# RELATIVE LOCALITY, METASTRING THEORY & MODULAR SPACE-TIME



LAURENT FREIDEL (PERIMETER), ROB LEIGH (URBANA), D.M. (VIRGINIA TECH)

1307.7080, 1405.3949, 1502.08005 ...

(A NEW FORMULATION OF QUANTUM GRAVITY & STRING THEORY; SHEDS LIGHT ON THE FOUNDATIONS OF QUANTUM TH.)

1) FOUNDATIONS: RELATIVE LOCALITY ⇝ (AMELINO-CAMELIA, L. FREIDEL, KOWALSKI-GLIKMAN, SMOLIN 2011)

(ALSO: THIRD RELATIVITY → WHEELER, FINKELSTEIN ... '80s)

—‖— : BORN DUALITY (RECIPROCITY) ⇝ BORN ... '30s ($x \to p$, $p \to -x$) ← CHOICE OF BASIS IN QM IMMATERIAL

LOCALITY: CENTRAL TO MODERN PHYSICS (BEING BASED ON QUANTUM THEORY & RELATIVITY (LOCALITY / CAUSALITY))

↳ (LOCAL EFFECTIVE FIELD THEORIES; STRUCTURE OF RG / SEPARATION OF SCALES)

HOWEVER: a) QUANTUM THEORY HAS NON-LOCAL FEATURES (HEISENBERG UNCERTAINTY $\Delta x \Delta p \sim \hbar$; ENTANGLEMENT / BELL INEQUALITIES; MEASUREMENT; INTERFERENCE, AHARONOV/BOHM, A/C...)

RESPECTS LOCAL CAUSALITY ⇒ QFT

b) GENERAL RELATIVITY HAS NON-LOCAL FEATURES

↳ (NO LOCAL DIFF-INVARIANT OBSERVABLES; HOLOGRAPHY ... NON-LOCAL PROBES: LOOPS (STRINGS))

MESSING WITH LOCALITY MUST BE DONE VERY CAREFULLY!

USUALLY, LOCALITY IS ABSOLUTE: ABSOLUTE LOCALITY; SPACE-TIME IS INDEPENDENT OF ITS PROBES (APPROPRIATE FOR SEMI-CLASSICAL (POINT-LIKE) PROBES)

↳ CONSIDER MAKING LOCALITY RELATIVE: RELATIVE LOCALITY; SPACE-TIME DEPENDS ON THE (QUANTUM) NATURE OF THE PROBE (I.E. ENERGY OR OTHER QUANTUM #!)

↕ WHY?

IN GENERAL, PHYSICS LOOKS DIFFERENT WHEN PROBED AT DIFFERENT ENERGIES!

IF LOCALITY IS RELATIVE ⇒ DIFFERENT OBSERVERS/PROBES "SEE"/PROBE DIFFERENT SPACE-TIMES (THIRD RELATIVITY)

WE NEED BOTH SPACE-TIME & ENERGY-MOMENTUM SPACE FOR COMPLETE DESCRIPTION

THUS, IF LOCALITY IS RELATIVE THE PROPER SETTING FOR QUANTUM GEOMETRY IS A PHASE SPACE $\mathcal{P}$

HOWEVER, SPACE-TIME IS CURVED, SO WE NEED CURVED ENERGY-MOMENTUM [USUALLY WE THINK OF IT AS A LINEAR SPACE]

ALSO, WE NEED TWO SCALES, $\lambda$ & $\epsilon$ : $\hbar = \lambda\epsilon$ ; $\frac{\lambda}{\epsilon} = \alpha'$ (⇝ STRING TENSION)

WORK WITH $\mathbb{X}^A = \begin{pmatrix} X^a/\lambda \\ Y_a/\epsilon \end{pmatrix} \in \mathcal{P}$ (2 SCALE RG) [CURVED E-M SPACE → M. BORN (1930s)!!]

(FREIDEL + LIVINE, 3D; NCFT: LANGMANN-SZABO, GROSSE-WULKENHAAR)

2) IMPLEMENTATION/REALIZATION: QUANTUM GRAVITY AS METASTRING THEORY → (STRINGS IN (DYNAMICAL) PHASESPACE OR STRINGS IN THE GEOMETRY OF QUANTIZATION)

ORIGINALLY, STRINGTHEORY DEVELOPED AS A PERTURBATIVE DEFINITION OF AN S-MATRIX THEORY FOR PARTICLE-LIKE STATES IN A FLAT SPACE-TIME BACKGROUND/TARGET-SPACE

(CFT CONSISTENCY ⟺ SPACE-TIME CONSISTENCY ⟺ (CAUSALITY, UNITARITY, LOCALITY))

(MUTUAL LOCALITY, MODULAR INVARIANCE, SYMMETRY) ⟷

RELAX INTERPRETATION: DO NOT ASSUME TARGET = SPACE-TIME, OR LOCALITY IN SPACE-TIME

IN PERTURBATIVE STRING THEORY ELEMENTARY PROBES ARE STRINGS AND DEGREES OF FREEDOM ARE MAPS: $X : \Sigma \to M$ ; $\Sigma$ - RIEMANN SURFACE $M$ - TARGET SPACE-TIME

HOWEVER THE IDENTIFICATION OF $M$ AS A SPACE-TIME BREAKS BORN RECIPROCITY $\begin{pmatrix} x \to p \\ p \to -x \end{pmatrix}$ I.E. BY TAKING $M$ TO BE SPACE-TIME WE HAVE CHOSEN A BASIS & THUS BROKEN BASIS INDEPENDENCE

BUT, THE UNDERLYING STRUCTURE OF THE STRING IS BR-SYMMETRIC $H: \frac{1}{2}(p^2 + \delta^2) = N + \tilde{N} - 2$ ; $D: \ p \cdot \delta = N - \tilde{N}$ $\Big\|$ $\begin{matrix} p \to \delta \\ \delta \to -p \end{matrix}$

LOCALITY IS BUILT IN A CHOICE OF A BND. CONDITION (I.E. IN THE PERIODICITY OF THE STRING)

ALLOW FOR MONODROMIES: $X^\mu(\sigma + 2\pi, \tau) = X^\mu(\sigma, \tau) + \delta^\mu$ $\left(\alpha' p = \int_C {*dX} \quad \delta = \int_C dX\right)$

SOLUTIONS OF STRING EQ. OF MOTION ARE CHIRAL: $X^a(\tau, \sigma) = X_L{}^a(\tau - \sigma) + X_R(\tau + \sigma)$

T-DUALITY MIXES THIS UP WITH $Y^a(\tau, \sigma) = X_L{}^a(\tau - \sigma) - X_R{}^a(\tau + \sigma)$ $\left(\delta = \alpha' \int_C {*dY}, \ p = \int_C dY\right)$

USE $\mathbb{X}^A = \begin{pmatrix} X^a/\lambda \\ Y_a/\epsilon \end{pmatrix}$ AND $S_{MS} = \frac{1}{4\pi} \int \left(\partial_\tau \mathbb{X}^A (\eta_{AB} + \omega_{AB}) \partial_\sigma \mathbb{X}^B - \partial_\sigma \mathbb{X}^A H_{AB} \partial_\sigma \mathbb{X}^B\right)$

WHERE THE PHASE SPACE DATA ARE: $H_{AB} = \begin{pmatrix} \ell & 0 \\ 0 & \ell^{-1} \end{pmatrix}$ ← SPACE-TIME ← (FLAT) ← & ENERGY-MOMENTUM ; $\omega_{AB} = \begin{pmatrix} 0 & \delta \\ -\delta^T & 0 \end{pmatrix}$ → SYMPLECTIC STRUCTURE (→ TOTAL DERIVATIVE IN $S_{MS}$)

TELLS US HOW TO DISTINGUISH X & P ⇐ POLARIZATION METRIC $\eta_{AB} = \begin{pmatrix} 0 & \delta \\ \delta^T & 0 \end{pmatrix}$ ← RELATIVE LOCALITY METRIC $\eta(\mathbb{X})$!

T-DUALITY: CANONICAL TRANSFORMATION ; SPACE-TIME NOT TARGET, BUT (LAGRANGIAN) SUBMANIFOLD OF PHASE SPACE $\mathcal{P}$

$(\mathcal{P}, \omega, H, \eta)$ - GEOMETRY OF QUANTIZATION! $H$ ← BORN RULE ; $\eta$ ← DEFORMATION QUANTIZATION

3) PREDICTION: QUANTUM SPACE-TIME IS MODULAR SPACE-TIME (MODULAR SPACE-TIME = QUANTUM LAGRANGIAN SUBMANIFOLD)

FUNDAMENTAL NON-COMMUTATIVITY $\boxed{\{\mathbb{X}^A(\tau, \sigma), \mathbb{X}^B(\tau, \sigma')\} = \eta^{AB} \theta(\sigma - \sigma')}$ !! ⇐ $S_{MS}$

LAGRANGIAN SUBMANIFOLD: COMMUTATIVE SUBALGEBRA OF THE FULL OPERATOR ALGEBRA QUANTUM

THIS COMMUTATIVE SUBALGEBRA GIVEN BY MODULAR VARIABLES (AHARONOV ET AL) INTERFERENCE VIA OPERATORS

MODULAR MOMENTUM $[\hat{p}] = \hat{p} \bmod (h/L)$ ; MODULAR POSITION $[\hat{x}] = \hat{x} \bmod (L)$

NOTE: $[\hat{x}]$ & $[\hat{p}]$ COMMUTE ; CLASSICAL SPACE-TIME: EXTENSIFICATION (PRESERVES AREA IN PHASE SPACE $\mathcal{P}$)

(MANY EXTENSIFICATIONS!) → "MODULI"

TECHNICAL DETAILS: POLYAKOV (FLAT METRIC) $*$ – 2d Hodge duality

$$S_P(X) = \frac{1}{2\alpha'}\int \eta_{\mu\nu} * d\underline{X}^\mu \wedge d\underline{X}^\nu$$

(ASSUME NON-COMPACTNESS); $(\sigma,\tau)\in[0,2\pi]\times[0,1]$ cylinder

$d\underline{X}(\sigma,\tau)$ is PERIODIC w.r.t. $\sigma$; PERIOD $2\pi$.

$\Rightarrow \underline{X}^\mu$ is QUASI-PERIODIC $\quad \underline{X}^\mu(\sigma+2\pi,\tau) = \underline{X}^\mu(\sigma,\tau) + \bar{P}^\mu$

$\bar{P}^\mu$ is THE QUASI-PERIOD OF $\underline{X}^\mu$. IF $\bar{P}^\mu$ NON-ZERO NO A-PRIORI SPACE-TIME INTERP.

INSTEAD, STRING PROPAGATES INSIDE A PORTION OF PHASE SPACE. (NON-GEOMETRIC BACKGROUND)

T-DUALITY AS A FOURIER TRANSFORM: $\boxed{\frac{i}{\lambda^2}(!)}$

$$|\psi\rangle: \quad \psi(x^\mu(\sigma)) = \int_{\underline{X}|\partial\Sigma = x} \mathcal{D}\underline{X} \int_{Met(\Sigma)} \mathcal{D}\gamma\, e^{\frac{i}{2\alpha'}\int_\Sigma \eta_{\mu\nu}(*d\underline{X}^\mu \wedge d\underline{X}^\nu)}$$

Here $\underline{X}$-periodic

$\boxed{\frac{i}{\epsilon^2}(!)}$

FOURIER TRANSFORM: (F.T.) $(x \to p,\ p \to -x)$

$$\tilde{\psi}(Y_\mu(\sigma)) = \int_{Y|_{\partial\Sigma = y}} \mathcal{D}Y \int_{Met(\Sigma)} \mathcal{D}\gamma\, e^{\frac{i}{2\pi\alpha'}\int_\Sigma \eta_{\mu\nu}(*dY_\mu \wedge dY_\nu)}$$

Here $Y$- quasiperiodic

$\underset{|||}{}$

T-DUALITY $\equiv$ BORN DUALITY

FIRST ORDER ACTION: $\hat{S} = \int_\Sigma \left(X^\mu dP_\mu + \frac{\alpha'}{2}\eta^{\mu\nu}(*P_\mu \wedge P_\nu)\right)$ : E.O.M. $*P_\mu = \frac{1}{\alpha'}\eta_{\mu\nu}dX^\nu$

$\hookrightarrow \hat{S} = -\frac{1}{\alpha'}S_P + \underbrace{\int_{\partial\Sigma} X^\mu P_\mu}_{\text{Kernel of the F.T.}}$

(Note: $P_\mu = dY_\mu$ by integrating $\underline{X}$.. to get $dP = 0$)
$\downarrow$
locally.

We can do this on genus $g$, $\Sigma_g$ etc. $\Rightarrow$ LORENTZIAN WORLDSHEETS!

Note: Vertex operators $V_{p_i}(x) \to e^{ip_i X(z)}$ & dual $\tilde{V}_{p_i}(Y) \to e^{i\sum p_i \int_{e_i} *dY_i}$

non-local operators

$\hookrightarrow$ $z$ - are endpoints of a graph $\Gamma$ with edges $e_i$)

DYONIC $\leftarrow V_{p,\bar{p}} \sim e^{ipX + i\bar{p}Y}$

Electro-magnetic duality on the world-sheet

$$\left.e_1 m_2 + e_2 m_1 \in \mathbb{Z}\right\} \Rightarrow \underline{O(D,D)} \to \text{USUAL T-DUALITY}$$

$e$: electric charges $(p)$; $m$: magnetic charges $(\bar{p})$

(DOUBLE FIELD TH.), (HULL, ZWIEBACH...) SIEGEL

STRING THEORY IN PHASE SPACE: CONSIDER FIRST ORDER ACTION

$$\frac{1}{\hbar}\hat{S} = \int_\Sigma \left[\frac{1}{\hbar} P_\mu \wedge dX^\mu + \frac{1}{2\epsilon^2}\eta^{\mu\nu}(*P_\mu \wedge P_\nu)\right]$$

(RECALL: $2\alpha' = \frac{\lambda}{\epsilon}$, $\hbar = \lambda\epsilon$)

INSPIRED BY HAMILTONIAN FORMALISM DECOMPOSE $P_\mu = \mathcal{P}_\mu d\sigma + Q_\mu d\tau$

$$\Rightarrow S = \int \left(\mathcal{P}_\mu \partial_\tau X^\mu - Q_\mu \partial_\sigma X^\mu + \frac{\hbar}{2\epsilon^2}(Q_\mu Q^\mu - \mathcal{P}_\mu \mathcal{P}^\mu)\right)$$

: EOM $\mathcal{P}^\mu = \frac{\epsilon}{\lambda}\partial_\tau X^\mu$; $Q^\mu = \frac{\epsilon}{\lambda}\partial_\sigma X^\mu$

INTEGRATE OUT Q: $S = \int \mathcal{P}\partial_\tau X - \frac{1}{2}\int\left(\frac{\lambda}{\epsilon}\mathcal{P}\cdot\mathcal{P} + \frac{\epsilon}{\lambda}\partial_\sigma X\cdot\partial_\sigma X\right)$; Let $\partial_\sigma Y^\mu \equiv \mathcal{P}^\mu$

(LIKE X, Y IS QUASIPERIODIC; $\underbrace{Y(2\pi) - Y(0)}_{\text{QUASIPERIOD}}$ = STRING MOMENTUM)

THEN $\frac{1}{\hbar}S \to \int\left(\frac{1}{\lambda\epsilon}\partial_\sigma Y\cdot\partial_\tau X - \frac{1}{2\epsilon^2}\partial_\sigma Y\cdot\partial_\sigma Y - \frac{1}{2\lambda^2}\partial_\sigma X\cdot\partial_\sigma X\right)$

AS SUGGESTED BY DOUBLE FIELD FORMALISM (HULL, SIEGEL, ZWIEBACH ...) FOR USUAL T-DUALITY

LET $\mathbb{X}^A \equiv \begin{pmatrix} X^\mu/\lambda \\ Y_\mu/\epsilon \end{pmatrix}$ $\quad \eta_{AB} = \begin{pmatrix} 0 & \delta \\ \delta^{-1} & 0 \end{pmatrix}$ $\quad \delta$ — DIAGONAL $(+,+,+\ldots+)$ D-DIMENSIONAL IDENTITY MATRIX

$H_{AB} \equiv \begin{pmatrix} h & 0 \\ 0 & h^{-1} \end{pmatrix}$ $\quad h_{\mu\nu}$ — D-DIMENSIONAL LORENTZ METRIC

(NOTE: $J = \eta^{-1}H \Rightarrow J^2 = 1$, $J^T \eta J = \eta$; $J$ — CHIRAL STRUCTURE)

$\omega$ — SYMPLECTIC STRUCTURE

IN GENERAL: $(\eta_{AB} + \omega_{AB})$

TSEYTLIN ACTION: $\frac{1}{\hbar} S_{MS} = \frac{1}{2}\int\left(\partial_\tau \mathbb{X}^A \partial_\sigma \mathbb{X}^B \eta_{AB} - \partial_\sigma \mathbb{X}^A \partial_\sigma \mathbb{X}^B H_{AB}\right)$

CURVED BACKGROUND: POLYAKOV: $S_P = \frac{1}{2\alpha'}\int G_{\mu\nu}(x) * dX^\mu \wedge dX^\nu + B_{\mu\nu} dX^\mu \wedge dX^\nu$

$\Rightarrow$ TSEYTLIN ACTION WITH

$\eta_{AB} \equiv \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\quad H_{AB} \equiv \begin{pmatrix} \hat{G}^{-1}_{\mu\nu}/\alpha' & -(\hat{G}^{-1}\hat{B})_\mu{}^\nu \\ (\hat{B}\hat{G}^{-1})^\mu{}_\nu & \alpha'[\hat{G} - \hat{B}\hat{G}^{-1}\hat{B}]^{\mu\nu} \end{pmatrix}$

DUAL $\hat{G}$, $\hat{B}$: $\hat{G}^{\mu\nu} + \hat{B}^{\mu\nu} = [(G+B)^{-1}]^{\mu\nu}$

SOME TECHNICAL DETAILS:

1) TSEYTLIN ACTION (GENERALIZED): (POLYAKOV STRING → A "SINGULAR" LIMIT)

$$S_{MS} = \frac{1}{4\pi}\int_\Sigma d^2\sigma \left( \partial_\tau \mathbb{X}^A (\eta_{AB} + \omega_{AB})^{(*)} \partial_\sigma \mathbb{X}^B - \partial_\sigma \mathbb{X}^A H_{AB}^{(*)} \partial_\sigma \mathbb{X}^B \right) \to \text{CHIRAL } J \equiv \eta^{-1}H,\ J^2=1$$

$\mathbb{X}^A = \begin{pmatrix} X^\mu/\lambda \\ Y_\mu/\epsilon\end{pmatrix}$   $X^\mu$ — "LENGTH"   $Y^\mu$ — "MOMENTUM"   $\{\partial_\sigma X^\mu(\sigma), Y_\mu(\sigma')\} = 2\pi\,\delta(\sigma,\sigma')$

PERIODIC DELTA FUNCTION

$(X^\mu(\sigma+2\pi,\tau) = X^\mu(\sigma,\tau) + \delta^\mu)$   $O(D,D)$   $O(2,2(D-1))$

IN GENERAL: $\boxed{\eta,\ \omega \ \&\ H \text{ DYNAMICAL!!}}$   (IF CONSTANT: $\eta_{AB} = \begin{pmatrix}0 & \delta\\ \delta^T & 0\end{pmatrix}$, $H_{AB} = \begin{pmatrix} \ell & 0 \\ 0 & \ell^{-1}\end{pmatrix}$

$\delta$ - IDENTITY   $\ell$ - D-DIMENSIONAL LORENTZ)

⇓

RG eqs: $\square_H \delta H_{AB}=0$; $\square_H \delta\eta_{AB}=0$

$\square_\eta \delta\eta_{AB}\neq 0$; $\square_\eta\delta H_{AB}\neq 0$

$$\omega_{AB} = \begin{pmatrix} 0&\delta\\ -\delta^T & 0\end{pmatrix}$$

[N.B.: DFT: $\eta_{AB}$ CONSTANT ($\omega_{AB}$ USUALLY IGNORED)]

CONSISTENT WITH BORN DUALITY

HAMILTON ← $H = \delta_\sigma{}^A H_{AB}\delta_\sigma{}^B = 0$ ;   $D = \delta_\sigma{}^A \eta_{AB}\delta_\sigma{}^B = 0$ → DIFFEO

2) LORENTZIAN WORLDSHEETS (GIDDINGS - WOLPERT - KRICHEVER - NOVIKOV - NAKAMURA)

(NEEDED FOR MODULAR INVARIANCE!)   $\Delta_\gamma^A \equiv 2\pi\int_\gamma \delta^A$ ;   $S_{\Delta_i} \equiv \frac{1}{4\pi}\int_\Sigma d^2\sigma \left( (\eta_{AB}+\omega_{AB})\delta_\tau{}^A\delta_\sigma{}^B - H_{AB}\delta_\sigma{}^A\delta_\sigma{}^B\right)$

[$\sigma$ & $\tau$ DISTINGUISHED IN THE TSEYTLIN ACTION!]   $\delta^A$ - CLOSED FORM WITH FIXED MONODROMY   $\delta^A \in C^1_\Lambda(\Sigma,\Delta_i)$;   $\Lambda$ - LATTICE FORMED BY $\frac{\Delta}{2\pi}$!

METASTRING AMPLITUDE:

$$A_\Sigma(x_i,x_0) = \sum_{\Delta_i/2\pi\in\Lambda}^{\sum\Delta_i=0} \int_{C_\Sigma} dm_\tau \int_{C^1_\Lambda(\Sigma,\Delta_i)} [D\delta]\, e^{iS_\Delta(\delta)}$$

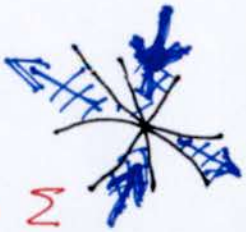
$(\forall \Delta_i/2\pi, \Delta_0/2\pi \in \Lambda)$

↓

$A_\Sigma(x_i+\Delta_i, x_0+\Delta_0)$

$\equiv$

$A_\Sigma(x_i,x_0)$



$m_\tau$ - MODULI SPACE OF CAUSAL STRUCTURES ON LORENTZIAN $\Sigma$

(LORENTZIAN ANALOG OF THE MODULI SPACE OF COMPLEX STRUCTURES)

$\int dm_\tau$ → SUM OVER "NAKAMURA GRAPHS" (FREIDEL, RAMGOOLAM..) → "STRIPS"

(GRAPHS THAT PRESERVE MODULAR INVARIANCE)

3) CLOSED (& OPEN) STRING WITH MONODROMIES!

GENERAL SYMPLECTIC FORMULATION: (SAY, CYLINDER, CUT OPEN ALONG $\sigma = 0, 2\pi$)

CLOSED STRING → SYMPLECTIC FLUX ACROSS THE CUT
BOUNDARY CONDITION HAS TO BE CONTINUOUS & INDEPENDENT OF WHERE WE MAKE THE CUT!

(NO "TORN" WORLDSHEET EMBEDDINGS!)

MONODROMY APPEARS AS A DISCONTINUITY ACROSS THE CUT
(THE CLOSED STRING BND CONDITION → REQUIRE THAT THE CUT IS INVISIBLE)
($\omega$ DOES NOT APPEAR IN THE SYMPLECTIC STRUCTURE)

OPEN STRING BND CONDITION: THE SYMPLECTIC FLUX MUST VANISH AT A PHYSICAL BOUNDARY

(FOR EXAMPLE,
- DIRICHLET $(\eta + \omega)\,\partial_\tau \mathbb{X}_e = 0$
- NEUMANN $(\eta + \omega)\, J \partial_\sigma \mathbb{X}_e = 0$
)

$\partial_\tau \mathbb{X} \in \tilde{L}$ → MOMENTUM SPACE
$\partial_\sigma \mathbb{X} \in L$ → SPACETIME
$L \oplus \tilde{L}$ → PHASE SPACE $\mathcal{P}$

4) STRINGY POISSON BRACKET:

$$\{\xi,\xi'\}_{SP} = \underbrace{\underbrace{\{\xi,\xi'\}_P}_{\text{USUAL POISSON}} + \oint d\sigma\, \eta_{AB}\, \xi^A(\sigma)\, \partial_\sigma \xi'^B(\sigma)}_{\text{COURANT BRACKET OF DFT}} + \overbrace{\oint\oint d\sigma d\sigma'\, \eta^{AB}\left[\partial_A \xi(\sigma)\, \Theta(\sigma-\sigma')\, \partial_B \xi'(\sigma')\right]}^{\text{MULTISTRING!}}$$

$\partial_A \xi \equiv \partial_A \xi_B(\mathbb{X})\, \partial_\sigma \mathbb{X}^B$

$\Theta(\sigma)$ – STAIRCASE DISTRIBUTION

$\{\sigma,\sigma'\}_{SP}$
↓
CLASSICAL ANOMALY
↓
QUANTIZATION!

"QUANTUM GROUP" STRUCTURE $(X,P)$, $(P,P)$, $(X,X)$

CLASSICAL STRINGY OBSERVABLES $\xleftarrow{\sigma}$ GENERATED BY A 1-FORM ON $\mathcal{P}$
"STRINGY GAUGE FIELD" $(A_M{}^a)$ ($a$ ← WORLDSHEET INDEX, $M$ ← TARGET INDEX)
(IN THE METASTRING SPECTRUM!)

MODULAR INVARIANCE → SELF-DUAL LATTICE $\Lambda$ → $\boxed{\text{MODULAR SPACETIME}}$
(UNIQUE → NO MODULI!)
(MONODROMIES IN ALL DIRECTIONS)
↓
CLASSICAL LIMIT → SPACETIME LAGRANGIAN

$\Lambda = \Pi_{1,25} \times \Pi_{1,25}$
↳ BORCHERDS ALGEBRAS

CONSTRAINTS: WEYL $\omega = 0$ LORENTZ $\mathcal{L} = \frac{1}{2}\, \$\$ + \frac{1}{2}\, \partial_\sigma \mathbb{X} \cdot (\eta^{-1})(\partial_\sigma \mathbb{X})$ ⑤

(WEYL & LORENTZ ON THE SAME FOOTING)

(WHERE $\$^A \equiv \partial_\tau \mathbb{X}^A - J(\partial_\sigma \mathbb{X})^A$ ; $J \equiv \eta^{-1} H$ ); $J^2 = 1 \Rightarrow L = \frac{1}{2}\, \$ \cdot \$$

HAMILTONIAN $H = \partial_\sigma \mathbb{X} \cdot J(\partial_\sigma \mathbb{X})$ Diffeo $D = \partial_\sigma \mathbb{X} \cdot \partial_\sigma \mathbb{X}$ (HAMILTONIAN & DIFFEO ON THE SAME FOOTING)

NOTE: LIOUVILLE MODES $(\theta, \varsigma)$ FOR LORENTZ & WEYL! [Diffeo x Weyl x Lorentz] $\Rightarrow$ NEW BETA FUNCTION EQS!

SPECTRUM AND NEW BACKGROUND FIELDS:

$(\lambda = 1,\ \epsilon = 1)$

H: $\frac{1}{2}(p^2 + \delta^2) = (N + \tilde{N} - 2)$

D: $p \cdot \delta = N - \tilde{N}$

} $N, \tilde{N}$ — LEFT & RIGHT NUMBER OPERATORS

BORN DUALITY $\left[\begin{matrix} p \to \delta \\ \delta \to -p \end{matrix}\right.$

DYONIC VERTEX OPERATORS (WAVE FUNCTIONS): $V(p,\delta) \equiv e^{ip \cdot X(\sigma)}\, e^{i\delta \cdot Y(\sigma)}$

COHERENT STATES OF DYONIC VERTEX OP. $\hookrightarrow$ CURVED PHASE SPACE!

$(X, Y)(\sigma + 2\pi) = (X + 2\pi\delta,\ Y + 2\pi p)$

BUT DIFFEOS $\rightarrow$ $V_{(p,\delta)}(\sigma + 2\pi) = V_{(p,\delta)}(\sigma)$

POLYAKOV: $\delta = 0$ & $M^2 = p^2$ PHASE SPACE (TSEYTLIN): $M^2 = p^2 + \delta^2$

[STRING SPECTRUM INVARIANT UNDER A DOUBLING OF THE LORENTZ GROUP: $p + \delta \to \Lambda(p + \delta)$, $p - \delta \to \bar{\Lambda}(p - \delta)$ ]

A PRIORI $\to$ NO SPACETIME INTERPRETATION

NEW EXCITATIONS: GENERALIZED KALB-RAMOND $\int \omega_{AB}\, dX^A \wedge dX^B$

(NON-SYMMETRIC H IN TSEYTLIN'S ACTION)

GENERALIZED DILATONS $\int \Phi(\mathbb{X})\, R + \int \hat{\Phi}(\mathbb{X})\, \hat{R}$ $\left(\begin{matrix} de^{\pm} \pm \omega \wedge (e^{\pm}) = 0 \\ d\omega = R \\ d{*}\omega = \hat{R} \end{matrix}\right)$

!! STRINGY GAUGE FIELD: $\int_{\Sigma} d^2\sigma\, A^a_A(\mathbb{X})\, \partial_a \mathbb{X}^A \Rightarrow$ DYONIC VERTEX OP.

MIXES WORLD-SHEET & TARGET SPACE $\Rightarrow$ VIOLATES LOCALITY (NO SPACETIME INTERPRETATION)

(APPEARS IN USUAL RG OF THE SIGMA MODEL $\to$ 3RD LOOP (OSBORN)) $\Rightarrow$ HAS TO BE IN THE SPECTRUM!!

Polyakov

SPACETIME NONLOCALITY! $\Leftarrow$ DYNAMICAL MOMENTUM SPACE! $\Leftarrow$

"WHY SHOULD I CARE ABOUT QUANTUM GRAVITY (QG)?" — [QUESTION POSED BY A PHENOMENOLOGIST FRIEND & COLLABORATOR]

i.e. WHAT CAN QG SAY ABOUT THE REAL WORLD?

{MANY PEOPLE TRIED: SORKIN ET AL .... CHANG, TAKEUCHI, D.M.}

1) THE VACUUM ENERGY PROBLEM: WHY IS THE OBSERVED VACUUM ENERGY SO DILUTED COMPARED TO $M_P^4$
$\left(\epsilon^4 \sim \left(\frac{M_{SM}^2}{M_P}\right)^4\right)$ ALSO, UV PHYSICS & IR PHYSICS MIXED
METASTRING WITH 2 SCALES & PHASE SPACE (COMPACT) NEEDED (FLM, ~~JUNE~~ 2014, TO APPEAR)

2) THE HIERARCHY PROBLEM: WHY IS $M_{SM}$ SO MUCH LOWER THAN $M_P$? (FLM, 2) RELATED TO 1)
(NO SUSY NEEDED!)

3) NON-LOCALITY IN THE STANDARD MODEL? "REWRITING" OF SM AS A THEORY ON A NON-COMMUTATIVE SPACE (CONNES ET AL ... ALSO NEEMAN, FAIRLIE .. SUPERCONNECTION SU(2|1))
(NEW SU(2|2) → PATI-SALAM) PHYSICS AT 4 TeV
AYDEMIR, TAKEUCHI, SUN, D.M (LRSM)

(METASTRING SHOULD NATURALLY LEAD TO A NON-LOCAL THEORY (AND NOT LOCAL EFFECTIVE FIELD THEORY) AT LOW ENERGIES (NO MSSM...)

4) NON-LOCALITY AND DARK MATTER: MILGROM'S SCALING IN GALACTIC ROTATION CURVES:
CAN BE EXPLAINED BY NON-LOCAL DARK MATTER (HO, NG, D.M & EDMONDS, FARRAH, HO, NG, TAKEUCHI, D.M.)
(STRINGY GAUGE FIELD?)

5) COSMOLOGY: NATURAL SETTING FOR THIRD RELATIVITY AND NON-LOCALITY

6) BLACK HOLE PHYSICS: ANOTHER NATURAL SETTING FOR NON-LOCALITY & THIRD RELATIVITY

7) GRAVITIZING THE QUANTUM & QUANTUM FOUNDATIONS:

QUANTUM MEASUREMENT & QG (AS IN PENROSE...)
"SPACETIME BIFURCATION"
"GRAVITIZING THE QUANTUM"

$\Box_H \delta H_{AB} = 0$; $\Box_H \delta g_{AB} = 0$ (LIGO?)

• GENERALIZED EINSTEIN EQS: $\Box_g \delta g_{AB} \neq 0$ ; $\Box_g \delta H_{AB} \neq 0$

"OUR MISTAKE IS NOT THAT WE TAKE OUR THEORIES TOO SERIOUSLY BUT THAT WE DO NOT TAKE THEM SERIOUSLY ENOUGH"
S. WEINBERG (THE FIRST THREE MINUTES)

"EVERYTHING THAT IS NOT FORBIDDEN IS COMPULSORY" M. GELL-MANN

[IS DYNAMICAL PHASE SPACE (AND THUS DYNAMICAL MOMENTUM SPACE) FORBIDDEN?]

TRIBUTE TO MAX BORN!!

STAY TUNED!

"WE ALL AGREE YOUR THEORY IS CRAZY. THE QUESTION WHICH DIVIDES US IS WHETHER IT IS CRAZY ENOUGH TO HAVE A CHANCE OF BEING CORRECT." (N. BOHR TO W. PAULI)