}, \mathcal{H}_{ij}$ are expressed in terms of real scalar superfield W_{ij} and its gauge covariant derivatives.

Constraints on W_{ij}

- $\Omega^{ij} W_{ij} = 0, \quad \bar{W_{ij}} = W^{ij} = \Omega^{ik}\Omega^{jl} W_{kl} = -\frac{1}{2}\varepsilon^{ijkl} W_{kl}$
- Specification of the concrete values for indices i, j, k, l and use of the matrix Ω yields the constraints for covariant spinor derivatives of W_{ij}. For example $\nabla_{\alpha 1} W_{12} = 0, \quad \nabla_{\alpha 2} W_{12} = 0$
- These constraints mean the very special dependence of W_{ij} on anticommuting coordinates

$$W_{12} = W_{12}(\theta^3, \theta^4, \bar{\theta}_2, \bar{\theta}_1), \quad W_{13} = W_{13}(\theta^2, \theta^4, \bar{\theta}_3, \bar{\theta}_1),$$

$$W_{24} = W_{24}(\theta^1, \theta^3, \bar{\theta}_4, \bar{\theta}_2), \quad W_{34} = W_{34}(\theta^1, \theta^2, \bar{\theta}_4, \bar{\theta}_3),$$

$$W_{14}(\theta^i, \bar{\theta}_i) = -W_{23}(\theta^i, \bar{\theta}_i)$$

(The coordinates x^m and z are not written down.)

- Result: W_{ij} with different indices i, j belong to different subspaces of full $USp(4), \mathcal{N} = 4$ superspace (short superfields).

Dependence on central charge

- Set of constraints completely fixes z-dependence of all strengths
- All z-dependent superstrengths are given by power series in central charge z. For example $W^{ij} = \sum_{k=0}^{\infty} W^{ij}_{(k)} z^k$
- The constraints allows us to express all coefficients in these expansions in terms of only two coefficients $W^{ij}_{(0)}, W^{ij}_{(1)}$. Consideration is analogous to one in $\mathcal{N}=2$ vector-tensor multiplet theory.
- If to switch off in the constraint $[\nabla_m, \nabla_z] = i\mathcal{V}_m$ all anticommuting coordinates and put $z=0$, one gets $D_m A_5 = V_m$. Here $A_5 = \Gamma_z^{(0)}|$ and $V_m = \mathcal{V}_z^{(0)}|$
- If to switch off in the constraint $[\nabla_z, W_{ij}] = -\mathcal{H}_{ij}$ all anticommuting coordinates and put $z=0$, one gets $H_{ij} = i[A_5, \phi_{ij}]$. Here $H_{ij} = \mathcal{H}_{ij}^{(0)}|$ and $\phi_{ij} = W_{ij}^{(0)}|$
- Result: constraints yield the expressions for auxiliary fields V_m and H_{ij} in terms of scalar field A_5 in SSW theory. In the other words, a role of scalar A_5 is played by z-independent scalar component of connection Γ_z.

$USp(4)/U(1) \times U(1)$ harmonic variables

- Harmonics: 4×4 unitary matrices u^I_j with unit determinant, preserving the antisymmetric matrix Ω^{ij}

$$u^I_i \bar{u}^i_J = \delta^I_J, \quad u^I_i \Omega^{ij} u^J_j = \Omega^{IJ}$$

$$u^{Ii} = \Omega^{ik} u^I_k = \Omega^{IK} \bar{u}^i_K, \quad u^I_i = \Omega_{ij} u^{Ij}, \quad \bar{u}^i_I = \Omega_{IJ} u^J_k \Omega^{ki}.$$

- Notations of harmonics in according with their $U(1)$ charges

$$u^1_i = u^{(+,0)}_i, \quad u^2_i = u^{(-,0)}_i, \quad u^3_i = u^{(0,+)}_i, \quad u^4_i = u^{(0,-)}_i$$

- Orthogonality and completeness conditions. Various identities. Complex conjugation, tilde conjugation.
- Derivatives with respect to harmonics, integral over harmonics

$\mathcal{N}=4$ central charge harmonic superspace

- Harmonic projections of anticommuting coordinates

$$\theta^I_\alpha = -\bar{u}^{Ii}\theta_{\alpha i} = u^I_i\theta^i_\alpha, \quad \bar{\theta}^I_{\dot\alpha} = u^I_i\bar{\theta}^i_{\dot\alpha} = -\bar{u}^{Ii}\bar{\theta}_{\dot\alpha i}$$

- Harmonic projections of superstrengths W_{ij}

$$u^{(+,0)}_i u^{(-,0)}_j W^{ij} = W^{(0,0)}_1, \quad u^{(+,0)}_i u^{(0,+)}_j W^{ij} = W^{(+,+)},$$

$$u^{(+,0)}_i u^{(0,-)}_j W^{ij} = W^{(+,-)}, \quad u^{(-,0)}_i u^{(0,+)}_j W^{ij} = W^{(-,+)},$$

$$u^{(-,0)}_i u^{(0,-)}_j W^{ij} = W^{(-,-)}, \quad u^{(0,+)}_i u^{(0,-)}_j W^{ij} = W^{(0,0)}_2 = -W^{(0,0)}_1$$

- Harmonic derivatives

$$\partial^{(\pm\pm,0)} = u^{(\pm,0)}_i \frac{\partial}{\partial u^{(\mp,0)}_i}, \qquad \partial^{(0,\pm\pm)} = u^{(0,\pm)}_i \frac{\partial}{\partial u^{(0,\mp)}_i}$$

$$\partial^{(\pm,\pm)} = u^{\pm,0}_i \frac{\partial}{\partial u^{0,\mp}_i} + u^{0,\pm}_i \frac{\partial}{\partial u^{\mp,0}_i}, \qquad \partial^{(\pm,\mp)} = u^{(\pm,0)}_i \frac{\partial}{\partial u^{0,\pm}_i} - u^{(0,\mp)}_i \frac{\partial}{\partial u^{\mp,0}_i}$$

Commutation relations are equivalent to $USp(4)$ algebra

- Harmonic projections of gauge covariant spinor operators

$$\nabla^{(\pm,0)}_{\hat\alpha} = u^{(\pm,0)}_i \nabla^i_{\hat\alpha}, \quad \nabla^{(0,\pm)}_{\hat\alpha} = u^{(0,\pm)}_i \nabla^i_{\hat\alpha}$$

- Algebra of covariant derivatives in terms of operators $\nabla^{(\pm,0)}_{\hat\alpha}, \nabla^{(0,\pm)}_{\hat\alpha}$

$$\{\nabla^{(+,0)}_{\hat\alpha}, \nabla^{(+,0)}_{\hat\beta}\} = 0, \quad \{\nabla^{(0,+)}_{\hat\alpha}, \nabla^{(0,+)}_{\hat\beta}\} = 0$$

$$\{\nabla^{(\pm,0)}_{\alpha}, \bar\nabla^{(\mp,0)}_{\dot\alpha}\} = \mp 2i\nabla_{\alpha\dot\alpha}, \quad \{\nabla^{(0,\pm)}_{\alpha}, \bar\nabla^{(0,\mp)}_{\dot\alpha}\} = \mp 2i\nabla_{\alpha\dot\alpha},$$
$$\{\nabla^{(+,0)}_{\hat\alpha}, \nabla^{(-,0)}_{\hat\beta}\} = 2i\varepsilon_{\hat\alpha\hat\beta}\nabla_z \pm 2i\varepsilon_{\hat\alpha\hat\beta} W^{(0,0)}_1,$$
$$\{\nabla^{(0,+)}_{\hat\alpha}, \nabla^{(0,-)}_{\hat\beta}\} = 2i\varepsilon_{\hat\alpha\hat\beta}\nabla_z \pm 2i\varepsilon_{\hat\alpha\hat\beta} W^{(0,0)}_2,$$
$$\{\nabla^{(+,0)}_{\hat\alpha}, \nabla^{(0,-)}_{\hat\beta}\} = \pm 2i\varepsilon_{\hat\alpha\hat\beta} W^{(+,-)}, \quad \{\nabla^{(+,0)}_{\hat\alpha}, \nabla^{(0,+)}_{\hat\beta}\} = \pm 2i\varepsilon_{\hat\alpha\hat\beta} W^{(+,+)},$$
$$\{\nabla^{(-,0)}_{\hat\alpha}, \nabla^{(0,+)}_{\hat\beta}\} = \pm 2i\varepsilon_{\hat\alpha\hat\beta} W^{(-,+)}, \quad \{\nabla^{(-,0)}_{\hat\alpha}, \nabla^{(0,-)}_{\hat\beta}\} = \pm 2i\varepsilon_{\hat\alpha\hat\beta} W^{(-,-)}$$

$\mathcal{N}=4$ central charge harmonic superspace

Harmonic projections of W_{ij} transform through each other under the action of $USp(4)$ generators

$$\partial^{(++,0)}W^{(-,+)} = W^{(+,+)}, \quad \partial^{(+,-)}W^{(-,+)} = -2W^{(0,0)},$$

$$\partial^{(--,0)}W^{(+,+)} = W^{(-,+)}, \quad \partial^{(0,--)}W^{(+,+)} = W^{(+,-)}, \quad \partial^{(-,-)}W^{(+,+)} = 2W_1^{(0,0)}$$

$$\partial^{(++,0)}W^{(+,+)} = 0, \quad \partial^{(+,-)}W^{(+,+)} = \partial^{(-,+)}W^{(+,+)} = 0.$$

Harmonic superspace and analytic subspaces

$USp(4), \mathcal{N}=4$ harmonic superspace is parameterized by coordinates $(x^m, z, \theta^{(\pm,0)}_{\hat\alpha}, \theta^{(0,\pm),}_{\hat\alpha}, u_i)$ ($\hat\alpha = (\alpha, \dot\alpha)$).

Bianchi identities and algebra of covariant derivatives in terms of operators $\nabla^{(\pm,0)}_{\hat\alpha}, \nabla^{(0,\pm)}_{\hat\alpha}$ show that each of the following harmonic projections $W^{(+,+)}, W^{(+,-)}, W^{(-,+)}, W^{(-,-)}$ lives on its own analytic subspace

$$\nabla^{(+,0)}_{\alpha} W^{(+,+)} = \nabla^{(0,+)}_{\alpha} W^{(+,+)} = \bar\nabla^{(+,0)}_{\dot\alpha} W^{(+,+)} = \bar\nabla^{(0,+)}_{\dot\alpha} W^{(+,+)} = 0$$

$$W^{(+,+)} = W^{(+,+)}(x, z, \theta^{(+,0)}, \theta^{(0,+)}, \bar\theta^{(+,0)}, \bar\theta^{(0,+)}, u)$$

$$\nabla^{(+,0)}_{\alpha} W^{(+,-)} = \nabla^{(0,-)}_{\alpha} W^{(+,-)} = \bar\nabla^{(+,0)}_{\dot\alpha} W^{(+,-)} = \bar\nabla^{(0,-)}_{\dot\alpha} W^{(+,-)} = 0$$

$$W^{(+,-)} = W^{(+,-)}(x, z, \theta^{(+,0)}, \theta^{(0,-)}, \bar\theta^{(+,0)}, \bar\theta^{(0,-)}, u)$$

Analogously

$$W^{(-,+)} = W^{(-,+)}(x, z, \theta^{(-,0)}, \theta^{(0,+)}, \bar\theta^{(-,0)}, \bar\theta^{(0,+)}, u)$$

$$W^{(-,-)} = W^{(-,-)}(x.z, \theta^{(-,0)}, \theta^{(0,-)}, \bar\theta^{(-,0)}, \bar\theta^{(0,-)} u)$$

Four different analytical subspaces of full harmonic superspace

$W^{(0,0)}$ is not analytical

Basic superstrength in $USp(4), \mathcal{N} = 4$ SYM theory

All strengths $G_{\alpha i}, \bar{G}_{\dot{\alpha} i}, F_{\alpha i m}, \bar{F}_{\dot{\alpha} i m}, \mathcal{V}_m, F_{mn}, \mathcal{H}_{ij}$ are expressed in terms of real scalar superfield W_{ij} and its gauge covariant derivatives. Then one introduces the corresponding harmonic projections. Therefore the harmonic projections of above strengths are expressed through the harmonic projections of W_{ij}. They are expressed in terms of $W^{(+,+)}$ with help of $USp(4)$ generators. As a result, the superfield $W^{(+,+)}$ can be considered as basic superstrength in $USp(4), \mathcal{N} = 4$ SYM theory.

Analytic basis

- Analytic subspace for $W^{(+,+)}$ is parameterized by the coordinates

$$\{\zeta^M, u\} = \{x_A^m, z_A, \theta_\alpha^{(+,0)}, \theta_\alpha^{(0,+)}, \bar\theta_{\dot\alpha}^{(+,0)}, \bar\theta_{\dot\alpha}^{(0,+)}, u_i^{(\pm,0)}, u_i^{(0,\pm)}\}$$

-

$$x_A^m = x^m - i\theta^{(-,0)}\sigma^m\bar\theta^{(+,0)} - i\theta^{(+,0)}\sigma^m\bar\theta^{(-,0)} - i\theta^{(0,-)}\sigma^m\bar\theta^{(0,+)} - i\theta^{(0,+)}\sigma^m\bar\theta^{(0,-)}$$

$$z_A = z + i\theta^{(-,0)\alpha}\theta_\alpha^{(+,0)} + i\theta^{(0,-)\alpha}\theta_\alpha^{(0,+)} - i\bar\theta_{\dot\alpha}^{(+,0)}\bar\theta^{(-,0)\dot\alpha} - i\bar\theta_{\dot\alpha}^{(0,+)}\bar\theta^{(0,-)\dot\alpha}$$

- Covariant spinor derivatives

$$short: \quad D_{\hat\alpha}^{(+,0)} = \frac{\partial}{\partial\theta^{(-,0)\hat\alpha}}, \quad D_{\hat\alpha}^{(0,+)} = \frac{\partial}{\partial\theta^{(0,-)\hat\alpha}},$$

$$long: \quad D_\alpha^{(-,0)}, \quad D_\alpha^{(0,-)}, \quad \bar D_{\dot\alpha}^{(-,0)}, \quad \bar D_{\dot\alpha}^{(0,-)}$$

- Basic property: if $\Phi^{(q_1,q_2)}$ is an analytic superfield, then it is harmonic analytic superfield, i.e. if $D_{\hat\alpha}^{(+,0)}\Phi^{(q_1,q_2)} = 0, D_{\hat\alpha}^{(0,+)}\Phi^{(q_1,q_2)} = 0$, then $D^{(++,0)}\Phi^{(q_1,q_2)} = 0, D^{(\pm,+)}\Phi^{(q_1,q_2)} = 0, D^{(0,++)}\Phi^{(q_1,q_2)} = 0$

Superfield action and its component form

Requirements to superfield action

- Dependence of $\mathcal{N}=4$ superfields
- Gauge invariance
- $\mathcal{N}=4$ supersymmetry
- Central charge invariance

A proposal for action

$$S \sim \mathrm{tr} \int d\zeta^{(-4,-4)} du ((\theta^{(+,0)})^2 - (\bar{\theta}^{(+,0)})^2)((\theta^{(0,+)})^2 - (\bar{\theta}^{(0,+)})^2)\mathcal{L}^{(2,2)}$$

$$\mathcal{L}^{(2,2)} = W^{(+,+)} W^{(+,+)}$$

$\mathcal{L}^{(2,2)}$ is an analytic and harmonically short superfield.

$$d\zeta^{(-4,-4)} du = d^4 x_A d^2\theta^{(+,0)} d^2\theta^{(0,+)} d^2\bar{\theta}^{(+,0)} d^2\bar{\theta}^{(0,+)} du$$

$d\zeta^{(-4,-4)} du = d^4 x_A d^2\theta^{(+,0)} d^2\theta^{(0,+)} d^2\bar{\theta}^{(+,0)} d^2\bar{\theta}^{(0,+)} du$ is an analytic superspace dimensionless integration measure.

Superfield action and its component form

- Gauge invariance is obvious
- $\mathcal{N}=4$ supersymmetry (analogously to $\mathcal{N}=2$ theory with central charge; N. Dragon, E. Ivanov, S. Kuzenko, E. Sokatchev, U. Theis, 1997, 1998) The $\mathcal{N}=4$ supersymmetry coordinate transformations are

$$\delta x^A_{\alpha\dot\alpha} = -2i(\epsilon^{(-,0)}_\alpha \bar\theta^{(+,0)}_{\dot\alpha} + \epsilon^{(0,-)}_\alpha \bar\theta^{(0,+)}_{\dot\alpha} + \theta^{(+,0)}_\alpha \bar\epsilon^{(-,0)}_{\dot\alpha} + \theta^{(0,+)}_\alpha \bar\epsilon^{(0,-)}_{\dot\alpha})$$

$$\delta z_A = 2i(\epsilon^{\alpha(-,0)}\theta^{(+,0)}_\alpha + \epsilon^{\alpha(0,-)}\theta^{(0,+)}_\alpha - \bar\epsilon^{(-,0)}_{\dot\alpha}\bar\theta^{(+,0)\dot\alpha} - \bar\epsilon^{(0,-)}_{\dot\alpha}\bar\theta^{(0,+)\dot\alpha}),$$

$$\delta\theta^{(+,0)}_{\hat\alpha} = \epsilon^{(+,0)}_{\hat\alpha}$$

Action transforms as follows

$$\delta S|_{\epsilon^{(-,0)}} = \int d\zeta^{(-4,-4)} du\{((\theta^{(+,0)})^2 - (\bar\theta^{(+,0)})^2)((\theta^{(0,+)})^2 - (\bar\theta^{(0,+)})^2)\times$$

$$\times\delta z_A \frac{\partial}{\partial z_A}\mathcal{L}^{(2,2)} - 2(\epsilon^{(+,0)}\theta^{(+,0)} - \bar\epsilon^{(+,0)}\bar\theta^{(+,0)})((\theta^{(0,+)})^2 - (\bar\theta^{(0,+)})^2)\mathcal{L}^{(2,2)}\}$$

After integrating by parts and using some identities, one gets $\delta S|_{\epsilon^{(-,0)}} = 0$. Analogously $\delta S|_{\epsilon^{(0,-)}} = 0$. Action is $\mathcal{N}=4$ supersymmetric.

Superfield action and its component form

- Central charge independence (analogously to $\mathcal{N}=2$ theory with central charge; N. Dragon, E. Ivanov, S. Kuzenko, E. Sokatchev, U. Theis, 1997, 1998)

$$\frac{\partial}{\partial z_A} S \sim i \int d\zeta^{(-4,-4)} du((\theta^{(0,+)})^2 - (\bar{\theta}^{(0,+)})^2)\times$$
$$\times(\partial^{(++,0)} - 2i\theta^{(+,0)}\sigma^m\bar{\theta}^{(+,0)}\partial_m)\mathcal{L}^{(2,2)} = 0$$

  The identities $D^{(++,0)}\mathcal{L}^{(2,2)} = D^{(0,++)}\mathcal{L}^{(2,2)} = 0$ have been used.
  Action is invariant under any central charge transformation.

- Action constructed form $\mathcal{N}=4$ superfields.

All requirements to action are fulfilled

Component form. Reproduction of SSW action.

- Integration rule

$$\int d\zeta^{(-4,-4)}du =$$

$$= \frac{1}{256}\int d^4x du^{(\pm,0)} du^{(0,\pm)} (\nabla^{(-,0)})^2 (\bar{\nabla}^{(-,0)})^2 (\nabla^{(0,-)})^2 (\bar{\nabla}^{(0,-)})^2$$

- First, integration over anticommuting coordinates $\theta^{(0,+)}, \bar{\theta}^{(0,+)}$

$$S \sim \mathrm{tr}\int d^4x du d^2\theta^{(+,0)} \mathcal{L}^{(++,0)} - \mathrm{tr}\int du d^4x d^2\bar{\theta}^{(+,0)} \mathcal{L}^{(++,0)}$$

$$\mathcal{L}^{(++,0)} = \frac{1}{4}\int du^{(0,\pm)} ((\nabla^{(-,0)})^2 - (\bar{\nabla}^{(-,0)})^2) \mathcal{L}^{(2,2)}$$

$$= -\frac{1}{2}\int du^{(0,\pm)} (G^{(+,0)\alpha} G^{(+,0)}_{\alpha} + \bar{G}^{(+,0)\dot{\alpha}} \bar{G}^{(+,0)}_{\dot{\alpha}} - 2iH^{(+,-)} W^{(+,+)}).$$

- Second, integration over harmonics $u^{(0,\pm)}$

$$\mathcal{L}^{(++,0)} = u^{(+,0)(i} u^{(+,0)j)} \mathcal{L}_{ij}$$

$$\mathcal{L}_{ij} = -\frac{1}{2}(G_i^{\alpha} G_{\alpha j} + \bar{G}_i^{\dot{\alpha}} \bar{G}_{\dot{\alpha} j} + \frac{i}{2} H_i^{\ k} W_{jk})$$

Superfield action and its component form

- Third, integration over harmonics $u_i^{(\pm,0)}$

$$S \sim -\frac{1}{20}\mathrm{tr}\int d^4x(\nabla^{\alpha(i}\nabla_\alpha^{j)} - \bar\nabla_{\dot\alpha}^{(i}\bar\nabla^{\dot\alpha j)})\mathcal{L}_{ij}|_{\theta=0} = \mathrm{tr}\int d^4x\mathcal{L},$$

$$\mathcal{L} = -\frac{1}{20}\{[\nabla^{\beta i}, \nabla_\beta^j]G_i^\alpha G_{\alpha j} + \nabla^{\beta j}G^{\alpha i}\nabla_{\beta i}G_{\alpha_j} - \nabla^{\beta i}G_i^\alpha\nabla_\beta^j G_{\alpha_j}$$
$$+[\nabla^{\beta i}, \nabla_\beta^j]\bar G_i^{\dot\alpha}\bar G_{\dot\alpha j} + \nabla^{\beta j}\bar G^{\dot\alpha i}\nabla_{\beta i}\bar G_{\dot\alpha_j} - \nabla^{\beta i}\bar G_i^{\dot\alpha}\nabla_\beta^j\bar G_{\dot\alpha_j}$$
$$+[\bar\nabla^{\dot\beta i}, \bar\nabla_{\dot\beta}^j]G_i^\alpha G_{\alpha j} + \bar\nabla^{\dot\beta j}G^{\alpha i}\bar\nabla_{\dot\beta i}G_{\alpha_j} - \bar\nabla^{\dot\beta i}G_i^\alpha\bar\nabla_{\dot\beta}^j G_{\alpha_j}$$
$$+[\bar\nabla^{\dot\beta i}, \bar\nabla_{\dot\beta}^j]\bar G_i^{\dot\alpha}\bar G_{\dot\alpha j} + \bar\nabla^{\dot\beta j}\bar G^{\dot\alpha i}\bar\nabla_{\dot\beta i}\bar G_{\dot\alpha_j} - \bar\nabla^{\dot\beta i}\bar G_i^{\dot\alpha}\bar\nabla_{\dot\beta}^j\bar G_{\dot\alpha_j}$$
$$+\frac{i}{2}[\nabla^{\alpha i}, \nabla_\alpha^j]H_i{}^l W_{jl} + \frac{i}{2}H_i{}^l[\nabla^{\alpha i}, \nabla_\alpha^j]W_{jl} + i\nabla_j^\alpha H_{il}\nabla_\alpha^i W^{jl} + i\nabla^{\alpha i}H_i{}^l\nabla_\alpha^j W_{jl}$$
$$+\frac{i}{2}[\bar\nabla^{\dot\alpha i}, \bar\nabla_{\dot\alpha}^j]H_i{}^l W_{jl} + \frac{i}{2}H_i{}^l[\bar\nabla^{\dot\alpha i}, \bar\nabla_{\dot\alpha}^j]W_{jl} + i\bar\nabla_j^{\dot\alpha} H_{il}\bar\nabla_{\dot\alpha}^i W^{jl} + i\bar\nabla^{\dot\alpha i}H_i{}^l\bar\nabla_{\dot\alpha}^j W_{jl}\}$$

Dependence on z and θ is absent here.

- Fourth, action (with common coefficient $\frac{1}{4}$ can be transformed (after tedious work) to the form

$$\mathcal{L} = -\frac{1}{4}F^{mn}F_{mn} - \frac{1}{2}V^m V_m + \frac{1}{8}H^{ij}H_{ij} + \frac{1}{8}\nabla^m W_{ij}\nabla_m W^{ij} +$$

$$+\frac{1}{16}[W_{ik}, W_j{}^{k}][W^i{}_l, W^{jl}] +$$

$$+iG^{\alpha i}\nabla_{\dot\alpha\alpha}\bar{G}^{\dot\alpha}_i + \frac{i}{2}[W_{ik}, G^{\alpha k}]G^i_\alpha + \frac{i}{2}[W_{ik}, \bar{G}^k_{\dot\alpha}]\bar{G}^{\dot\alpha i}$$

∇_m is the covariant derivative of component fields.

This expression contains all terms corresponding to SSW action (1)
Identification of fields in above action with fields in SSW action:
$W_{ij} = 2(\sqrt{2})\phi_{ij}, H_{ij} = 2H'_{ij}, G^i_\alpha = (i)\lambda^i_\alpha$.

Result: the component form of superfield action exactly reproduces the SSW action

Summary

- Harmonic superspace formulation of $\mathcal{N}=4$ SYM theory with central charge.
- Gauge theory in $USp(4)$, $\mathcal{N}=4$ superspace.
- Lagrange multiple A_5 and expressions of auxiliary fields V_m and H_{ij} in terms of A_5, used in SSW theory, have a natural algebraic origin as the consequences of the Bianchi identities for superfield strengths.
- $USp(4)$, $\mathcal{N}=4$ harmonic superspace and several corresponding analytic subspaces.
- It is proved that Bianchi identities in conventional and harmonic $\mathcal{N}=4$ superspaces with central charge provide the superspace treatment of all components of the SSW model.
- The harmonic superspace under consideration allows us to introduce the analytic superfield strength $W^{(+,+)}$, which is a basic object for analytic superfield Lagrangian $\mathcal{L}^{(2,2)}$.
- Gauge invariant, $\mathcal{N}=4$ supersymmetric action, invariant under central charge transformations is proposed and it is proved that this action exactly reproduces the component action of SSW model

Summary

Problems

- Superfield action is not a functional of independent superfield potentials. Unclear how to write the superfield equations of motion.
- Problem of formulation in terms of independent unconstrained superfield potentials open.

THANK YOU VERY MUCH!