Geometry of the Universe in CDT

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Outline

1. Basics of Causal Dynamical Triangulations
2. Geometry of spatial slices
3. Fractal structure of spatial slices
4. Background geometry
5. Radial propagation in spacetime
6. Causal diffusion waves
7. Conclusions

Model of Causal Dynamical Triangulations

Causal Dynamical Triangulations (CDT) is a background independent approach to quantum gravity.

- The partition function of quantum gravity is defined as a formal integral over all geometries weighted by the Einstein-Hilbert action.
- Discretization is used as regularization.
  The gravitational path integral is written as a nonperturbative sum over all causal triangulations $\mathcal{T}$.

$$Z = \int \mathcal{D}[g] e^{i S^{EH}[g]} \rightarrow \sum_{\mathcal{T}} e^{-S^{R}[\mathcal{T}]}$$

$$S = -\frac{1}{G} \int \mathrm{d}t \int \mathrm{d}\Omega \sqrt{g} (R - 6\lambda) \rightarrow -K_0 N_0 + K_4 N_4 + \Delta (N_{14} - 6 N_0)$$

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- **C**ausal **D**ynamical **T**riangulations assume global proper-time foliation. Time-like links and spatial-like links are distinguishable, and the Wick rotation is well defined.
- CDT defines the class of admissible spacetime geometries which contribute to the transition amplitude.
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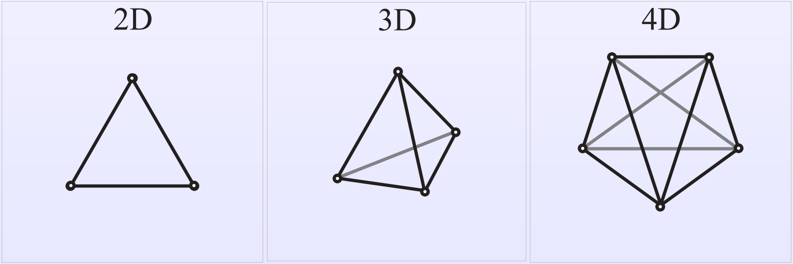
- Spacetime geometries contributing to the path integral are discretized via **simplicial manifolds**.
- A **4-dimensional** configuration in CDT consists of consecutive 3-dimensional slices connected by **four-simplices**.
- Each slice is built from equilateral **tetrahedra** and forms a **3-dimensional** simplicial manifold with topology S^3.The slices are numerated by integer time label t.
- Manifolds of topology $S^3 \times S^1$ have a causal structure.



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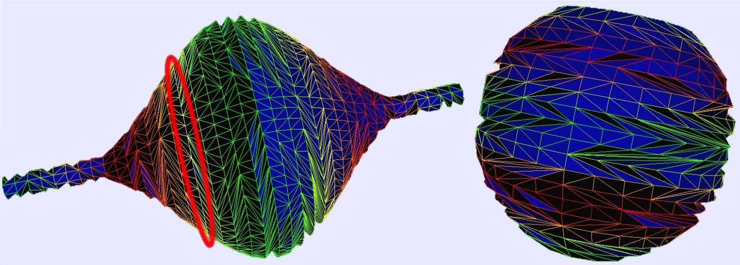

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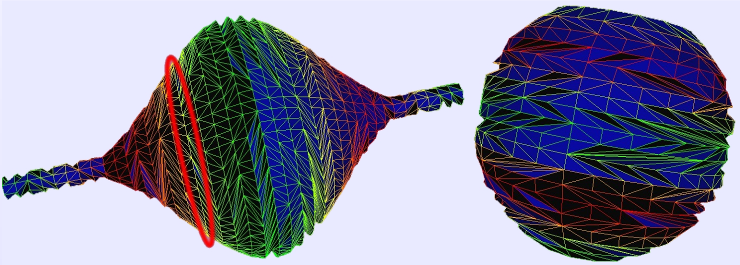

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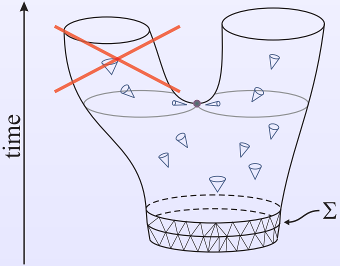


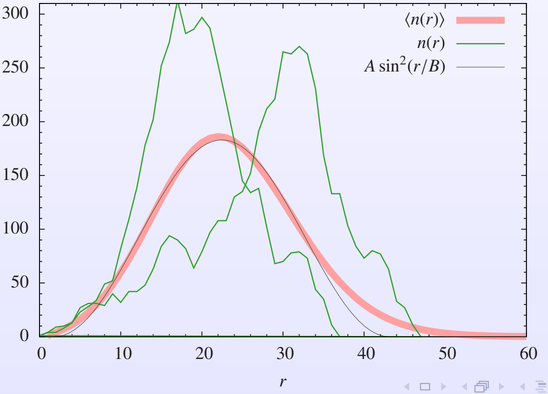
Geometry of spatial slices

- By construction the spatial slices - of fixed time t - have a S^3 topology and are built of regular tetrahedra.
- However, their geometry may significantly vary from S^3 geometry.
- **How the geometry of constant time surface looks like?**

The time foliation of spacetime is present in many other approaches to quantum gravity. The understanding of the geometric structure of constant time surface may be important for further development of quantum gravity theories.

3D spatial slices - radial volume distribution

- Within one slice, the radial volume $n(r)$ measures the number of tetrahedra at a three-dimensional geodesic distance r from some initial tetrahedron. (Average over configurations, slices, initial points).
- For S^3 we would expect $n(r) \propto \sin^2(r/r_0)$.

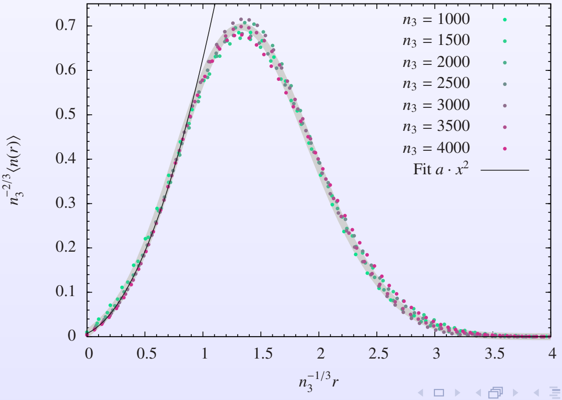


3D spatial slices - Hausdorff dimension

- For Hausdorff dimension d_H following scaling of $n(r)$ with the total slice volume n_3 is expected:

$$r \to n_3^{-1/d_H} \cdot r, \qquad n(r) \to n_3^{-1+1/d_H} \cdot n(r)$$

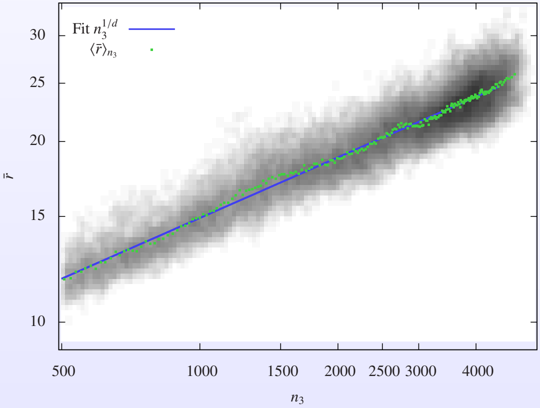
- The results are consistent with $d_H = 3$. (2.98 ± 0.05)



3D spatial slices - Hausdorff dimension

- The average linear extent $\bar{r} \equiv \frac{1}{n_3} \sum_r r \cdot n(r)$ also scales with $d_H = 3$:

$$\bar{r} \propto n_3^{1/d_H}$$



3D spatial slices - spectral dimension

The spectral dimension d_s is defined by the diffusion phenomena

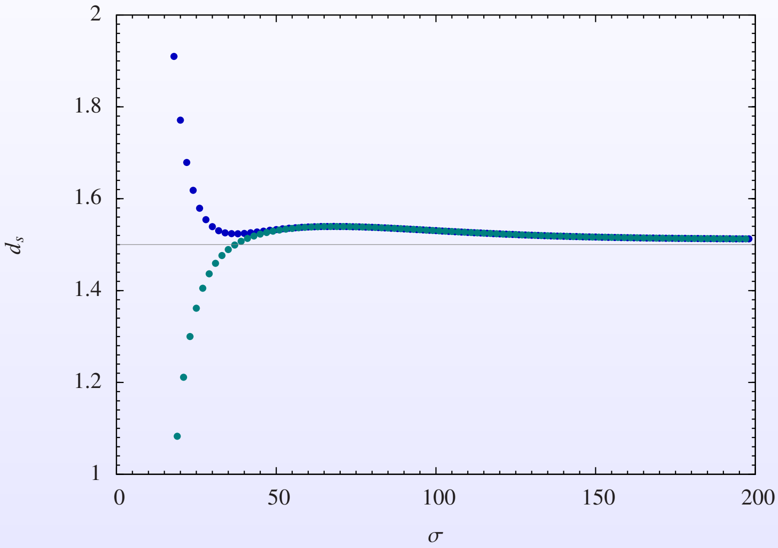
$$d_s = -2 \frac{\mathrm{d} \log P(\sigma)}{\mathrm{d} \log \sigma}$$

where

- σ - diffusion time
- $P(\sigma)$ - return probability after time σ

For S^3 and short diffusion times, the spectral dimension $d_s = 3$.

3D spatial slices - spectral dimension

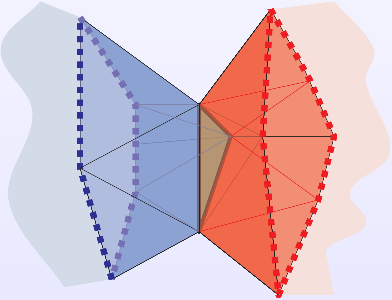


3D spatial slices - spectral dimension

- The measured spectral dimension $d_S \approx 1.5$ is significantly smaller than the Hausdorff dimension $d_H \approx 3$.
- The difference between d_H and d_S is an indication of a fractal nature of slices (determined by foliation).
- **Is the character of quantum geometry fractal?**
- **Can it be described by Gaussian fluctuations around an average geometry?**

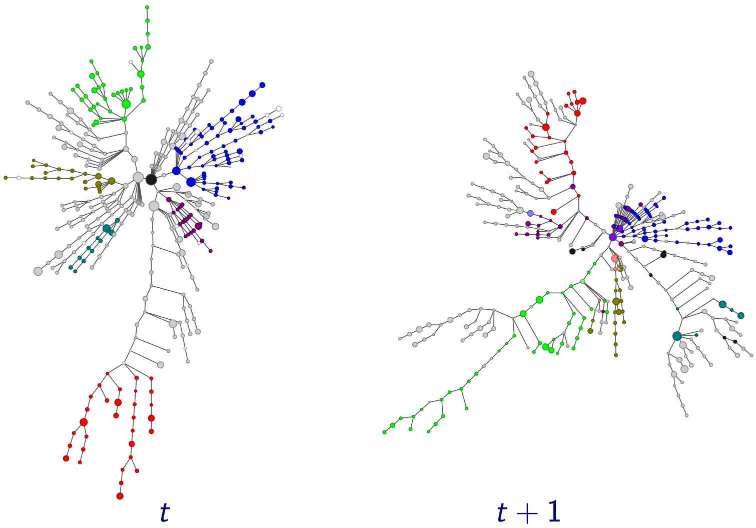
3D spatial slices - fractal structure

- Spatial slices are built from tetrahedra and have S^3 topology.
- We can look for *minimal necks* which separate a triangulation into *almost* disconnected parts. (none for *smooth* triangulation)



3D spatial slices - fractal structure

Tree of minimal necks



Structure of spatial slices

- The quantum geometry has a nontrivial microstructure
- Spatial slices reveal a fractal nature, completely different from smooth S^3
- Similarity to *branched polymers*
- Quantum fluctuations can not be described by Gaussian deviations from background geometry
- The specified surfaces of fixed time are not physical

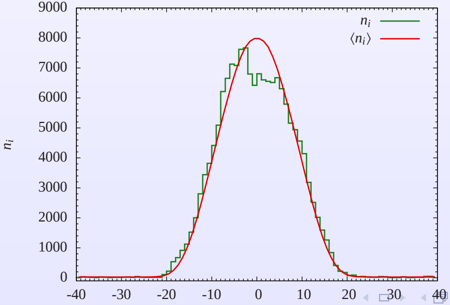
Four-dimensional geometry of spacetime

- So far, only the inside of three-dimensional spatial slices was considered.
- From the point of view of e.g. scalar field propagator more important is the geometry of four-dimensional spacetime.
- The foliation determines the surface of constant time and allows introducing the **spatial volume** operator $\mathbf{n_i}$.
- Spatial volume is defined as the number of tetrahedra building a three-dimensional slice $i = 1 \dots T$ and is the simplest observable giving information about the geometry.

4D de Sitter spacetime as a background geometry

- The spatial volume profile n_i is bell-shaped, the time translation symmetry is spontaneously broken.
- The average profile $\langle n_i \rangle$ agrees with the Euclidean **de Sitter** space (S^4), a classical **vacuum solution**.
- For different total volume N_4, $\langle n_i \rangle$ scales as a genuine **four-dimensional** *Universe*.

$$\langle n_i \rangle = H \cos^3 \left(\frac{i}{W} \right)$$



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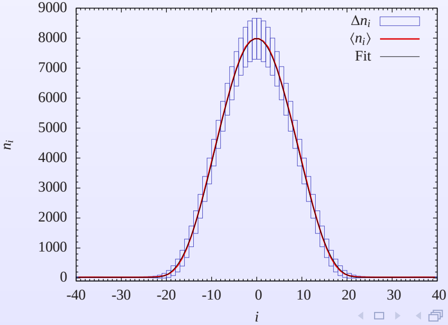

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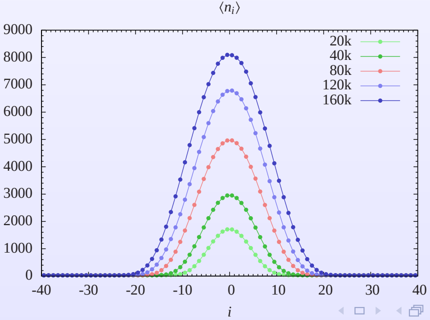

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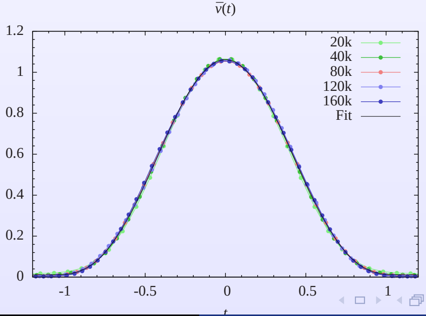
$$t = N_4^{-1/4} i, \quad \bar{v}(t) = N_4^{-3/4} \langle n_i \rangle = \frac{3}{4\omega} \cos^3\left(\frac{t}{\omega}\right).$$



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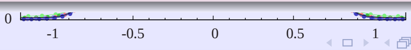
Minisuperspace model

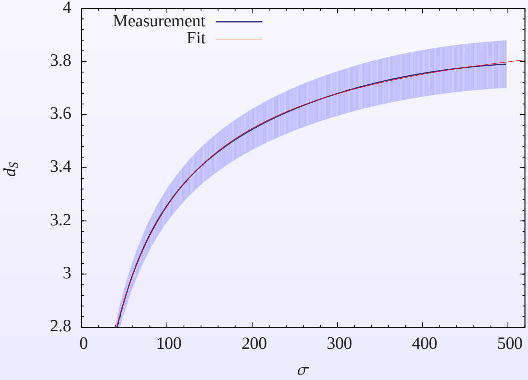
- The expectation value $\langle v(t) \rangle \propto \cos^3\left(\frac{t}{\omega}\right)$ corresponds to S^4 and is a classical solution of maximally symmetric model,

$$\mathrm{d}s^2 = \mathrm{d}t^2 + v^{2/3}(t)\mathrm{d}\Omega_3^2 \ \Rightarrow \ S = \frac{1}{G}\int \frac{\dot{v}^2}{v} + v^{\frac{1}{3}} - \lambda v \mathrm{d}t.$$

- The topology of spacetime dynamically transmutes from $S^3 \times S^1$ to S^4.
- Quantum fluctuations of $v(t)$ are also well described by the effective action.



Simulations of the diffusion process allow to compute spectral dimension d_s.



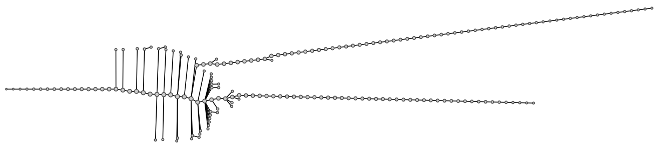
Extrapolation of the results gives short and long range behavior

$$d_s(\sigma \to 0) = 1.95 \pm 0.10, \quad d_s(\sigma \to \infty) = 4.02 \pm 0.10,$$

where σ is a fictitious diffusion time.

Radial propagation in four dimensions

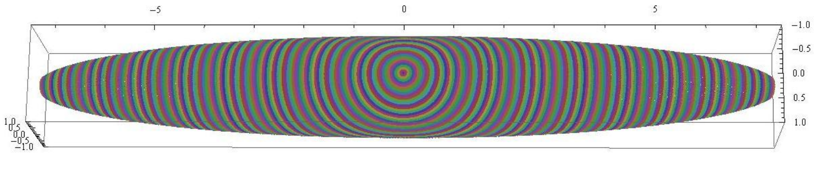
In analogy to the tree of minimal necks one can analyze a **tree of connected components**



- No fractal structure
- Shape of an elongated (in time direction) spheroid
- From the shortest circumference one can estimate the semi-minor axis of the spheroid (c.a. 4)
- From the volume distribution $v(t)$ one can estimate the semi-major axis of the spheroid (c.a. 40)

Two-dimensional spheroid

- Curves of equidistant points from a point on a spheroid
- Top view:



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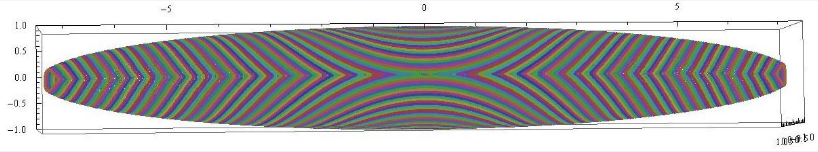

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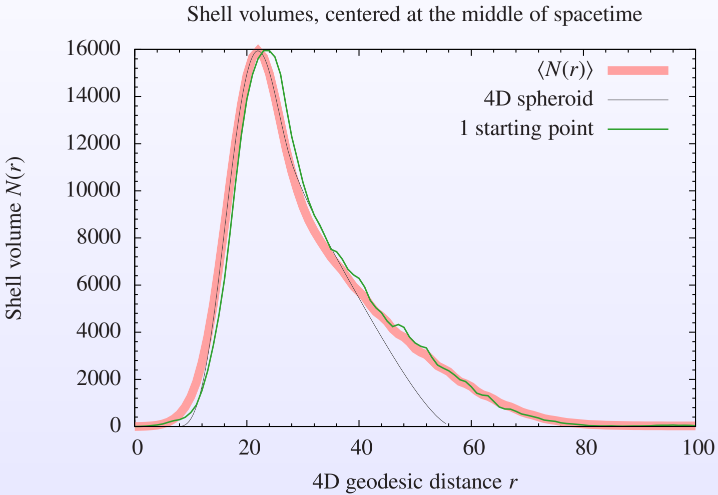


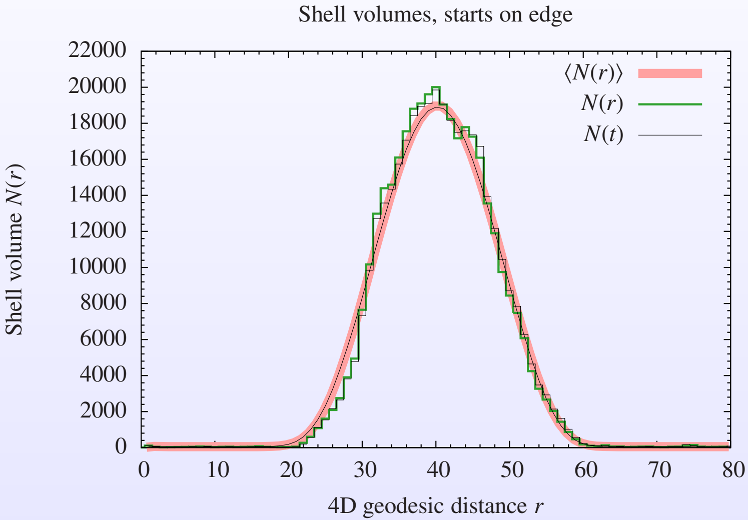
Four-dimensional geometry of spacetime

- The foliation and imposition of causal structure allows to distinguish between the length of time and spatial links.
- This resulted in an elongated shape of spacetime and decomposition of radial shells into two parts.
- Fluctuations of volumes of radial shells are much smaller then in the three-dimensional case.
- 4D geometry looks much smoother than 3D geometry.
- Connected components tree behaves classically.

Another definition of surfaces of constant time

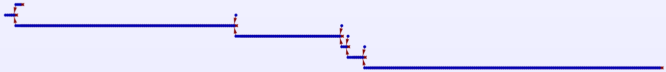
- Since the surfaces of fixed time are not physical, one is motivated to change the definition of *time*.
- As a definition of a constant time surface one can assume the front of a wave released from *the beginning of time* - causal diffusion waves.
- After averaging over configurations the spherical symmetry would be reproduced.
- The topology of the wave front is preserved.
- The radial propagation corresponds to propagation of light rays.

Causal diffusion waves



Causal diffusion waves

- With high accuracy, the front of the wave overlaps with the original foliation.
- No branching of the wave front is observed - preserved topology (causality).



- The choice of the foliation is not important.

Conclusions

1. The model of Causal Dynamical Triangulations is manifestly **diffeomorphism-invariant** (only geometric invariants) and **nonperturbative**. The contribution coming from the action is as important as the entropy of geometric configurations.
2. The **three-dimensional** geometry of constant time surfaces has a **fractal structure** and is governed by **large fluctuations**:
   - Hausdorff dimension $d_H \approx 3$
   - Spectral dimension $d_S \approx 1.5$
3. The shape of the spacetime resembles an elongated **four-dimensional spheroid**:
   - Hausdorff dimension $d_H \approx 4$
   - Spectral dimension from $d_S \approx 2$ on short distances to $d_S \approx 4$ on long distances.
4. Quantum fluctuations of four-dimensional radial volumes are smaller than in three-dimensional case - a smooth four-dimensional spacetime is reconstructed.

Thank you for your attention