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Area-law from Loop Quantum Gravity

In collaboration with

A. Hamma, J. Hung & M. Zhang

"Area law in LQG" arXiv:1506.01623

"CS with area law in LQG" soon on the arXiv

"Thermalization in LQG" in preparation

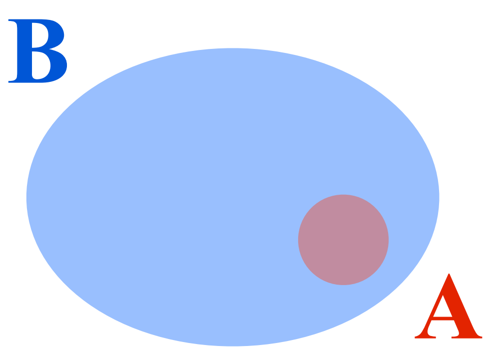
Summary

- I. Quantum entanglement and its quantification/measurement
- II. Area-law in gravity
- III. Complexity of entanglement and thermalization properties
- IV. Requirements on LQG states from the area-law, and new CS
- V. States with area-law in the Hilbert space of LQG
- VI. Conclusions & Outlook

I. Reduced density matrix and EE

Reduced density matrix

$$\rho = |\psi\rangle\langle\psi|, \qquad \rho_A = \mathrm{Tr}_B \rho$$



Entanglement Entropy (von Neumann)

$$S_{\mathrm{EE}} = -\mathrm{Tr}_A(\rho_A \ln \rho_A)$$

I. A simple example: 2 spin system

i) 4 basis states

$$|\uparrow_1\uparrow_2\rangle, \quad |\uparrow_1\downarrow_2\rangle, \quad |\downarrow_1\uparrow_2\rangle, \quad |\downarrow_1\downarrow_2\rangle,$$

ii) Consider the reduced density matrix from tracing out spin 2

iii) For quantum states that are direct product of states

$$S_{EE}(|\uparrow_1\uparrow_2\rangle) = 0$$

iv) EPR single state:

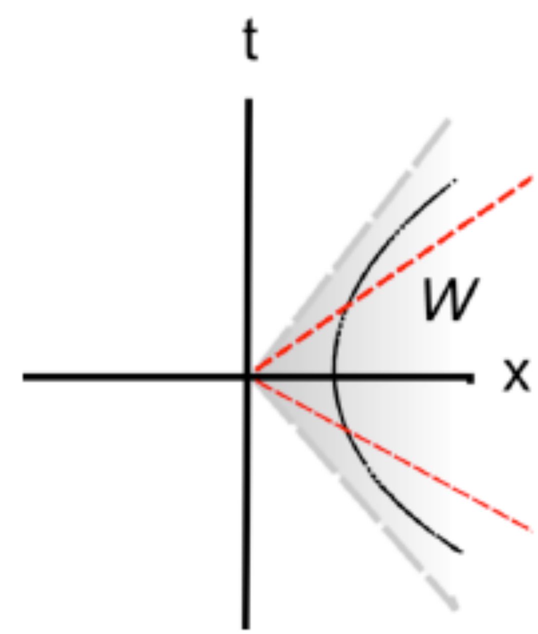
$$S_{EE}(|\uparrow_1\downarrow_2\rangle - |\downarrow_1\uparrow_2\rangle) = \ln 2$$

v) The reduced density matrix looks like a mixed state!

I. Rindler observers

i) Proper coordinates are Rindler coordinates (acceleration along x-axis ~ 1/R)

$$x^{\pm} = \chi e^{\pm\tau/R}, \qquad x^{\pm} = x \pm t$$



ii) An accelerated observer sees an apparent thermal (mixed) state. His density matrix is the reduced density matrix of the half space:

$$A = \{x; x > 0\}$$

I. Renyi entropy: where does it come in?

Proposed by Renyi, from the perspective of information theory

$$S_n = \frac{1}{1-n} \ln \sum_i p_i^n$$

Set of probability distribution $\{p_i\}$

Most general function that quantifies the uncertainty in a system, the outcome of which is characterized by a set of probability distribution

ON MEASURES OF ENTROPY AND INFORMATION

ALFRÉD RÉNYI
MATHEMATICAL INSTITUTE
HUNGARIAN ACADEMY OF SCIENCES

1. Characterization of Shannon's measure of entropy

Let $\mathcal{P} = (p_1, p_2, \cdots, p_n)$ be a finite discrete probability distribution, that is, suppose $p_k \geqq 0 (k = 1, 2, \cdots, n)$ and $\sum_{k=1}^{n} p_k = 1$. The amount of uncertainty of the distribution $\mathcal{P}$, that is, the amount of uncertainty concerning the outcome of an experiment, the possible results of which have the probabilities $p_1, p_2, \cdots, p_n$, is called the *entropy* of the distribution $\mathcal{P}$ and is usually measured by the quantity $H[\mathcal{P}] = H(p_1, p_2, \cdots, p_n)$, introduced by Shannon [1] and defined by

$$H(p_1, p_2, \cdots, p_n) = \sum_{k=1}^{n} p_k \log_2 \frac{1}{p_k}. \tag{1.1}$$

I. Renyi entropy

i) in the context of quantifying quantum entanglement, we may use

$$S_n = \frac{1}{1-n} \ln \mathrm{Tr} \rho_A^n$$

ii) Von Neumann entanglement entropy is a limiting case of the Renyi entropy by analytically continuing n

$$S_{EE} = -\mathrm{Tr}(\rho_A \ln \rho_A) = \lim_{n \to 1} S_n = -\partial_n \ln \mathrm{Tr} \rho_A^n|_{n=1}$$

iii) replica trick as a useful way of computing the (Von Neumann) entanglement entropy, especially when the density matrix is a big matrix!

$$S_{EE} := S_1$$

iv) the Renyi entropy decreases as n increases:

$$S_2 < S_{EE} := S_1$$

I. General properties of EE

i) strong subadditivity

$$S_1(A+B) + S_1(B+C) \geq S_1(A+B+C) + S_1(B)$$

ii) finite temperature, large A $\longrightarrow$ thermal entropy **Senthil & Swingle**

Universal crossover functions between EE and low temperature thermal entropy in gapless quantum many-body systems.

iii) existence of some special features that entanglement entropy of the ground state of a local Hamiltonian satisfies: large size limit entails

$$S_1(A) = \frac{A_{D-2}}{\epsilon^{D-2}} + \cdots + \mathcal{C}\ln\frac{R}{\epsilon} + \text{regular}$$

Arad, Kitaev, Landau, Vazirani, Swingle, Mcgreevy, Calabrese, Cardy...

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II. What about theories that involve gravity?

i) Black hole entropy satisfies an area-law:

$$S_{BH} = \frac{A}{4G_N}$$

Bekenstein, Hawking...

ia) black hole entropy = entanglement entropy

Sorkin '83 Bombelli, Koul, Lee & Sorkin '86; Srednicki; Frolov & Novikov, Jacobson, Susskind, Uglum

ib) entanglement entropy renormalization by matter fields

$$\Delta S_{EE} = A \Delta \frac{1}{4G_N}$$

see e.g. emergent gravity theories

ii) Holographic entanglement entropy: EE in d-dim CFT as a minimal surface in AdS d+1

Ryu,Takayanagi, Lewkowicz, Maldacena

iii) Analogy with spin systems: EE of GS of local Hamiltonian has area law

Arad, Kitaev, Landau, Vazirani, Swingle, Mcgreevy...

iv) Negative heat capacity & area law

Hu, Oppenheim

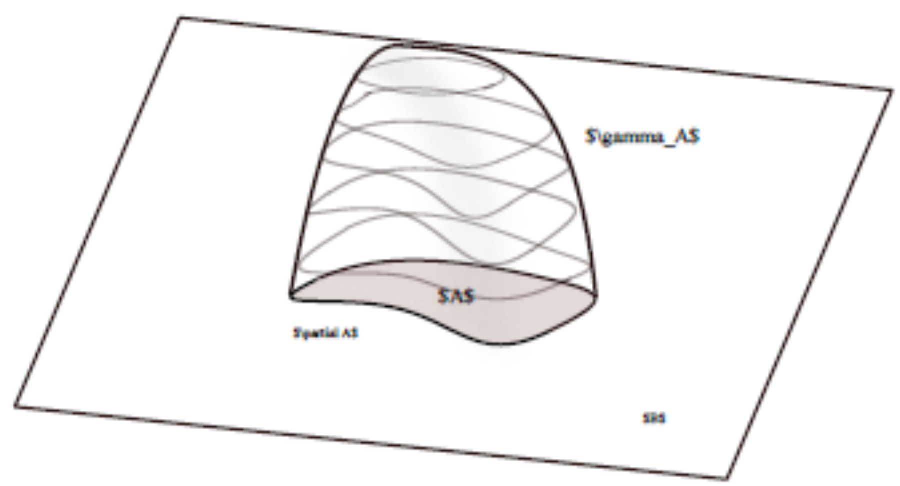
II. Holographic Entanglement Entropy

A proposal inspired by black hole physics:

$$S_1 = \frac{1}{4G_N} \min\{\mathcal{A}_{D-1}\}|_{\partial\mathcal{A}=\partial A}$$

Ryu & Takayanagi

It has been recently proved



Lewkowycz & Maldacena

II. Gravity, heat capacity & the area law

Gravity & non equilibrium thermodynamics of classical matter

Gravity has a negative heat capacity: Antonsov's 1962 "gravothermal catastrophe"

Lynden-Bell, R. Wood,

In presence of long-range interactions some classical thermodynamical laws break down

...Villain, Hu

i) energy and entropy no more extensive

ii) inequivalence of (microcanonical and canonical) ensembles

iii) slow relaxation towards thermodynamic equilibrium
and convergence towards quasi-stationary states.

Typical of: gravitational systems, two-dimensional hydrodynamics, two-dimensional elasticity, charged and dipolar systems.

II. Analog spin models & the area law

Black holes have negative heat capacity, as gravitating gas

Page, Penrose, Smolin, Sorkin

The entropy of a spherically symmetric distribution of matter in self-equilibrium:

Oppenheim

i) neglect gravitational effects: entropy proportional to volume; gravitational self-interactions become more important: the entropy no longer purely volume scaling

ii) when boundary of the system approaches its event horizon, total entropy of the system proportional to its area

The scaling laws of the system's thermodynamical quantities are identical to those of a black hole, even though the system does not possess an event horizon.

iii) analog spin-spin model and thermodynamics: from Ising model to black hole

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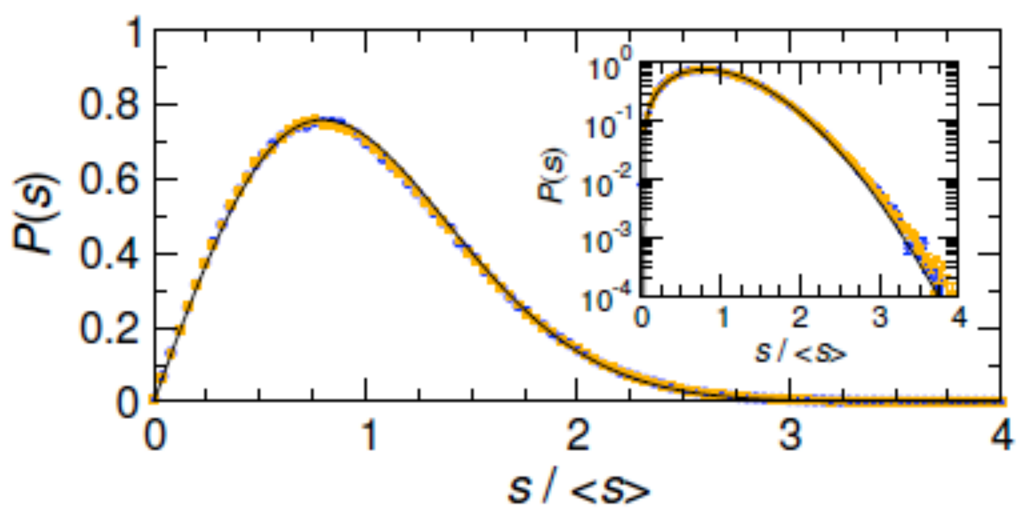
III. EE as a way to quantify complexity

i) Given the eigenvalues of the density matrix ordered decreasingly, define the ratio of consecutive spacing

$$r_i := \frac{\epsilon_{i+1}}{\epsilon_i}\,, \qquad \epsilon_i := \lambda_i - \lambda_{i+1}$$

ii) Three different scenarios are found out of their distribution (P, WD and indetermediate)

$$P_{\rm WD}(r) = \frac{27}{8}\frac{(r+r^2)^\beta}{(1+r+r^2)^{1+\frac{3}{2}\beta}} \longrightarrow \textbf{Random Matrices} \longrightarrow \textbf{Thermality}$$



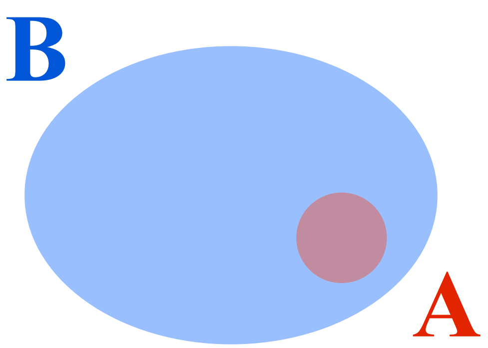
Chamon, Hamma & Mucciolo '14

Schaffer, Chamon, Hamma & Mucciolo '14

III. Thermalization in closed quantum systems

i) Consider a closed quantum system, and a pure state $|\Psi\rangle$

ii) Despite $|\Psi\rangle$ **is not thermal, the trace over a sub region in which** $|\Psi\rangle$ **is defined could be thermal**



$$\widehat{\Psi} := |\Psi\rangle\langle\Psi|\,, \qquad \widehat{\Psi}_A = \mathrm{Tr}_B \widehat{\Psi}$$

iii) But if area law is found for GS, then thermalization can still happen "at the boundary"

$$\cancel{\widehat{\Psi}_A} \quad \textbf{but probably} \quad \widehat{\Psi}_{\partial A}$$

In spin systems, EE of the GS of local Hamiltonians satisfy area law

Arad, Kitaev, Landau, Vazirani, Swingle, Mcgreevy...

III. "ETH" and for LQG?

i) Consider an eigenstate $|\Psi\rangle$ at "high energy", and again $\widehat{\Psi} := |\Psi\rangle\langle\Psi|$ **,and** $\widehat{\Psi}_A = \mathrm{Tr}_B \widehat{\Psi}$

ii) $\widehat{\Psi}_A$ **is thermal if:**

$$\langle \Psi_A | \mathcal{O}_A | \Psi_A \rangle \sim \mathrm{Tr}\left(\mathcal{O}_A \frac{e^{-\beta H}}{Z} \right)$$

a) Thermalization already contained in the eigenstates (kinematics)

b) After a large time t, $\rho_0 \to \rho_t$ **the final state being thermal (dynamics)**

iii) We can revert ETH, and find for non-zero energies

$$\langle \mathcal{O} \rangle_{|E\rangle} = \mathrm{Tr}\left(\mathcal{O} \frac{e^{-\beta(E) H}}{Z} \right)$$

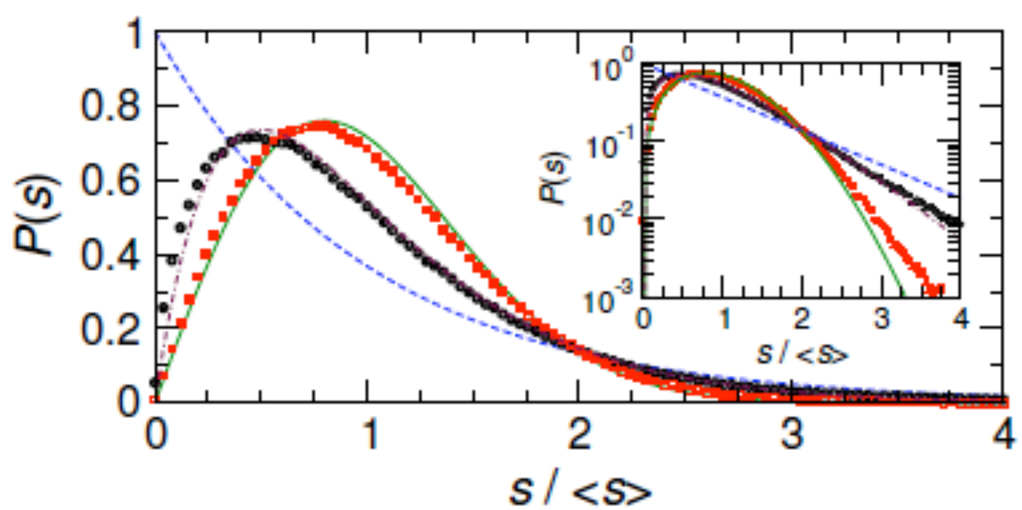
$$E = \mathrm{Tr}\left(H \frac{e^{-\beta H}}{Z} \right)$$

$$\beta = \beta(E)$$

III. “ETH” and the ground state of LQG

iv) what if $|\Psi\rangle$ is exactly the ground state of the system?

We can still say something about the GS, if we look at the distribution of the ratio “r”



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IV. Set up for the analysis: LQG

i) SU(2) spin-network state: a triplet $|\mathcal{S}\rangle = |\Gamma, j_l, i_n\rangle$

Γ given proper graph with L oriented links and N nodes

$j_l \in \mathbb{Z}^+/2$ is an assignment of an SU(2) unitary irreducible representation to each link l

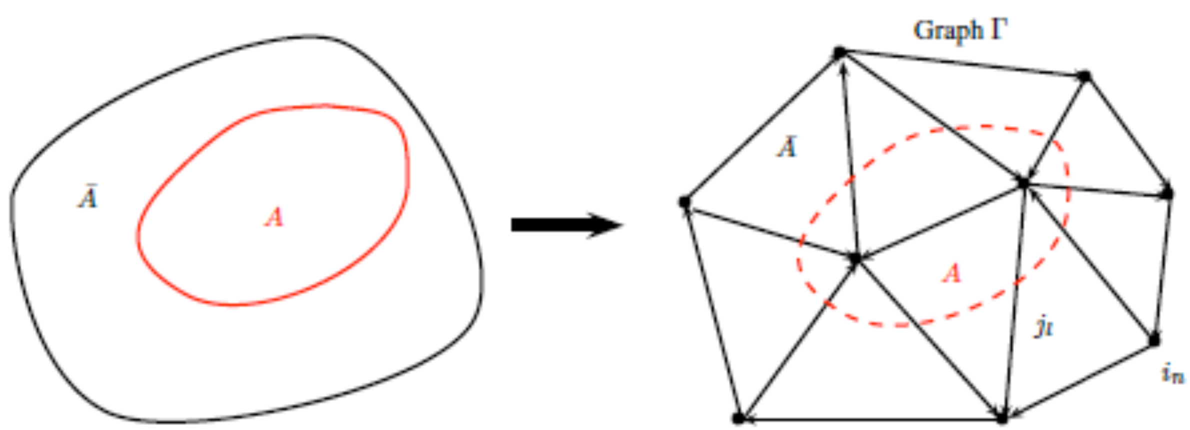
i_n is an assignment of an SU(2) intertwiner to each node n

The spin-j representation is isomorphic to a SL(2,C) representation $(\gamma j_l, j_l)$ selected by the linear version of the simplicity constraints

$$\vec{K} = \gamma \vec{L}$$

Engle, Pereira, Rovelli & Livine

ii) SU(2) spin-networks correspond to quantum 3-geometries: twisted geometry



Freidel & Speziale

IV. Some questions from the perspective of EE

Can entanglement entropy teach us something about recovering a classical geometry?

i) Area-law (of ground states) for the gravitational degrees of freedom

ii) Gauge invariance of the Entanglement Entropy

iii) No macroscopic quantum superpositions (no room for STSC)

$$\downarrow$$

Modifying coherent states, necessary to look at the asymptotic-limit

$$\Psi_{\Gamma,H_{ab}} \simeq \sum_{j_{ab}} \left(\prod_{ab} (2j_{ab}+1) e^{-\frac{(j_{ab}-j^0_{ab})^2}{2\sigma^0_{ab}}} \right) \Psi_{\Gamma,j_{ab},\Phi_a(\vec{n}_{ab})}(h_{ab})$$

Thiemann, Winkler, Rovelli, Bianchi, Magliaro, Perini

IV. Area-law and the ground state

i) States of LQG are "particular" from the perspective of many-body system: annihilated by the scalar constraint

$$\mathcal{H}_{\rm S}|\Psi\rangle = 0$$

ii) GS of a local Hamiltonian with area law, gauge invariance and no STSC

$$S_A = \boxed{\frac{\mathcal{A}}{4\ell_p^2}} + \mu L - \frac{3k}{2}\ln\frac{\mathcal{A}}{\ell_p^2}$$ **area law**

$$|\Psi\rangle = \frac{1}{N}\sum_{\{j_l\}}\int_{SU(2)} dh \left(\prod_l^L d_{j_l} e^{-(j_l-J_l)^2 t_l}\right) \sum_{\{m_l\}\{k_l\}}\sum_{\{n_l\}}\prod_l^L F^{j_l}_{m_l n_l} \boxed{D^{j_l}_{n_l k_l}(h)} |j_l, m_l, k_l^\dagger\rangle$$ **gauge invariance**

$$\mathcal{I}(A|B) := S(A) + S(B) - S(A|B)\,, \qquad \boxed{\mathcal{I}_\infty = 0}$$ **no STSC**

iii) The construction can be generalized to "N-body" states

$$|\Psi_N\rangle = \sum_{\{j_l\},\{i_n\}} c_{\{j_l\}\{i_n\}} \prod_n^N |\Psi_n\rangle, \qquad |\Psi_n\rangle \equiv |\{j_{l\in n}\}, i_n\rangle, \qquad c_{\{j_l\}\{i_n\}} \equiv \frac{1}{\mathcal{N}}\prod_{l,n}\Delta^{j_l}_{(t_l,J_l)} F^{j_l}\cdot\overline{v}_{i_n}$$

IV. Coherent states and asymptotic limit

i) Coherent states can be recovered considering a suitable "correction" encoding area-law

$$|\psi_{CS}\rangle = \frac{1}{N_j}\sum_{mn} D_{mn}\left(g e^{-\pi\gamma L_z + i\phi L_z - \frac{\exp(1-2\pi\gamma L_z)}{4\pi\gamma}}\tilde{g}^\dagger\right)|jm\rangle\langle jn|$$

new CS

ii) Expectation values of area and holonomy operators can be evaluated in the asymptotic limit

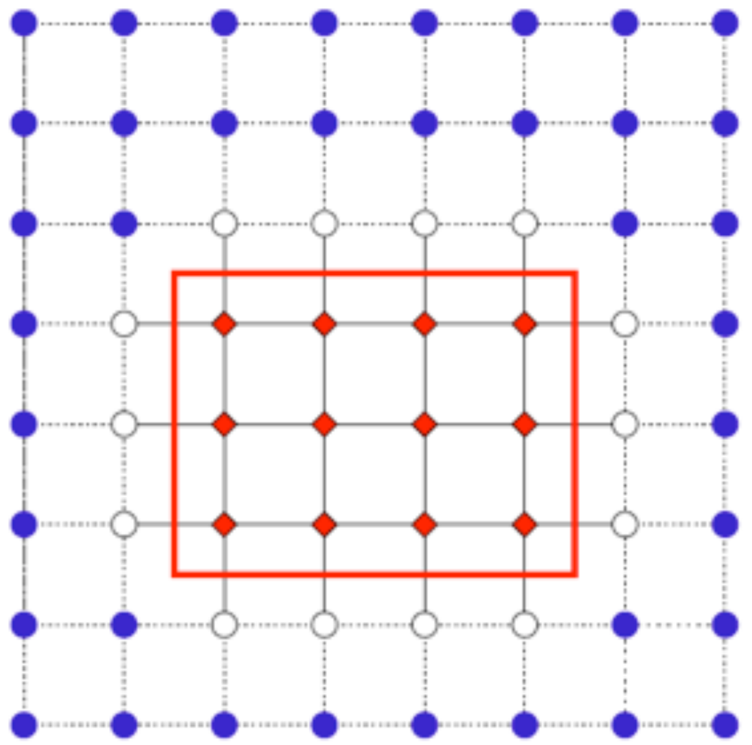
$$\boxed{\begin{gathered}\Delta^2\langle A\rangle = (\gamma\ell_p^2)^2\left(\frac{1}{4t} - \frac{e^{-cJ}}{4t} - \frac{e^{-cJ}}{2Jt} + O\left(\frac{1}{Jt}\right)\right) \\ \Delta^2\langle W\rangle = 2t\sin^2\frac{\phi}{2} - ie^{-cJ}\sin\phi\end{gathered}}$$

In the asymptotic limit uncertainty relations are "saturated"

iii) States can indeed be decomposed in the new CS: resolution of the identity

IV. Lattice gauge theories (Kitaev model)

Graph with orientation $\pm$, and qbits on the edges



Consider a discrete many-body system on a square lattice with n/2 vertices and plaquettes, and Hamiltonian

$$H = \sum_v (1 - A(v)) + \sum_p (1 - B(p))$$

and a finite dimensional group G, and the Hilbert space $\mathcal{H} \simeq C[G]$ with the orthornormal basis for the q-bits$\{|g\rangle \ : g \in G\}$, and$|G| = d$

Relevant operators are L and T:

$$L^g_+|z\rangle = |gz\rangle, \qquad L^g_-|z\rangle = |zg^{-1}\rangle,$$
$$T^h_+|z\rangle = \delta_{h,z}|z\rangle, \qquad T^h_-|z\rangle = \delta_{h^{-1},z}|z\rangle$$

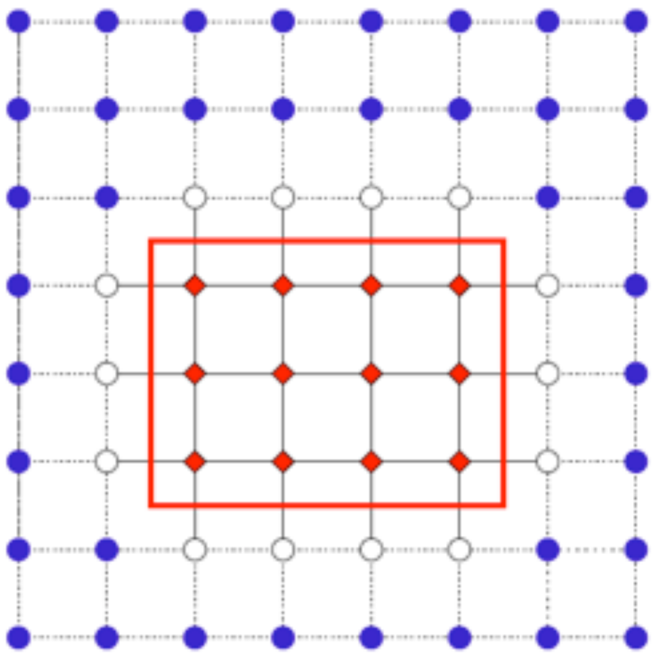
Gauge transformations:

$$A_g(v) = \prod_{l \in v} L^g(l,v), \qquad B_e(p) = \sum_{h_1 h_2 h_3 h_4 = e} \prod_{m=1}^{4} T^{h_m}(j_m, p)$$

Star and plaquette operators:

$$A(v) = |G|^{-1} \sum_{g \in G} A_g(v), \qquad B(p) = B_1(p)$$

IV. Kitaev model: GS and EE



$$[A(v), B(p)] = [A(v), A(v')] = [B(p), B(p')] = 0$$

Zero energy GS must satisfy: $\mathcal{S} = \{|\xi\rangle \in \mathcal{H};\ A(v)|\xi\rangle = B(p)|\xi\rangle = |\xi\rangle \quad \forall v, p\}$

$$|\psi\rangle = \frac{1}{\sqrt{|G|}} \sum_{g \in G} g|0\rangle$$

Imagine the bipartition of the Hilbert space: $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$, and the set biorthonormal $\{|h_A\rangle \otimes |h_B\rangle\}$

i) The reduced density matrix is unitarily equivalent to a matrix that only addresses the degrees of freedom on the boundary of the partition

$$\rho_A = \tilde{\rho}_A \otimes \rho_A^{(\mathrm{bulk})},$$
$$\tilde{\rho}_A = \frac{1}{\sqrt{|G|}} \sum_{h \in G_{AB}} h_A |0\rangle\langle 0|_a h_A$$

ii) Entanglement nearly shows an area law because there is a global constraint on the boundary

$$\rho_A^{\mathrm{area}} = \bigotimes_{v=1}^{n} \rho_v = \tilde{\rho}_A \otimes \rho_A^{\mathrm{top}}$$

Bounds on the value of the entanglement entropy can be computated for the easiest cases

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V. States enjoying Bisognano-Wichmann

These are the states:

$$|\Psi\rangle = \sum_j \alpha_j \exp(-\gamma K)_{mn} |j, m, n\rangle$$

$$|j, m, n\rangle = |j, m\rangle \otimes |j, n\rangle^\dagger$$

Bianchi & Myers '12

Chirco, Rovelli & Ruggiero '14

with reduced density matrix proportional to the exponentiation of the boost operator K

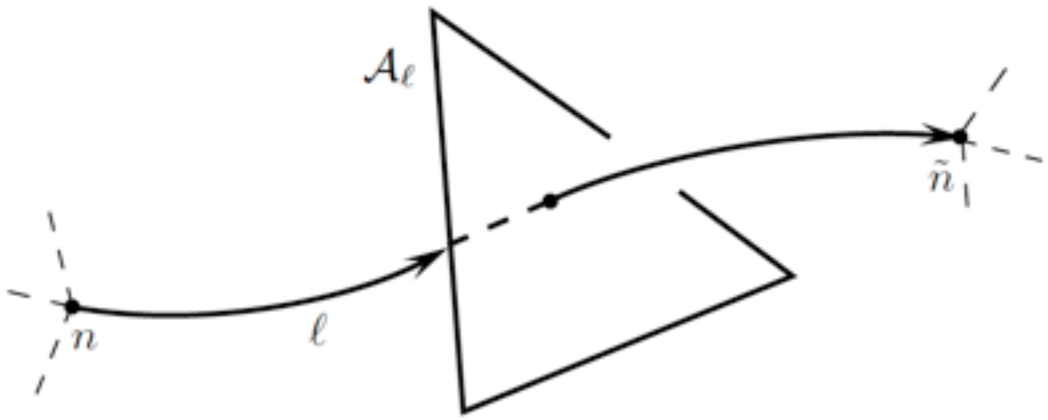
$$\rho = \mathrm{Tr}_t |\Psi\rangle\langle\Psi| = e^{-2\pi \vec{K}_s \cdot \vec{n}}$$

One can check that this state does not satisfy the area-law in the large j-limit

This is a crucial ingredient to recover the semi-classical limit

V. "One-body" states with area law

One-link states, with target and source indices specified



we choose a state in the diagonalized basis, with fixed spin j, such that the reduced density matrix satisfies

$$\rho_A \equiv \sum_{m=-j}^{j} p(m)|j,m\rangle\langle j,m|, \qquad p(m) \equiv \frac{f(m)}{\sum_{m=-j}^{j} f(m)}$$

which in the large j limit, when $0 < j^{-1} \ll c \ll j^{-\frac{1}{2}} \ll 1$

$$f(m) = \exp\left[-c\,m - \frac{\exp(1-c\,m)}{c}\right]$$

state that satisfies the area law!

$$|\Psi\rangle \equiv \sum_{mn} \langle j,m|e^{-\pi\gamma L_z - \frac{\exp(1-2\pi\gamma L_z)}{4\pi\gamma}}|j,n\rangle \; |j,m,n\rangle$$

V. EE between two half links I

$$S = \lim_{n\to 1} \frac{1}{1-n} \ln \frac{1}{\mathcal{N}^n} \chi_j \left[\exp n \left(-2\pi\gamma L_z - \frac{\exp(1-2\pi\gamma L_z)}{2\pi\gamma} \right) \right]$$

How to estimate normalization:

$$\mathcal{N} = \chi_j \left[\exp \left(-2\pi\gamma L_z - \frac{\exp(1-2\pi\gamma L_z)}{2\pi\gamma} \right) \right]$$

- Insert the resolution of identity of coherent state into the character χ_j

$$1_j = d_j \int \mathrm{d}g |j, g\rangle\langle j, g|$$

- Expand $\exp(1-2\pi\gamma L_z)$ into Taylor series.

$$\mathcal{N} = d_j \sum_k^{\infty} \frac{(-)^k \, \mathrm{e}^k}{k!(2\pi\gamma)^k} \int \mathrm{d}g \; \mathrm{e}^{S_k}$$

$$S_k = 2j \ln\langle\uparrow |g^\dagger e^{-2\pi\gamma(1+k)L_z} g| \uparrow\rangle$$

V. EE between two half links II

- Rescale $j \to \lambda j$ and take $\lambda >> 1$, the leading contribution of $\mathcal{N}$ is controlled by the solutions of the equations of motion

$$\delta_g S_k = 0$$

- Solutions:

$$g L_z g^\dagger = \epsilon L_z, \quad \epsilon = \pm$$

- The leading contribution $\mathcal{N}$ is

$$\mathcal{N} = \sum_{\epsilon=\pm} \mathcal{N}^\epsilon$$

with

$$\mathcal{N}^\epsilon \sim \exp\left(-1 - \frac{\exp(1 - \epsilon 2\pi\gamma j)}{2\pi\gamma}\right)$$

When $\gamma j >> 1$, $\mathcal{N}^+$ is much larger than $\mathcal{N}^-$.

$$\mathcal{N} = \mathcal{N}^+ \sim \exp\left(-1 - \frac{\exp(1 - 2\pi\gamma j)}{2\pi\gamma}\right)$$

V. EE between two half links II

$$\boxed{S = \lim_{n\to 1} \frac{1}{1-n} \ln \frac{1}{\mathcal{N}^n} \chi_j \left[\exp n \left(-2\pi\gamma L_z - \frac{\exp(1-2\pi\gamma L_z)}{2\pi\gamma} \right) \right]}$$

- Same analysis for $I \equiv \chi_j \left[\exp n \left(-2\pi\gamma L_z - \frac{\exp(1-2\pi\gamma L_z)}{2\pi\gamma} \right) \right]$. The leading contribution is controlled by the same saddle point as in $\mathcal{N}$

$$I = I^+ \sim \frac{1}{n} \exp \left[-1 + (1-n) 2\pi\gamma j - n \frac{\exp(1-2\pi\gamma L_z)}{2\pi\gamma} \right]$$

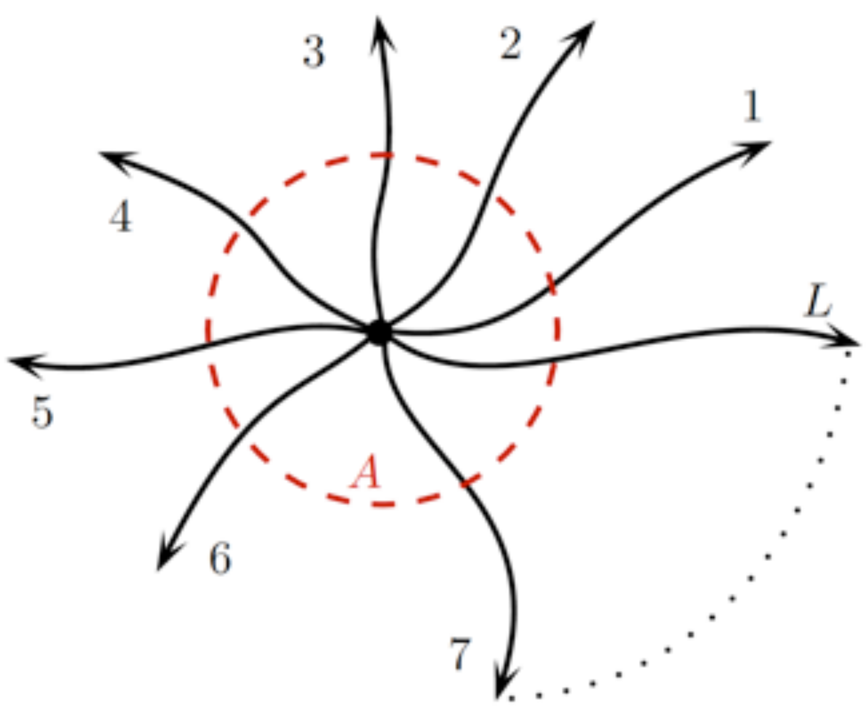
The major contribution of the entanglement entropy is

$$S = \lim_{n\to 1} \frac{1}{1-n} \ln \frac{I^+}{\mathcal{N}^n} = \lim_{n\to 1} \frac{1}{1-n} \ln \frac{1}{n} \mathrm{e}^{(1-n)(2\pi\gamma j - 1)} = 2\pi\gamma j$$

V. “One-body” with many links

Spin-network states, expanded as $|\mathcal{S}_{\Gamma,\{j_l\},\{i_n\}}\rangle \equiv \sum_{\{m\}\{k\}} \prod_n^N v_{i_n\ \{k_n\}}^{\{m_n\}} \prod_l^L |j_l, m_l, k_l^\dagger\rangle$

with $|j_l, m_l, k_l^\dagger\rangle \equiv |j_l, m_l\rangle \otimes \langle j_l, k_l| \in \mathcal{H}_{j_l}$



Can entanglement entropy teach us something about recovering a classical geometry?

$$|\Psi\rangle \equiv \frac{1}{\mathcal{N}} \sum_{\{j_l\},\{i_n\}} \prod_{l,n} \Delta^{j_l}_{(t_l,J_l)} F^{j_l} \cdot \overline{v}_{i_n} |\Gamma, j_l, i_n\rangle$$

$$\Delta^{j_l}_{(t_l,J_l)} \equiv d_{j_l} e^{-(j_l-J_l)^2 t_l}$$

$$F^{j_l} \equiv D^{j_l}\left(g_l e^{-\pi\gamma L_z + \imath\phi_l L_z - \frac{\exp(1-2\pi\gamma L_z)}{4\pi\gamma}} \tilde{g}_l^\dagger\right)$$

$$S_A = \frac{\mathcal{A}}{4\ell_p^2} + \mu L - \frac{3k}{2} \ln \frac{\mathcal{A}}{\ell_p^2}$$

V. “Many-body” states

Quasi-adiabatic continuation can be applied to recover “many-body” states that preserves area law

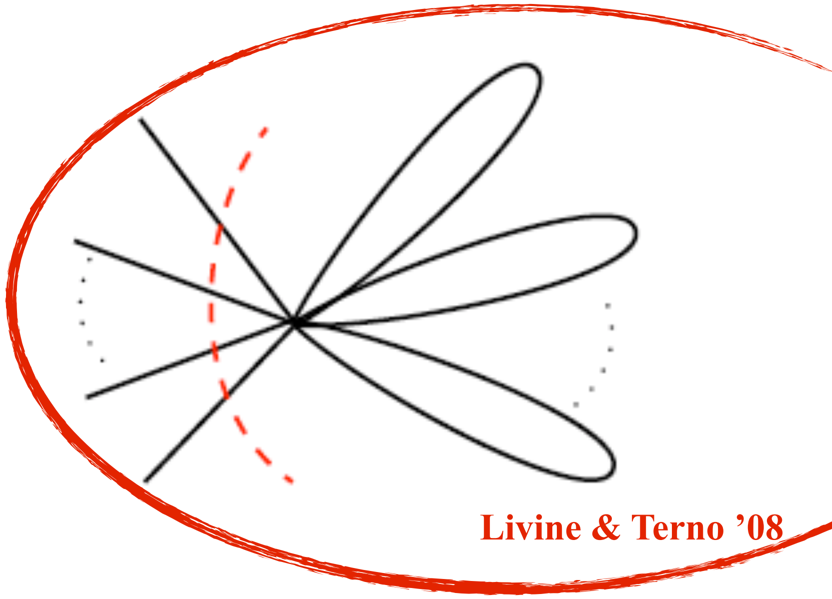
$$\boxed{|\Psi(\lambda)\rangle = U(\lambda)\,|\Psi(0)\rangle}$$

$$\tilde{H}(s) = \imath \int dt F(t) e^{\imath H(s)t} \partial_s H(s) e^{-\imath H(s)t}$$

$$U(\lambda) = \mathcal{T} \exp\{-i \int_0^\lambda \tilde{H}(s) ds\}$$

$\boxed{H(\lambda) = \sum_X \hat{T}_{X\subset\Gamma}(\lambda)}$ **local Hamiltonian, sum of local operators**

$\hat{T}_X(\lambda)$ **operators with support only on the subgraph X**



No spacetime Schroedinger’s cats!

$$\mathcal{I}(A|B) := S(A) + S(B) - S(AB)$$

Mutual information

$$S := S_2 = -\log \mathrm{Tr}\rho_A^2$$

Log of purity, preserved by quasi-adiabatic continuation

States with non-zero $\mathcal{I}_\infty$ between two distant macroscopic regions must be ruled out!

Summary

- I. Quantum entanglement and its quantification/measurement
- II. Area-law in gravity
- III. Complexity of entanglement and thermalization properties
- IV. Requirements on LQG states from the area-law, and new CS
- V. States with area-law in the Hilbert space of LQG
- VI. Conclusions & Outlook

Conclusion & Outlook

- i) EE and a new way to address thermalization
- ii) Area-law as a distiller of potential classical geometries
- iii) Area-law and local entanglement Hamiltonian
- iv) Relation between Hamiltonian constraint, EE and Einstein equations?
- v) Asymptotic limit and modified coherent states
- vi) EE as suitable probe/description of gravity?

Dziękuje! **Thank you!**

谢谢 **Grazie!**