Is purity eternal at the Planck scale?

Michele Arzano

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MARIE CURIE ACTIONS

September 9, 2015 The Planck Scale II

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PHYSICAL REVIEW D VOLUME 14, NUMBER 10 15 NOVEMBER 1976

Breakdown of predictability in gravitational collapse*

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$h_{\alpha\beta}$ is a hermitian matrix of constants and Q^α form a basis of hermitian matrices

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ABSTRACT

Motivated by Hawking's proposal that the quantum-mechanical density matrix ρ obeys an equation more general than the Schrödinger equation, we study the general properties of evolution equations for ρ. We argue that any more general equation for ρ violates either locality or energy-momentum conservation.

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end of the story?

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"The fascinating possibility that purity may not be eternal is still out of reach."

<u>Goal of this talk</u>: show that **Planck scale deformations of translations** naturally lead to the possibility of generalized quantum time evolution of Lindblad type!

MA: 1403.6457; Phys. Rev. D 90, 024016 (2014)

Outline

- **Topological particles and group momentum space in 3d gravity**
- **Group valued momenta: classical vs. quantum systems**
- **Deformed translations and Lindblad evolution in 3d**
- **de Sitter momentum space: beyond von Neumann evolution in 4d**
- **Conclusions and outlook**

Born's legacy

A suggestion for unifying quantum theory and relativity

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$$\Gamma = \underbrace{G}_{q} \times \underbrace{\mathbb{R}^{n,1}}_{p}$$

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Point particles coupled to three dimensional gravity...

(Matschull and Welling, Class. Quant. Grav. **15**, 2981 (1998) [gr-qc/9708054].)

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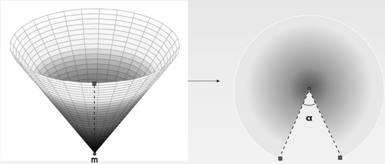
(3d Newton's constat G $\sim 1/M_{\mathrm{Planck}}$)



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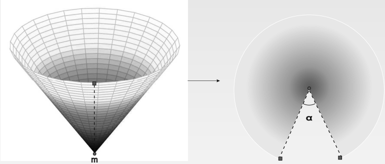

Particle's mass (**rest energy**) can be read off evaluating the **holonomy** of the flat connection around the defect and results in a **rotation** $h_\alpha \in SL(2,\mathbb{R})$

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- Deformed **mass-shell** condition:

$$\tfrac{1}{2}\mathrm{Tr}(h) = \cos(4\pi G m)$$

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- massive and massless co-dimension 2 defects in $3+1$ dimensions used as building blocks of 't Hooft *"piecewise flat gravity"* (see arXiv:0804.0328)

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Parametrize group elements: $h = u\mathbb{1} + \frac{p^\mu}{\kappa}\gamma_\mu$ with $\kappa = (4\pi G)^{-1}$

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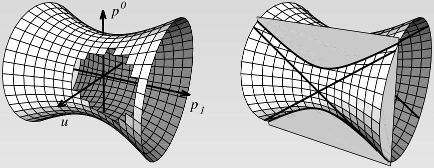

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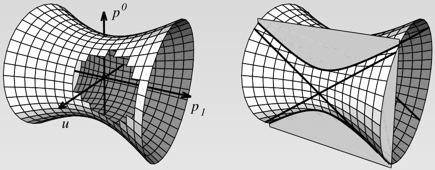

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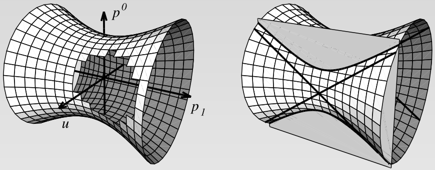

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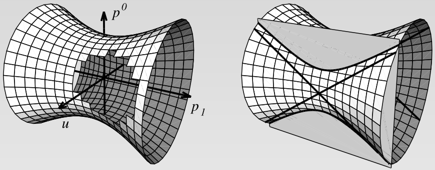
$SL(2,\mathbb{R})$ momentum space: embedding coordinates

Parametrize group elements: $h = u\mathbb{1} + \frac{p^\mu}{\kappa}\gamma_\mu$ with $\kappa = (4\pi G)^{-1}$

The unit determinant condition $u^2 + p^2/\kappa^2 = 1 \Longrightarrow$



p^μ are *embedding coordinates on AdS space*; basic relativistic properties:

- **mass-shell condition**: $p^2 = \kappa^2 \cos 4\pi G m = m_\kappa$
- **Lorentz transformation**: $h' = ghg^{-1}$, undeformed on p^μ e.g. boost in the 1-direction $g = e^{\frac{1}{2}\eta\gamma_2}$

$$\begin{cases} p'^0 = p^0 \cosh\eta - p^1 \sinh\eta \\ p'^1 = p^1 \cosh\eta - p^0 \sinh\eta \\ p'^2 = p^2 \end{cases}.$$

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Define **non-abelian addition** $\mathcal{P}_\mu(\pi \cdot \pi') \equiv \mathcal{P}_\mu(\pi) \oplus \mathcal{P}_\mu(\pi')$ via *group homorphism*; the **total momentum**

$$\boxed{\mathcal{P}_\mu(\pi_1, \pi_2) \equiv \mathcal{P}_\mu(\pi_1, \mathbb{1}) \oplus \mathcal{P}_\mu(\mathbb{1}, \pi_2) = \mathcal{P}_\mu(\mathbb{1}, \pi_2) \oplus \mathcal{P}_\mu(\pi_1, \mathbb{1})}$$

since $\Gamma_{\pi 1} \times \{\mathbb{1}\} \simeq \Gamma_{\pi 1} \Rightarrow$ total momentum $=$ **abelian sum of individual momenta**.

MA and F. Nettel: in preparation

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to illustrate *deformed quantum kinematics* let's recall some basic reps' theory...

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i.e. just the familiar **adjoint action**... **N.B.** Using the spectral theorem any operator can be written in terms of a combination of projectors $|k\rangle\langle k|$

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Key point: the action on operators will be deformed accordingly

Deformed translations and Lindblad evolution in three dimensions

For the deformed translation generators associated to $SL(2,\mathbb{R})$ momentum space:

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with "decoherence" matrix given by

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In our case *h is not* **positive definite** *nor* **real**

Further work needed to establish properties of our Lindblad evolution...

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Can the picture be generalized to the four-dimensional case?

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κ-**momenta**: coordinates on **Lie group** $AN(3)$ obtained form the Iwasawa decomposition of $SO(4,1) \simeq SO(3,1)AN(3)$, sub-manifold of dS_4

$$-p_0^2 + p_1^2 + p_2^2 + p_3^2 + p_4^2 = \kappa^2 \, ; \qquad p_0 + p_4 > 0$$

with $\kappa \sim E_{Planck}$

dS momentum space without DSR

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\Delta(P_0) &= P_0 \otimes \mathbb{1} + \mathbb{1} \otimes P_0 + \frac{1}{\kappa} P_m \otimes P_m \,, \\
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In embedding coordinates we have *ordinary relativistic kinematics* at the **one-particle** level...all non-trivial structures confined to "co-algebra" sector

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A straightforward calculation of $\mathrm{ad}_{P_0}(\rho)$ leads to a *non-symmetric* **Lindblad equation**

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- the adjoint actions of N_i and P_0 satisfy

$$\mathrm{ad}_{\mathrm{ad}_{N_i}(P_0)}(\cdot) = \mathrm{ad}_{N_i}(\mathrm{ad}_{P_0})(\cdot) - \mathrm{ad}_{P_0}(\mathrm{ad}_{N_i})(\cdot)$$

  in this sense the κ-Lindblad equation follows a **deformed notion of covariance**

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- *Phenomenology* of κ-Lindblad evolution? (Ellis et al. "Search for Violations of Quantum Mechanics," Nucl. Phys. B **241**, 381 (1984)); bounds on κ using **precision measurements of neutral kaon systems** (nearby KLOE-2 experiment)?