The DSR-deformed relativistic symmetries and the relative locality of 3D quantum gravity

Stefano Bianco

Max Planck Institute for Gravitational physics

Quantum gravity

- Quantum gravity $\rightarrow$ discover quantum properties of gravitational phenomena and describe them

Quantum gravity

- Quantum gravity $\rightarrow$ discover quantum properties of gravitational phenomena and describe them
- Theoretical and experimental challenges

Quantum gravity

- Quantum gravity $\rightarrow$ discover quantum properties of gravitational phenomena and describe them
- Theoretical and experimental challenges
- Interesting to look at theories which describe just some properties of quantum gravity and provide suitable amplifiers which make these properties manifest

3D gravity

Gravity in 3D in absence of matter is a topological theory: it has 0 local degrees of freedom.

[1] Matschull and Welling, Class. Quant. Grav. 15 (1998) 2981 .
[2] Bais, Muller and Schroers, Nucl. Phys. B640 (2002) 3 .

3D gravity

Gravity in 3D in absence of matter is a topological theory: it has 0 local degrees of freedom.
If one couples point particles the situation gets interesting.
First ideas by Jackiw and 't Hooft and later calculations in metric [1] and Chern-Simons formulations [2] show that the momentum space of a particle gets deformed by the presence of the evergy scale given by G^{-1}.

[1] Matschull and Welling, Class. Quant. Grav. 15 (1998) 2981 .
[2] Bais, Muller and Schroers, Nucl. Phys. B640 (2002) 3 .

3D gravity

Gravity in 3D in absence of matter is a topological theory: it has 0 local degrees of freedom.
If one couples point particles the situation gets interesting.
First ideas by Jackiw and 't Hooft and later calculations in metric [1] and Chern-Simons formulations [2] show that the momentum space of a particle gets deformed by the presence of the evergy scale given by G^{-1}.
In particular the momentum space is described no more by a vector space but by the Lie group $SL(2, R)$.

[1] Matschull and Welling, Class. Quant. Grav. 15 (1998) 2981 .

[2] Bais, Muller and Schroers, Nucl. Phys. B640 (2002) 3 .

3D gravity

Moreover in the spin foam approach it is possible to couple a scalar field to gravity, integrate out gravity and obtain an effective field theory for matter characterized by the momentum space $SL(2, R)$ or in a dual picture by a noncommutative field theory on the spinning spacetime $[X^\mu, X^\nu] = \varepsilon^{\mu\nu}{}_\rho X^\rho$[3].

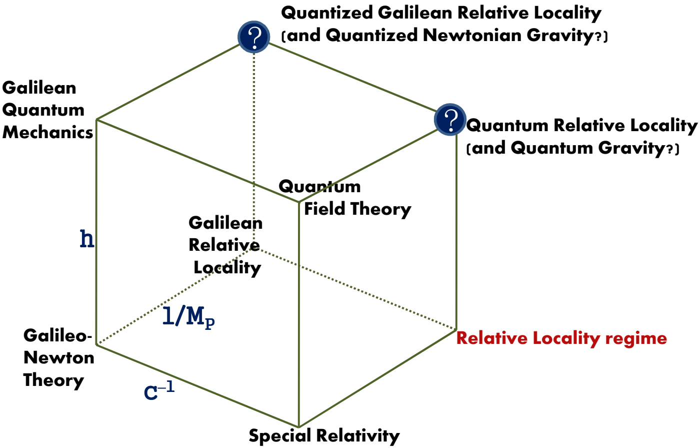
As a consequence of the deformation of the momentum space also the symmetries gets deformed. They are given no more by the Poincarè group, but by a deformation in G of it: the quantum group called Lorentz double, in which the action of translations is deformed, while rotation and boosts are the special-relativistic ones[4].

[3] Freidel and Livine Phys. Rev. Lett. 96 (2006) 221301 .

[4] Bais, Muller and Schroers, Nucl. Phys. B640 (2002) 3 [hep-th/0205021].

The relative-locality framework

The relative-locality framework describes a regime in which the effects due to $\hbar$ and G are negligible, but the effects due to their ratio $\frac{\hbar}{G}$ are relevant. Formally $\hbar \to 0$, $G \to 0$, with $\frac{\hbar}{G}$ fixed.



The relative-locality framework

The relative-locality framework incorporates two ideas of quantum gravity research:

The relative-locality framework

The relative-locality framework incorporates two ideas of quantum gravity research:

- momentum space can be a curved manifold (Born, 3D gravity, noncommutative geometry)

The relative-locality framework

The relative-locality framework incorporates two ideas of quantum gravity research:

- momentum space can be a curved manifold (Born, 3D gravity, noncommutative geometry)
- special-relativistic symmetries may be deformed (DSR-studies, 3D gravity, noncommutative geometry)

The relative-locality framework

- The curvature of the momentum space and the deformation of the special relativistic symmetries are controlled by the Planck mass $M_p = \sqrt{\frac{\hbar}{G}}$

The relative-locality framework

- The curvature of the momentum space and the deformation of the special relativistic symmetries are controlled by the Planck mass $M_p = \sqrt{\frac{\hbar}{G}}$
- It is a relativistic framework with invariant constants c and M_p: an extension of special relativity

Geometric structures on momentum space

- the metric $g^{\mu\nu}$ encodes the on-shell relation

$$D^2(0, p_\mu) = m^2,$$

where $D(0, p_\mu)$ is the geodesic distance of the momentum p_μ from the origin 0.

Geometric structures on momentum space

- the metric $g^{\mu\nu}$ encodes the on-shell relation

$$D^2(0, p_\mu) = m^2,$$

where $D(0, p_\mu)$ is the geodesic distance of the momentum p_μ from the origin 0.

- the connection $\Gamma^{\mu\nu}_{\rho}$ encodes the composition laws of two momenta

$$p_\mu \oplus q_\mu = p_\mu + q_\mu + \Gamma_\mu^{\nu\rho}(0) p_\nu p_\rho.$$

Describing the kinematics of a system of particles

For every particle a term

$$\int_{s_0}^{s_1} dx \left(x^\mu \dot{p}_\mu + \mathcal{N}_p[D^2(0,p) - m_p^2]\right)$$

Describing the kinematics of a system of particles

For every particle a term

$$\int_{s_0}^{s_1} dx \left(x^\mu \dot{p}_\mu + \mathcal{N}_p [D^2(0,p) - m_p^2] \right)$$

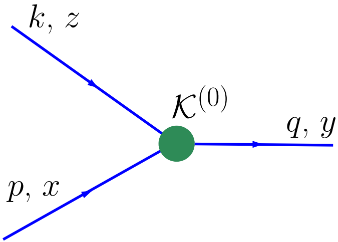
For every interaction a possibly-deformed conservation law imposed by a boundary term of the type

$$\xi^\mu \mathcal{K}_\mu(p_\mu, q_\mu, k_\mu, ...),$$

where $\mathcal{K}_\mu(p_\mu, q_\mu, k_\mu, ...)$ is a possibly-non-linear function of the momenta of the interacting particles.

These boundary terms encode the translational symmetry of the theory.

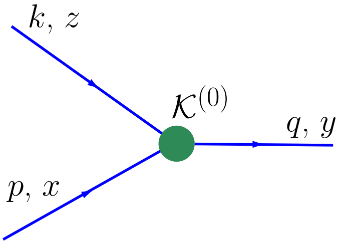
The relativity of locality



$$\mathcal{K}^{(0)}_\mu = (k \oplus p)_\mu \ominus q_\mu = 0$$

$$z^\mu(s_0) = \xi^\nu_{(0)} \frac{\partial \mathcal{K}^{(0)}_\nu}{\partial k_\mu}\ , \quad x^\mu(s_0) = \xi^\nu_{(0)} \frac{\partial \mathcal{K}^{(0)}_\nu}{\partial p_\mu}\ , \quad y^\mu(s_0) = -\xi^\nu_{(0)} \frac{\partial \mathcal{K}^{(0)}_\nu}{\partial q_\mu}\ .$$

The relativity of locality



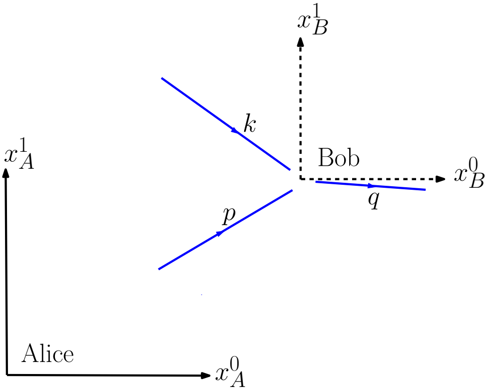
$$\mathcal{K}^{(0)}_\mu = (k \oplus p)_\mu \ominus q_\mu = 0$$

$$z^\mu(s_0) = \xi^\nu_{(0)} \frac{\partial \mathcal{K}^{(0)}_\nu}{\partial k_\mu} \ , \quad x^\mu(s_0) = \xi^\nu_{(0)} \frac{\partial \mathcal{K}^{(0)}_\nu}{\partial p_\mu} \ , \quad y^\mu(s_0) = -\xi^\nu_{(0)} \frac{\partial \mathcal{K}^{(0)}_\nu}{\partial q_\mu} \ .$$

We note that in general $\frac{\partial \mathcal{K}^{(0)}_\nu}{\partial k_\mu} \neq \frac{\partial \mathcal{K}^{(0)}_\nu}{\partial p_\mu} \neq \frac{\partial \mathcal{K}^{(0)}_\nu}{\partial q_\mu}$. Then if $\xi^\nu_{(0)} \neq 0$ we have that $z^\mu(s_0) \neq x^\mu(s_0) \neq y^\mu(s_0)$ and they are different from 0.

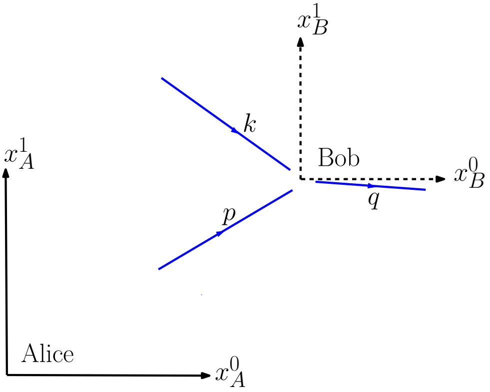
The relativity of locality

Then an observer, say Alice, far from the interaction, i.e. $\xi^{\nu}_{(0)} \neq 0$, describes it as non-local.



The relativity of locality

Then an observer, say Alice, far from the interaction, i.e. $\xi^{\nu}_{(0)} \neq 0$, describes it as non-local.



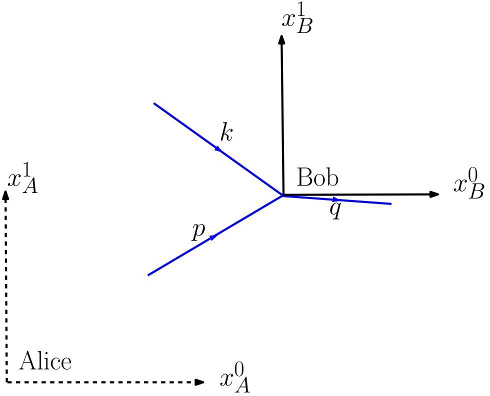
But if we consider the translated observer characterized by

$z^{\mu}(s_0)_B = z^{\mu}(s_0)_A - \xi^{\nu}_{(0)} \frac{\partial \mathcal{K}^{(0)}_{\nu}}{\partial k_{\mu}}$, $x^{\mu}(s_0)_B = x^{\mu}(s_0)_A - \xi^{\nu}_{(0)} \frac{\partial \mathcal{K}^{(0)}_{\nu}}{\partial p_{\mu}}$,

$y^{\mu}(s_0)_B = y^{\mu}(s_0)_A + \xi^{\nu}_{(0)} \frac{\partial \mathcal{K}^{(0)}_{\nu}}{\partial q_{\mu}}$,

The relativity of locality

then $z^\mu(s_0)_B = x^\mu(s_0)_B = y^\mu(s_0)_B = 0$. So an observer near the interaction describes it to be local [5][6]

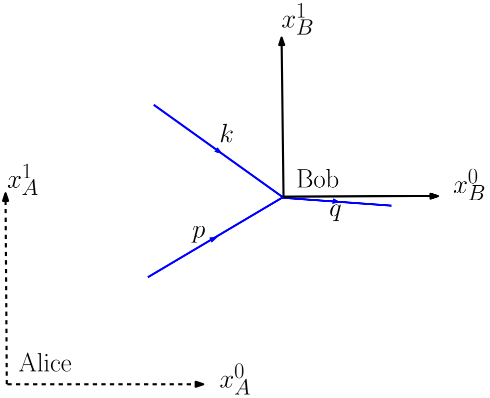


[5] G. Amelino-Camelia et al., Phys.Rev.Lett., 2011

[6] G. Amelino-Camelia et al., Phys. Rev.D, 2011

The relativity of locality

then $z^{\mu}(s_0)_B = x^{\mu}(s_0)_B = y^{\mu}(s_0)_B = 0$. So an observer near the interaction describes it to be local [5][6]



We note that inferences made by distant observers are misleading.

[5] G. Amelino-Camelia et al., Phys.Rev.Lett., 2011

[6] G. Amelino-Camelia et al., Phys. Rev.D, 2011

Metric and connection from 3D gravity

The momentum space is $SL(2,R)$, for which we take coordinates $\mathbf{p}_\mu$ such that

$$\mathbf{p} = u\mathbb{I} - 2\ell p_\mu X^\mu, \tag{1}$$

where X^μ are a basis of $sl(2,R)$

$$X^0 = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, X^1 = \frac{1}{2}\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}, X^2 = \frac{1}{2}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}. \tag{2}$$

From the constraint $\det \mathbf{p} = 1$ we find $u^2 - \ell^2 p^\mu p_\mu = 1$, from which we read that the our momentum space is Anti-deSitter.

Metric and connection from 3D gravity

The momentum space is $SL(2,R)$, for which we take coordinates $\mathbf{p}_\mu$ such that

$$\mathbf{p} = u\mathbb{I} - 2\ell p_\mu X^\mu, \tag{1}$$

where X^μ are a basis of $sl(2,R)$

$$X^0 = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, X^1 = \frac{1}{2}\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}, X^2 = \frac{1}{2}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}. \tag{2}$$

From the constraint $\det \mathbf{p} = 1$ we find $u^2 - \ell^2 p^\mu p_\mu = 1$, from which we read that the our momentum space is Anti-deSitter. Defining the metric as the one induced by flat metric in the embedding space we find

$$g^{\mu\nu} = \eta^{\mu\nu} - \frac{\ell^2 p^\mu p^\nu}{1 + \ell^2 p^\mu p_\mu} \ , \tag{3}$$

from which the on-shell relation is

$$\ell^{-2}\left(\arcsin\left(\sqrt{-\ell^2 p^\mu p_\mu}\right)\right)^2 = m^2$$

.

Metric and connection from 3D gravity

The composition law is defined by the group multiplication: considering two group elements **p** and **q**

$$\begin{aligned} \mathbf{p} &= \sqrt{1+\ell^2 p^\mu p_\mu}\,\mathbb{I} - 2\ell p_\mu X^\mu \\ \mathbf{q} &= \sqrt{1+\ell^2 q^\mu q_\mu}\,\mathbb{I} - 2\ell q_\mu X^\mu \end{aligned} \tag{4}$$

we obtain a new element **pq**

$$\begin{aligned} \mathbf{pq} = &\left(\sqrt{1+\ell^2 p^\mu p_\mu}\sqrt{1+\ell^2 q^\nu q_\nu} + \ell^2 p^\mu q_\mu\right)\mathbb{I} \\ &- 2\ell\left(\sqrt{1+\ell^2 q^\nu q_\nu}\,p_\mu + \sqrt{1+\ell^2 p^\nu p_\nu}\,q_\mu - G\varepsilon_\mu{}^{\nu\rho} p_\nu q_\rho\right) X^\mu . \end{aligned} \tag{5}$$

At first order in ℓ we have

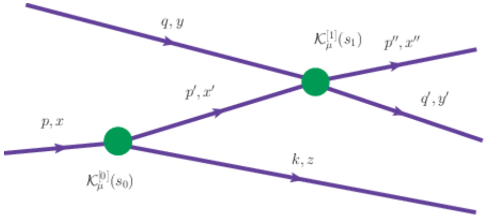
$$(p \oplus q)_\mu \simeq p_\mu + q_\mu - \ell\varepsilon_\mu{}^{\nu\rho} p_\nu q_\rho ,$$

from which we read the connection coefficients

$$\Gamma_\mu^{\nu\rho}(0) = -\ell\varepsilon_\mu{}^{\nu\rho} \; . \tag{6}$$

Case study

We study the propagation of a high-energy photon [7]



The boundary terms enforcing the conservation law at the vertices are characterized by

$$\mathcal{K}^{[0]}_\mu(s_0) = (q \oplus p)_\mu - (q \oplus p' \oplus k)_\mu$$

$$\mathcal{K}^{[1]}_\mu(s_1) = (q \oplus p' \oplus k)_\mu - (p'' \oplus q' \oplus k)_\mu$$

[7] G. Amelino-Camelia et al., Class. Quant. Grav. 2013

The relativistic symmetries

We will work at first order in ℓ. Then the onshell relation is

$$p^\mu p_\mu = m^2$$

and the composition law is

$$(p \oplus q)_\mu \simeq p_\mu + q_\mu - \ell \varepsilon_\mu{}^{\nu\rho} p_\nu q_\rho .$$

The relativistic symmetries

We will work at first order in ℓ. Then the onshell relation is

$$p^\mu p_\mu = m^2$$

and the composition law is

$$(p \oplus q)_\mu \simeq p_\mu + q_\mu - \ell \varepsilon_\mu{}^{\nu\rho} p_\nu q_\rho .$$

We see that standard rotation and boosts transformations are compatible with the above onshell relation and composition law.

The relativistic symmetries

We will work at first order in ℓ. Then the onshell relation is

$$p^\mu p_\mu = m^2$$

and the composition law is

$$(p \oplus q)_\mu \simeq p_\mu + q_\mu - \ell \varepsilon_\mu{}^{\nu\rho} p_\nu q_\rho.$$

We see that standard rotation and boosts transformations are compatible with the above onshell relation and composition law. Translational symmetry is given by the total momentum, for example for the worldline x'^μ we have

$$x'^\mu_B = x'^\mu_A + b^\nu \{(q \oplus p' \oplus k)_\nu, x'^\mu\}.$$

The deformation of the composition law imply deformed translational symmetry.

Dual-gravity lensing

we want to look for the dual-gravity lensing effect recently proposed [8]. We consider two observers Alice and Bob, purely

translated with respect to each other and aligned along their common x-axis.

[8] Freidel and Smolin, arxiv:1103.5626

Dual-gravity lensing

we want to look for the dual-gravity lensing effect recently proposed [8]. We consider two observers Alice and Bob, purely

translated with respect to each other and aligned along their common x-axis.We suppose an high-energy particle emitted in the spacetime of Alice:

$$\begin{aligned} x'^{1}_{A} &= x'^{0}_{A} \cos\theta \ , \\ x'^{2}_{A} &= -x'^{0}_{A} \sin\theta \ , \end{aligned} \tag{7}$$

and we want to check for which value of θ that this particle arrives at Bob's space origin.

[8] Freidel and Smolin, arxiv:1103.5626

Dual-gravity lensing

Since the description of distant observer can be misleading, we need to use Bob's description of the wordline, obtained by translating Alice's worldline, and impose that the particle passes in the space origin of Bob. Doing this one finds

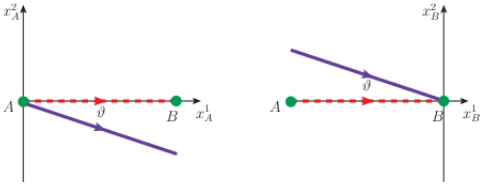
- $x_B^0 = 0$, there is no time delay between photons with different energies

Dual-gravity lensing

Since the description of distant observer can be misleading, we need to use Bob's description of the wordline, obtained by translating Alice's worldline, and impose that the particle passes in the space origin of Bob. Doing this one finds

- $x_B^0 = 0$, there is no time delay between photons with different energies
- $\theta = \ell(k_1 - q_1 + k_0 - q_0)$

In order for the particle to reach Bob, which is aligned along the x-axis of Alice, it should be emitted by Alice with a non zero angle



We note that the angle θ is extremely small

Future developments

- study actual observability of the dual-gravity lensing

[9] Amelino-Camelia, JCAP 2010
[10] Ballesteros et al, Phys. Rev. B 2014
[11] G. Amelino-Camelia et al. Phys. Rev. D, 2013

Future developments

- study actual observability of the dual-gravity lensing
- study extend this treatment to the case of 3D gravity with non-zero cosmological constant [9][10]

[9] Amelino-Camelia, JCAP 2010

[10] Ballesteros et al, Phys. Rev. B 2014

[11] G. Amelino-Camelia et al. Phys. Rev. D, 2013

Future developments

- study actual observability of the dual-gravity lensing
- study extend this treatment to the case of 3D gravity with non-zero cosmological constant [9][10]
- study the interplay between quantum and relative-locality effects [11]

[9] Amelino-Camelia, JCAP 2010

[10] Ballesteros et al, Phys. Rev. B 2014

[11] G. Amelino-Camelia et al. Phys. Rev. D, 2013