Sorkin's causal set

Manifold

$p \prec q \iff$ you can travel from p to q without going faster than the speed of light

Manifold $\implies (\prec \quad \leftrightarrow \quad g_{\mu\nu}/|g|)$

Discreteness $\leftrightarrow |g|$

(Discrete $\prec$) $\leftrightarrow$ (Discrete $g_{\mu\nu}$)

General

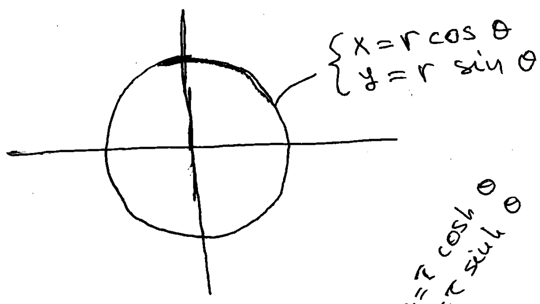
$\prec$ well defined, $g_{\mu\nu}$ is not

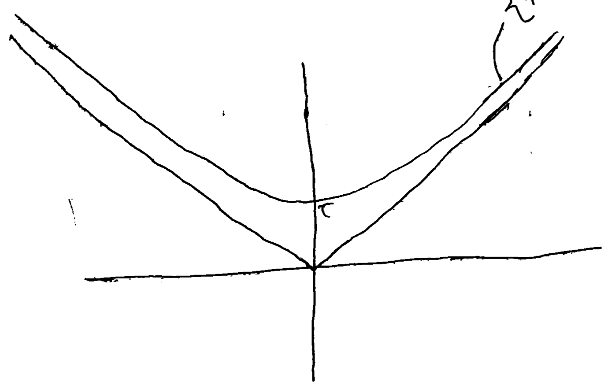
(Some conditions on $\prec$) $\implies$ Manifold $\implies$ Existence of $g_{\mu\nu}$

Above conditions on $\prec$ are unknown!

Key motivation : Manifold structure breaks down on small scales due to quantum fluctuations

Non-locality of Lorentz neighborhood





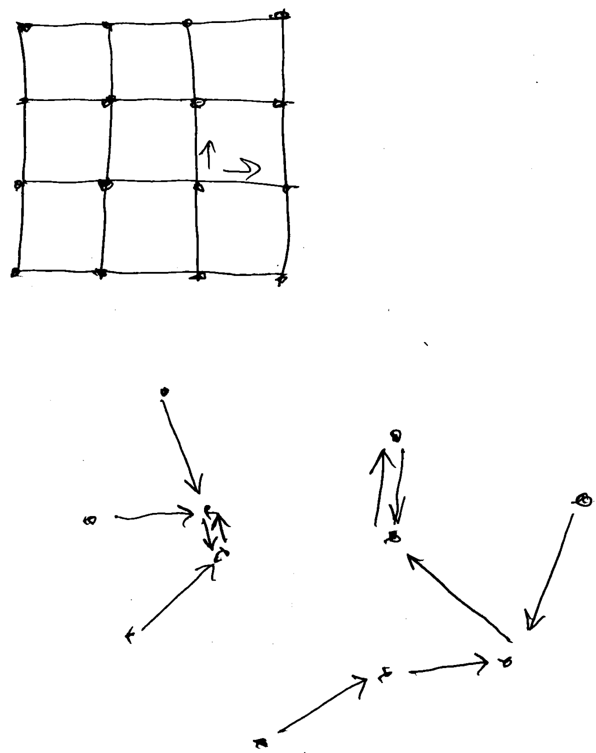
$$t = \sqrt{\tau^2 + r^2} = r\sqrt{1 + \frac{\tau^2}{r^2}} = r\left(1 + \frac{\tau^2}{2r^2}\right) + 0(\tau^2) = r + \frac{\tau^2}{2r} + 0(\tau^2) \tag{1}$$

$$t > r + (1 - \epsilon)\frac{\tau^2}{2r} \tag{2}$$

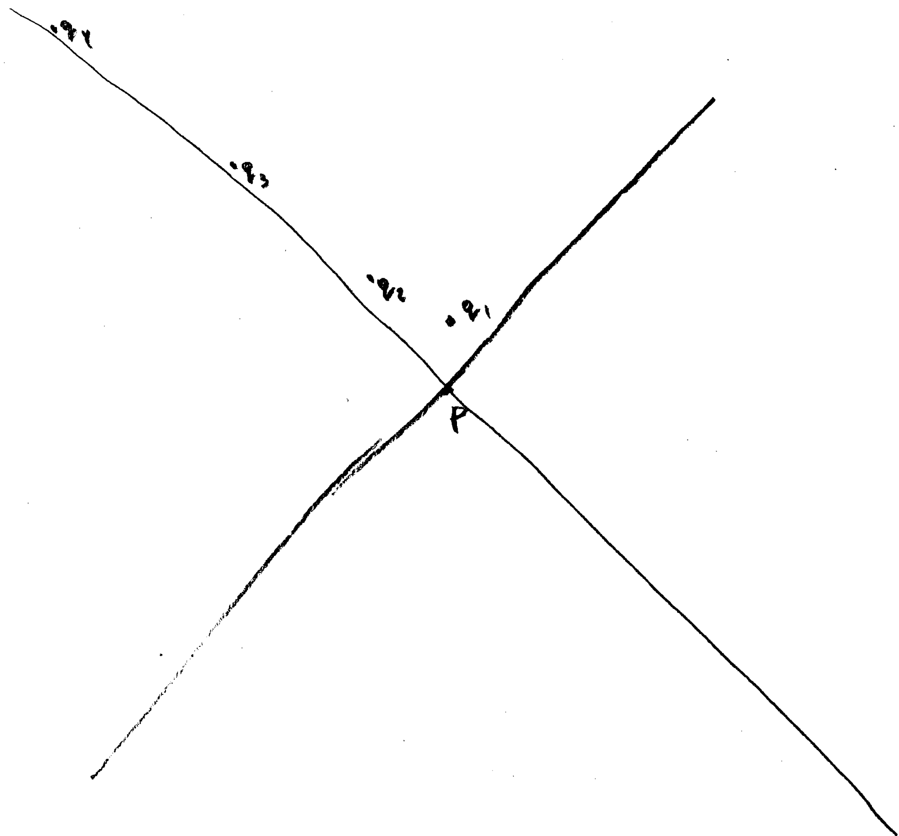
$$\int (t - x)dx > \frac{(1 - \epsilon)\tau^2}{2} \int \frac{dx}{x} = \infty \tag{3}$$

$$\int (t - r)4\pi r^2 dr > \frac{4\pi(1 - \epsilon)\tau^2}{2} \int r dr = \infty \tag{4}$$

"Local" discrete theories result in "preferred frame"



In Lorentzian case "can't choose" nearest neighbor



Problem with discrete theories

Finite (1) Lorentz (2) neighborhood is non-local (NO 3)

Therefore

Discreteness (1), Lorentz covariance (2) and Locality (3) can NOT co-exist

Continuum QFT: Keep 2 and 3; discard 1

— Continuity $\Longrightarrow$ No nearest neighbor $\Longrightarrow$ no preferred frame

Lattice theory: Keep 1 and 3; discard 2

— Nearest neighbor defines preferred frame

Causal set theory: Keep 2 and 3; discard 1m

— Lightcone nonlocality $\Longrightarrow$ No nearest neighbor $\Longrightarrow$ no preferred frame

Violation of Lorentzian (!!!) distance itself!

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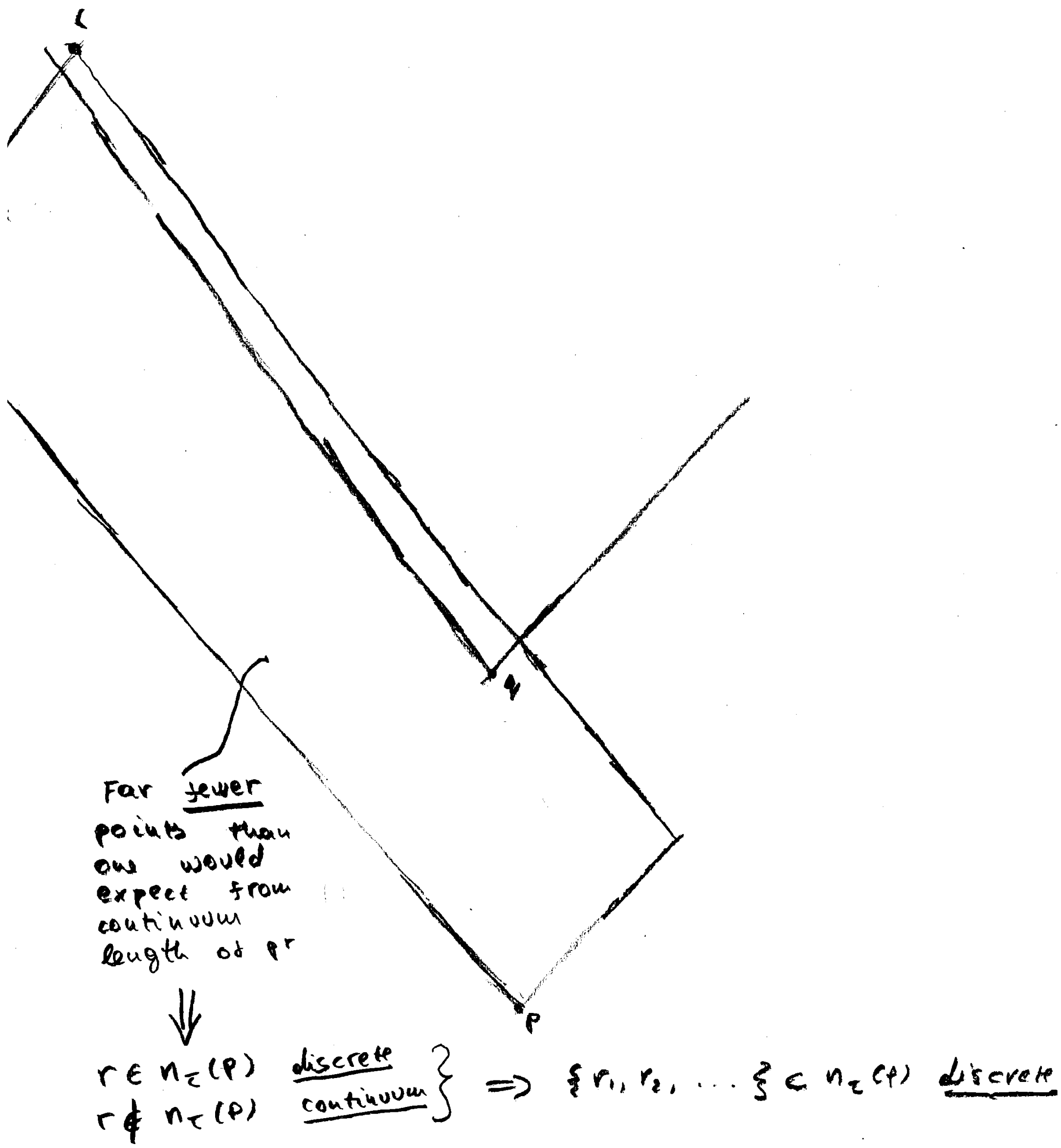


Replace Sorkin with Caianello!

Sorkin: Elements of S are "points" x^μ

Caianello: Elements of S are (x^μ, v^μ), where v^μ is a timelike, future-directed, tangent vector at x^μ

Sorkin: $a \prec b$ if and only if one can go from a to b without going faster than the speed of light

Caianello: $(x^\mu, u^\mu) \prec (y^\mu, v^\mu)$ if and only if one can start at x^μ, with velocity u^μ and arrive at y^μ, with velocity v^μ, while the Lorentzian acceleration, $|d^2x/d\tau^2|$, never exceeds $a_{\max}$ along the way. Here, $a_{\max}$ is aforegiven constant representing "maximal acceleration", similar to c representingn "maximal speed".

Question: How can we define the continuous based acceleration in Caianello's case?

Answer:

1. In Sorkin's case one can ask the same question about "speed" of signals between DIRECT neighbors

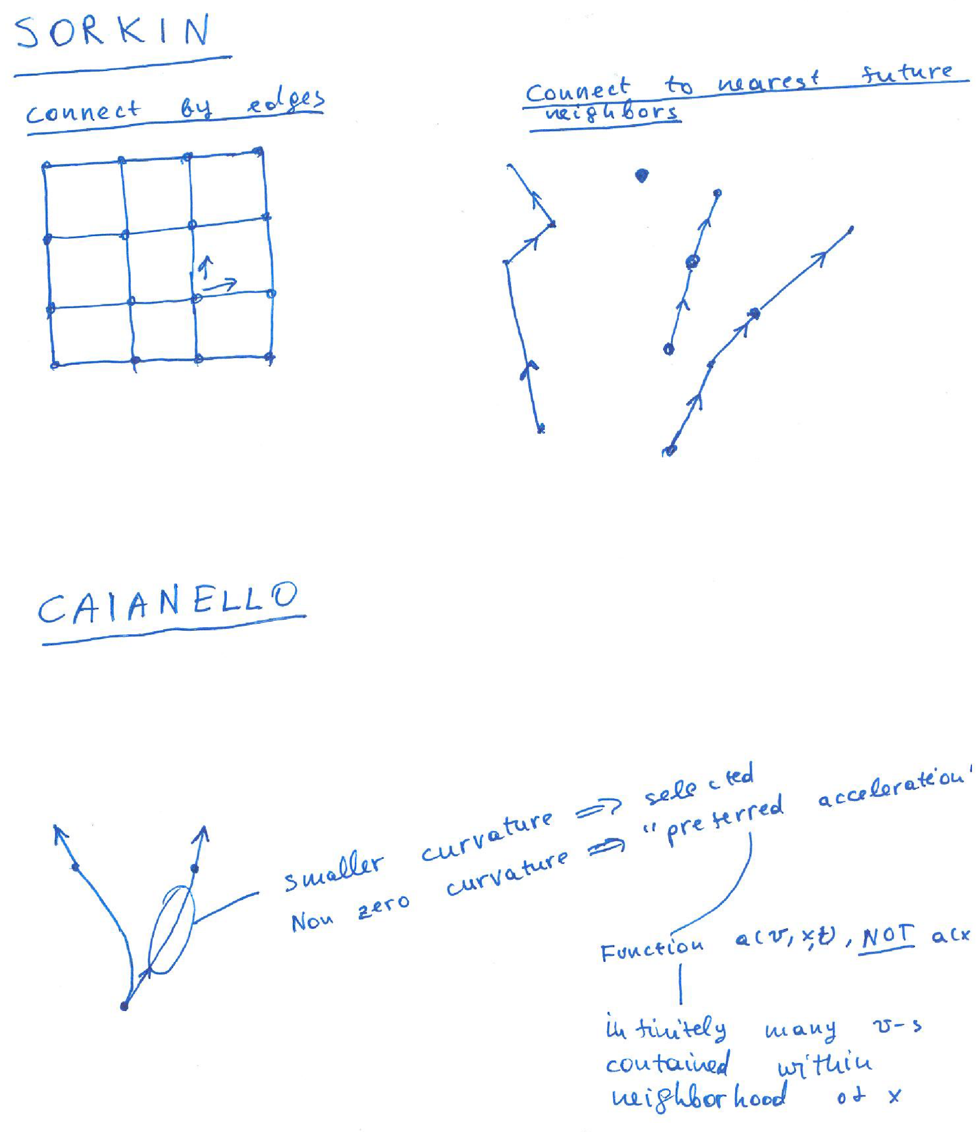
2. The way to handle it is to define "proper map" $f\colon S \mapsto$ (continuous geometry). *The focus is NOT S itself BUT f being "proper" or not!*

a) $f\colon S \mapsto \mathcal{M}$ is Sorkin's proper map: $a \prec b$ if and only if one can go from $f(a)$ to $f(b)$ without going faster than the speed of light

b) f is Caianello's proper map: $a \prec b$ if and only if, whenever $f(a) = (x^\mu, u^\mu)$ and $f(b) = (y^\mu, w^\mu)$, one can start from x^μ with velocity u^μ and reach y^μ with velocity v^μ while making sure that Lorentzian acceleration does not exceed $a_{\max}$.

Improtant: One only makes reference to velocity and acceleration on $\mathcal{M}$; NOT on S.

Nearest neighbors and preferred frame



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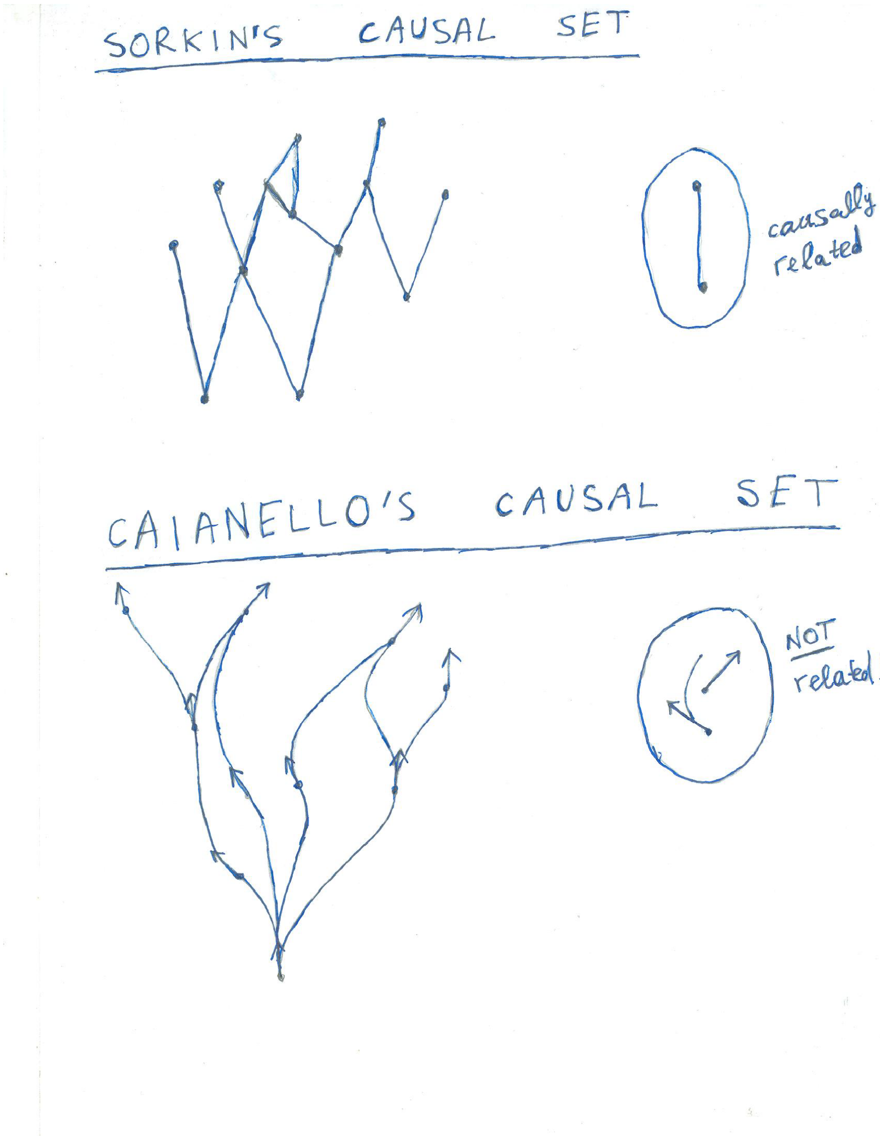
Sorkin's vs Caianello's causal set'



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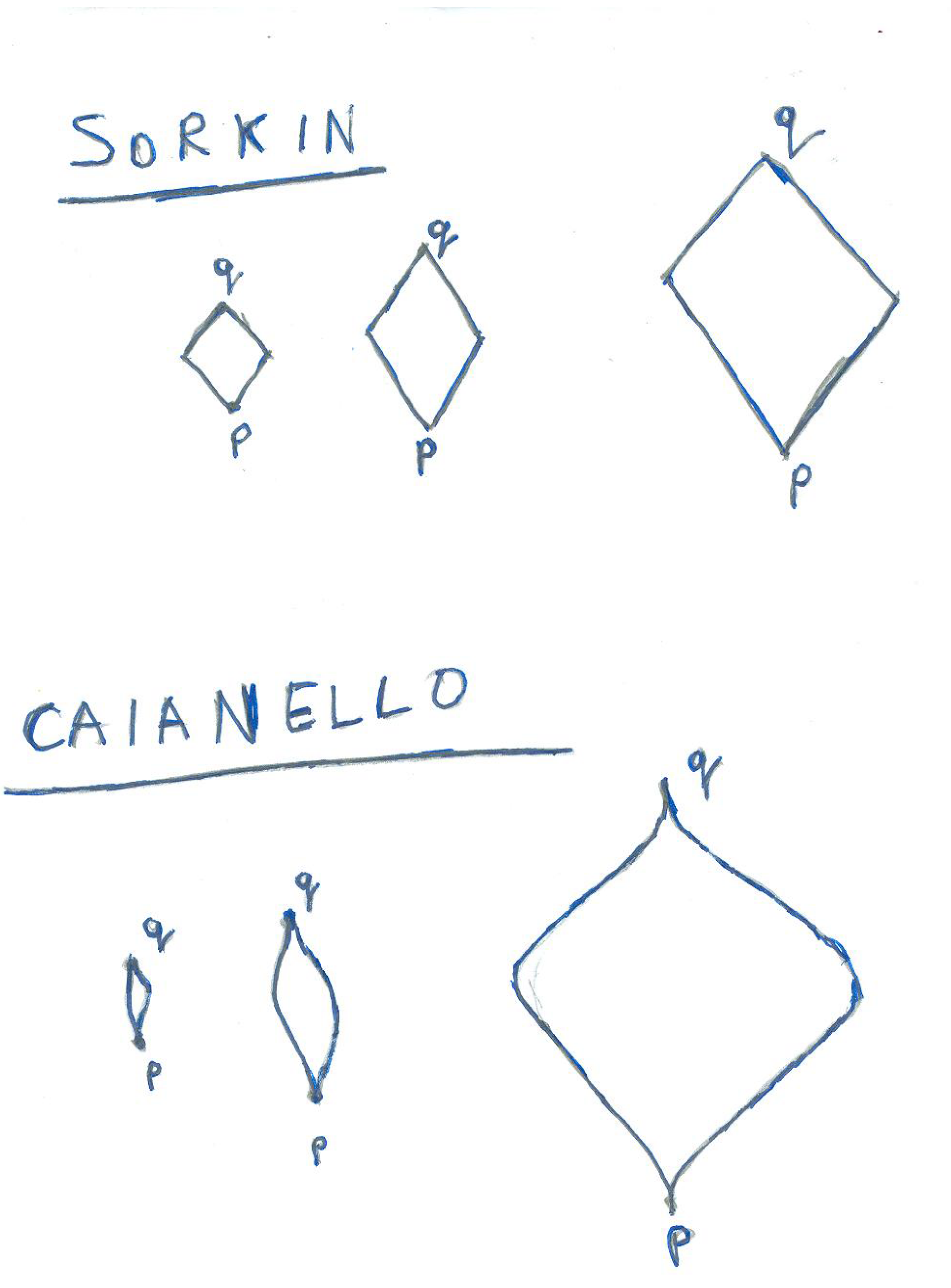
Sorkin's vs Caianello Alexandrov set



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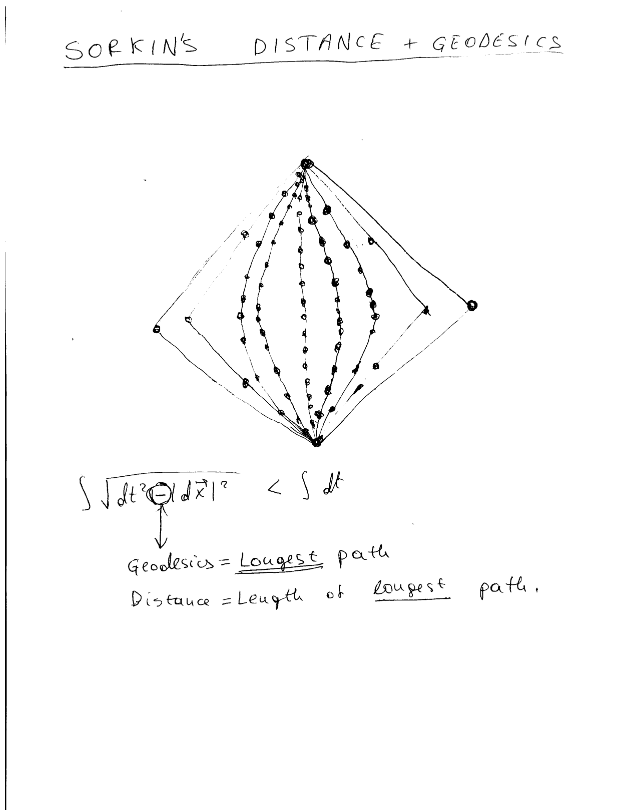
Sorkin's geodesics

For any given p and q, the geodesics connecting these two points is the longest path $p \prec r_1 \prec \cdots \prec r_{n-1} \prec q$



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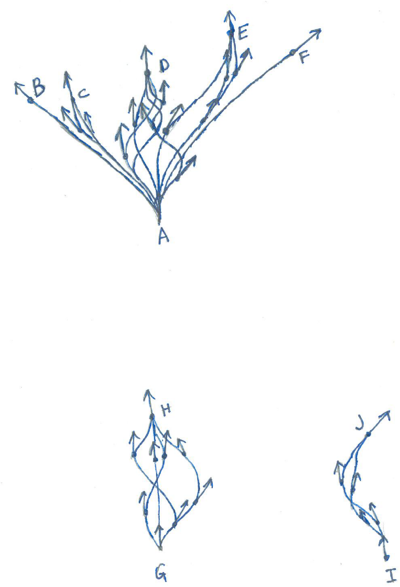
Caianello's geodesics

We say that $q \in F_n(p)$ if and only if the number of n-chains starting at p and ending at q is greater than said number for any other point $q' \succ p$

We say that $q \in P_n(p)$ if and only if the number of n-chains starting at q and ending at p is greater than said number for any other point $q' \prec p$



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Differences between Sorkin's and Caianello's geodesics

Sorkin Between any two points there is a geodesic

Caianello In order for geodesic to exist, two conditions have to be met:

a) The "earlier" point has to "almost" point *towards* the "later" point

b) The "later" point has to "almost" point *away from* the "earlier" point

Sorkin There is arbitrary many geodesics passing through every given point

Caianello

a) We can still have more than one geodesics passing through a given point due to discreteness

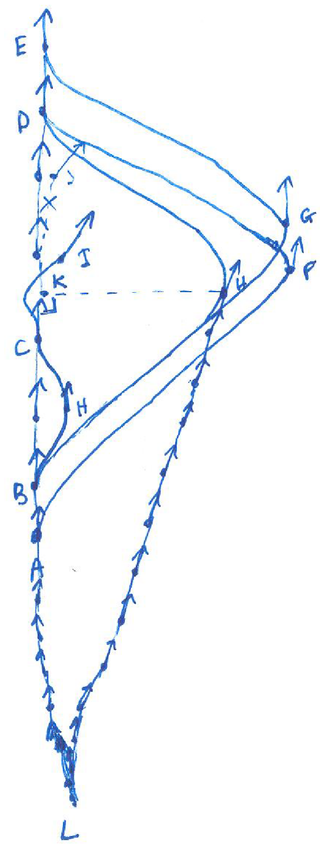
b) Any two geodesics passing through a given point are positioned at the small angle to each other

Sorkin + Caianello agree: We can have more than one geodesic passing through TWO points, but these two geodesics have to approximate each other

Sorkin There is arbitrary many "high acceleration" curves that "accidentally" look like geodesic due to Lorentz non-compactness

Caianello Such "accident" is impossible since Caianello geometry is compact

Distances and angles in Caianello's case



$$\tanh KLH = \frac{KH}{LK} = \frac{BD/2}{LK} = \frac{1}{2}\,\frac{BD}{LK}$$

2.pdf ;

Scalar field

Sorkin + Caianello: $\phi\colon S \mapsto \mathbb{R}$

Sorkin $\Longrightarrow x^\mu \in S \Longrightarrow \phi(x) \Longrightarrow$ GOOD

Caianello $\Longrightarrow (x^\mu, v^\mu) \in S \Longrightarrow \phi(x,v) \Longrightarrow$ PROBLEM

Resolving Caianello's situation

Constraint A: $\phi(x, v_1) = \phi(x, v_2)$

Discrete case $\Longrightarrow$ there is no $(x, v_2) \Longrightarrow$ no need for constraint?

Minus signs in Klein Gordon's Lagrangian

Constraint B: Derivative of ϕ is zero with respect to direction orthogonal to v

Problem: Contradiction between Constraint A and Constraint B

Solution: Drop Constraint A

Problem: Unwanted degrees of freedom $\Longrightarrow$ Harmonics with p^μ NOT being parallel to v^μ (here, p^μ is gradient of ϕ w.r.t. x)

Solution: Make unwanted harmonics "really heavy"

Problem: The harmonics where p^μ "ALMOST coincides" with v^μ have mass $2m$ instead of m

Solution: Introduce IMAGINARY component of mass for "unwanted" harmonics

Problem: Spacetime curvature $\Longrightarrow$ parallel transport around the loop will produce "unwanted" v^μ

Solution:

a) Make constraint B approximate rather than exact

b) Upper bound on curvature

Interesting question: Relationship between curvature and the mass we would get after resonances

Vector field on causal set: Sorkin's case

Vector field: $a\colon S \times S \mapsto \mathbb{R}$

$$a(p,q) = \int_{\Gamma(p,q)} g_{\mu\nu} A^{\mu} dx^{\nu} \tag{5}$$

$\Gamma(p,q)$ is geodesic connecting p and q

Linear case $\Longrightarrow$ lots of extra degrees of freedom

General case: Extra degrees of freedom can be accounted for by variation of A^{μ} "BETWEEN POINTS"

Problem: We are assuming the field is linear "between points" when we are defining Lagrangian on the basis of a

Vector field on causal set: Caianello's case – version 1

$$a(p,q) = \int_{\Gamma(p,q)} g_{\mu\nu} A^{\mu} dx^{\nu} \tag{6}$$

NOTE: $\Gamma(p,q)$ does NOT have to be a geodesic – especially since sometimes geodesic doesn't exist. BUT, we CAN say that Γ is "longest sequence of points" which might APPROXIMATE geodesic if the points are spaced far apart.

Constraint: $a((x^{\mu}, u_1^{\mu}), (y^{\mu}, v_1^{\mu})) \approx a((x^{\mu}, u_2^{\mu}), (y^{\mu}, v_2^{\mu}))$

Discreteness $\Longrightarrow (x^{\mu}, u_2^{\mu})$ does not exist $\Longrightarrow$ the only concern is $(x^{\mu} + \delta x, u_2^{\mu})$ BUT the field can shift between x^{μ} and $x^{\mu} + \delta x$!!! $\Longrightarrow$ no need for constraint?

Problem: $\{(x_1^{\mu}, u_{21}^{\mu}), (x_2^{\mu}, u_{22}^{\mu}), \cdots\}$ where $x_n^{\mu} \to x^{\mu}$ as $n \to \infty$.

Solution: u_{2n} approaches speed of light as $n \to \infty$. RESTRICT the above constraint to where velocities are within Λ-CUTOFF of each other.

Problem: Extra degrees of freedom outside of Λ-cutoff YET their behavior is restricted within the cutoff

Vector field on causal set: Caianello's case – version 2

$$b(x^\mu, v^\mu) = g_{\mu\nu}(x) v^\mu A^\nu(x) \tag{7}$$

Constraint: If we know $b(x^\mu, v_1^\mu)$, $b(x^\mu, v_2^\mu)$, $b(x^\mu, v_3^\mu)$ and $b(x^\mu, v_4^\mu)$ then we can find out $b(x^\mu, v_5^\mu)$

Discreteness $\Longrightarrow$ v_1^μ is the only vector that exists at x^μ $\Longrightarrow$ the only concern is $x^\mu + \delta x^{mu}$ BUT the field can shift between x^μ and $x^\mu + \delta x^\mu$ $\Longrightarrow$ no need for constraint?

Problem: $\{(x_1^\mu, v_1^\mu), (x_2^\mu, v_2^\mu), \cdots\}$ where $x_n^\mu \to x^\mu$ as $n \to \infty$.

Solution: v_n approaches speed of light as $n \to \infty$. RESTRICT the above constraint to where velocities are within Λ-CUTOFF of each other.

Problem: Extra degrees of freedom outside of Λ-cutoff YET their behavior is restricted within the cutoff

Gravity

Sorkin + Caianello: Gravity = $\prec$

Sorkin: Gravity is read off from propagation of “classical” “massless” particles

Caianello: Gravity is read off from propagation of “classical” “massive” particles

Fields on a causal set: Sorkin vs Caianello

Scalar field:

1. In both cases $\phi\colon S \mapsto \mathbb{R}$

2. In Caianello's case extra constraint $\psi(x,u) \approx \psi(x',v)$ if

a) u and v are within UV cutoff of each other

b) The coordinate displacement between x and x' is small in the rest frame of u^μ as well as rest frame of v^μ

Electromagnetic field

1. In Sorkin's case, $a\colon S \times S \mapsto \mathbb{R}$

2. In Caianello's case can be EITHER $a\colon S \times S \mapsto \mathbb{R}$, OR $a\colon S \mapsto \mathbb{R}$; interesting ideas of spin 0 – spin 1 unification in the second case.

Gravitational case

1. In both cases identified with $\prec$

2. In Caianello's case contains extra information about structure of a tangent plane that is assumed to be trivial in "conventional" GR

Common problem: Need for non-linearity to rationalize unwanted degrees of freedom and AT THE SAME TIME need for linearity to define Lagrangian

Conclusions

1. Locality, relativity and discreteness can not co-exist

2. To make them co-exist we have to introduce velocity coordinate, thus making "Lorentz violation" by a point analogous to "translational violation" of the same

3. Velocity coordinate introduces extra degrees of freedom that need to be dealt with

4. Getting rid of extra degrees of freedom involves some constraints that create further problems

5. Abandoning uncertainty principle $\Longrightarrow$ utilizing extra degrees of freedom

6. Gravity + Parallel transport $\Longrightarrow$ extra degrees of freedom