Wroclaw 6.7.2016

Hopf-algebra symmetries and cosmological neutrinos

$$[P_\mu, P_\nu] = 0 \, ,$$

$$\Delta P_0 = P_0 \otimes \mathbb{1} + \mathbb{1} \otimes P_0 \, , \quad \Delta P_j = P_j \otimes \mathbb{1} + e^{-P_0/\kappa} \otimes P_j \, ,$$

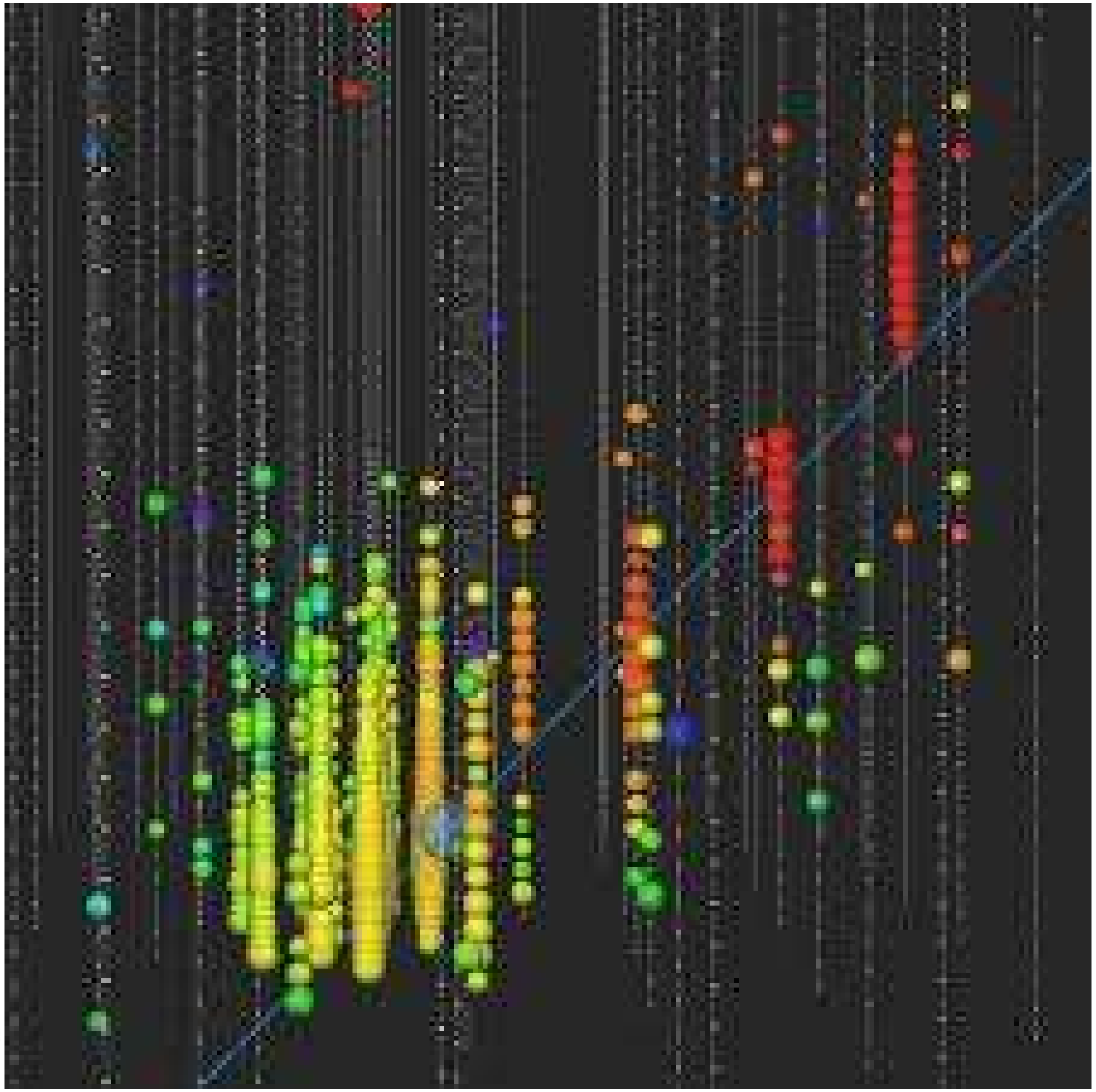
$$\varepsilon(P_\mu) = 0 \, , \qquad S(P_0) = -P_0 \, , \qquad S(P_j) = -e^{P_0/\kappa} P_j \, ,$$

$$[N_j, P_k] = i\,\delta_{jk} \left(\frac{\kappa}{2}(1 - e^{-2P_0/\kappa}) + \frac{1}{2\kappa}|\vec{P}|^2 \right) - \frac{i}{\kappa} P_j P_k \, ,$$

$$[N_j, P_0] = i\,P_j \, , \quad [R_j, P_k] = i\,\epsilon_{jkl} P_l$$

$$\Delta N_k = N_k \otimes \mathbb{1} + e^{-P_0/\kappa} \otimes N_k + \frac{i}{\kappa}\epsilon_{klm} P_l \otimes R_m \, , \quad \Delta R_j = R_j \otimes \mathbb{1} + \mathbb{1} \otimes R_j \, ,$$

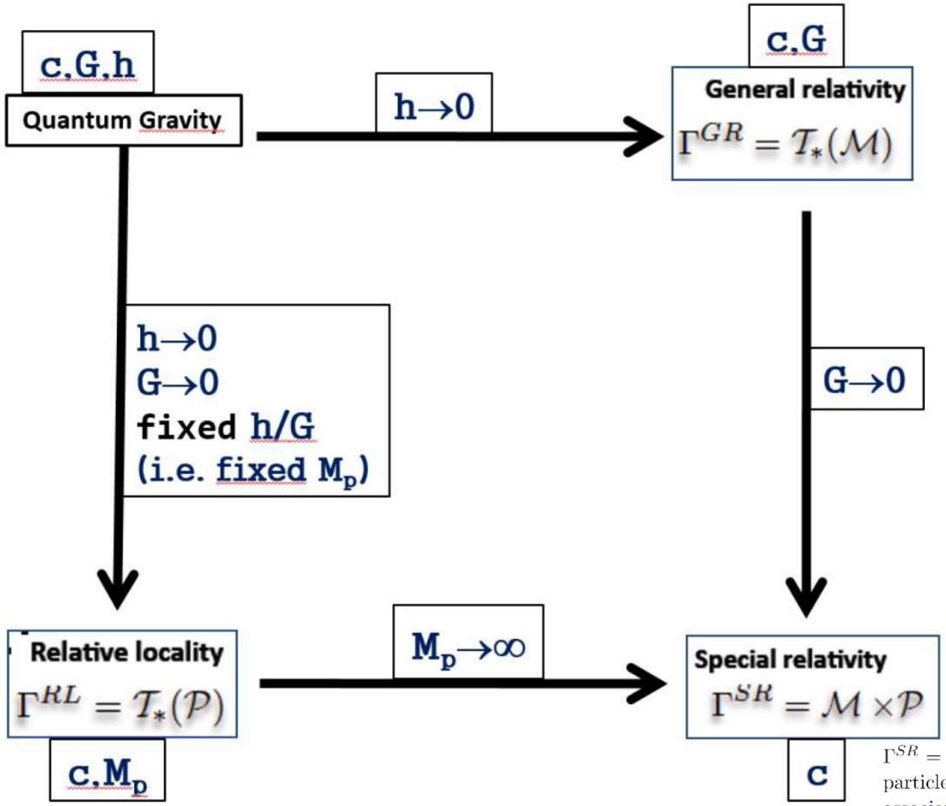
$$\varepsilon(N_j) = 0 \, , \quad \varepsilon(R_k) = 0 \, , \quad S(N_j) = -e^{P_0/\kappa} N_j + \frac{i}{\kappa}\epsilon_{jkl} e^{\lambda P_0} P_k R_l \, , \quad S(R_k) = -R_k$$



GAC+Freidel+KowalskiGlikman+Smolin, PhysRevD84,084010 (2011)

relative-locality regime:

$$\hbar \to 0\ , \quad G_N \to 0\ , \quad \text{but with fixed} \sqrt{\frac{c^3\,\hbar}{G_N}} = E_p$$



$\Gamma^{SR} = \mathcal{M} \times \mathcal{P}$. In general relativity, the spacetime manifold $\mathcal{M}$ has a curved geometry, and the particle phase space is no longer a product. Instead, there is a separate momentum space, $\mathcal{P}_x$ associated to each spacetime point $x \in \mathcal{M}$. This is identified with the cotangent space of $\mathcal{M}$ at x, so that $\mathcal{P}_x = \mathcal{T}^*_x(\mathcal{M})$. The whole phase space is the cotangent bundle of $\mathcal{M}$, *i.e.* $\Gamma^{GR} = \mathcal{T}^*(\mathcal{M})$. In a regime dual to the general-relativity regime one would have a momentum space $\mathcal{P}$ that is curved. Then there must be a separate spacetime $\mathcal{M}_p$ (flat since, in this regime we envision, one has sent $G \to 0$) for each value of momentum, $\mathcal{M}_p = \mathcal{T}^*_p(\mathcal{P})$, and the whole phase space is the cotangent bundle over momentum space, *i.e.* $\Gamma^{RL} = \mathcal{T}^*(\mathcal{P})$. From this perspective the quantum-gravity problem is the problem of finding a formalization of the more general theory, admitting as limiting cases the regime symbolized by $\Gamma^{GR} = \mathcal{T}^*(\mathcal{M})$ and the regime symbolized by $\Gamma^{RL} = \mathcal{T}^*(\mathcal{P})$.

GAC+Freidel+KowalskiGlikman+Smolin, PhysRevD84,084010 (2011)

mass of a particle with four-momentum p_μ is determined by the metric geodesic distance on momentum space from p_μ to the origin of momentum space

$$m^2 = d_\ell^2(p,0) = \int dt \sqrt{g^{\mu\nu}(\gamma^{[A;p]}(t))\dot{\gamma}_\mu^{[A;p]}(t)\dot{\gamma}_\nu^{[A;p]}(t)}$$

where $\gamma^{[A;p]}_\mu$ is the metric geodesic connecting the point p_μ to the origin of momentum space

with $A^{\mu\nu}{}_\lambda$ the Levi-Civita connection

$$\frac{d^2\gamma_\lambda^{[A]}(t)}{dt^2} + A^{\mu\nu}{}_\lambda \frac{d\gamma_\mu^{[A]}(t)}{dt}\frac{d\gamma_\nu^{[A]}(t)}{dt} = 0$$

the affine connection on momentum space determines the law of composition of momenta,

$q \oplus k$

and it might not be the Levi-Civita connection of the metric on momentum space

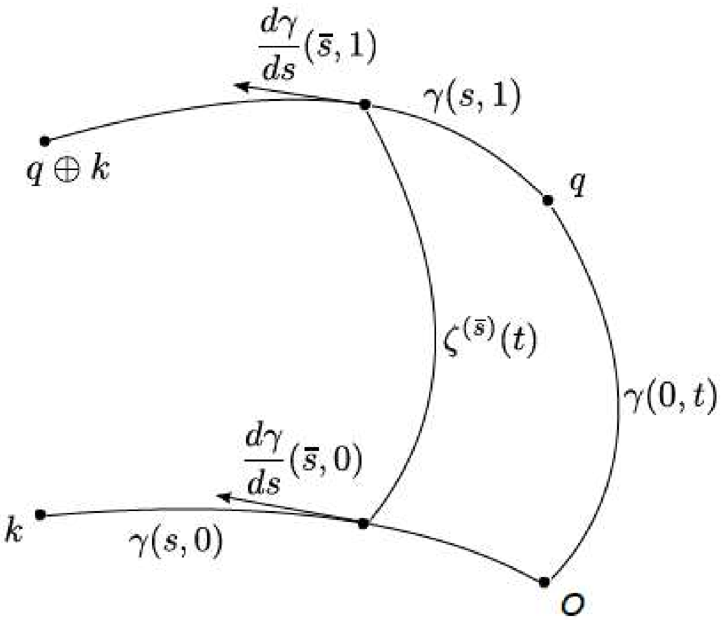


Figure 1. We determine the law of composition of momenta from the affine connection by associating to the points q and k of momentum space the connection geodesics $\gamma^{(q)}$ and $\gamma^{(k)}$ which connect them to the origin of momentum space. We then introduce a third curve $\bar{\gamma}(s)$, which we call the parallel transport of $\gamma^{(k)}(s)$ along $\gamma^{(q)}(t)$, such that for any given value $\bar{s}$ of the parameter s one has that the tangent vector $\frac{d}{ds}\bar{\gamma}(\bar{s})$ is the parallel transport of the tangent vector $\frac{d}{ds}\gamma^{(k)}(\bar{s})$ along the geodesic connecting $\gamma^{(k)}(\bar{s})$ to $\bar{\gamma}(\bar{s})$. Then the composition law is defined as the extremal point of $\bar{\gamma}$, that is $q \oplus_\ell k = \bar{\gamma}(1)$.

notably if momentum space has dS or AdS geometry then the theory can be formulated as "DSR relativistic", i.e. it is a relativistic theory with two non-trivial relativistic invariants, a high speed scale (speed-of-light scale "c") and a high momentum scale (Planck scale)

GAC, grqc0012051, IntJournModPhysD11,35
hepth0012238,PhysLettB510,255
KowalskiGlikman,hepth0102098,PhysLettA286,391
LectNotesPhys669,131
Magueijo+Smolin,hepth0112090,PhysRevLett88,190403
grqc0207085,PhysRevD67,044017
GAC,grqc0207049,Nature418,34

Hopf-algebra symmetries (such as kappa-Poincarè) arise by suitable choice of non-metric affine connection on such a momentum space

Several other choices of affine connection have been shown to also produce a DSR-relativistic theory, and this in particular occurs if one takes as affine connection the metric connection
(a key example of DSR-relativistic theory which is not linked to Hopf-algebra mathematics)

GAC+Gubitosi+Palmisano,arXiv13077988,IJMPD25,1650027
Banburski+Freidel,arXiv13080300,PRD90,076010

in 3D quantum gravity

see, e.g., Freidel+Livine, PhysRevLett96,221301(2006)

consider a matter field ϕ coupled to gravity,

$$Z = \int Dg \int D\phi \, e^{iS[\phi,g]+iS_{GR}[g]}, \qquad (1)$$

where g is the space-time metric, $S_{GR}[g]$ the Einstein gravity action and $S[\phi, g]$ the action defining the dynamics of ϕ in the metric g.

integrate out the quantum gravity fluctuations and derive an *effective action* for ϕ taking into account the quantum gravity correction:

$$Z = \int D\phi e^{iS_{eff}[\phi]}.$$

the effective action obtained through this constructive procedure gives matter fields in a noncommutative spacetime (similar to, but not exactly given by, kappa-Minkowski) and with curved momentum space, as signalled in particular by the deformed on-shellness

$$\cos(E) - e^{\ell E} \frac{\sin(E)}{E} P^2 = \cos(m)$$

(anti-deSitter momentum space….”Lorentz-double” Hopf algebra…)

κ-Poincarè Hopf algebra

Lukierski+Nowicki+Ruegg+Tolstoy,PLB(1991)
Nowicki+Sorace+Tarlini,PLB(1993)
Majid+Ruegg,PLB (1994)
Lukierski+Ruegg+Zakrzewski, AnnPhys(1995)

$$[P_\mu, P_\nu] = 0 \ ,$$

$$\Delta P_0 = P_0 \otimes \mathbb{1} + \mathbb{1} \otimes P_0 \ , \quad \Delta P_j = P_j \otimes \mathbb{1} + e^{-P_0/\kappa} \otimes P_j \ ,$$

$$\varepsilon(P_\mu) = 0 \ , \qquad S(P_0) = -P_0 \ , \qquad S(P_j) = -e^{P_0/\kappa} P_j \ ,$$

$$[N_j, P_k] = i\,\delta_{jk} \left(\frac{\kappa}{2}(1 - e^{-2P_0/\kappa}) + \frac{1}{2\kappa}|\vec{P}|^2 \right) - \frac{i}{\kappa} P_j P_k \ ,$$

$$[N_j, P_0] = i\,P_j \ , \quad [R_j, P_k] = i\,\epsilon_{jkl} P_l$$

$$\Delta N_k = N_k \otimes \mathbb{1} + e^{-P_0/\kappa} \otimes N_k + \frac{i}{\kappa} \epsilon_{klm} P_l \otimes R_m \ , \quad \Delta R_j = R_j \otimes \mathbb{1} + \mathbb{1} \otimes R_j \ ,$$

$$\varepsilon(N_j) = 0 \ , \quad \varepsilon(R_k) = 0 \ , \quad S(N_j) = -e^{P_0/\kappa} N_j + \frac{i}{\kappa} \epsilon_{jkl} e^{\lambda P_0} P_k R_l \ , \quad S(R_k) = -R_k \ ,$$

equivalent curved-momentum-space picture

$$d_\ell^2 \quad \leftrightarrow \quad g_{dS}^{\mu\nu} = \begin{pmatrix} 1 & 0 \\ 0 & e^{2\ell p_0} \end{pmatrix}$$

$$\oplus_\ell \quad \leftrightarrow \quad \Gamma_\lambda^{\mu\nu} = \ell \delta_0^\mu \delta_1^\nu \delta_\lambda^1$$

A powerful perspective for phenomenology:

Calculation of the energy dependence of speed of ultrarelativistic particles in a given model used to be troublesome and cumbersome.
Those calculations have ben streamlined now that we understand those results as dual redshift on Planck-scale-curved momentum spaces:

these results so far are fully understood for the case of
[maximally symmetric curved momentum space] ⊗ [flat spacetime]

it turns out that there is a duality between this and the familiar case of
[maximally-symmetric curved spacetime] ⊗ [flat momentum space]

ordinary redshift in deSitter spacetime implies in particular that massless particles emitted with same energy but at different times from a distant source reach the detector with different energy

dual redshift in deSitter momentum space implies that massless particles emitted simultaneously but with different energies from a distant source reach the detector at different times

GAC+Barcaroli+Gubitosi+Loret,
Classical&QuantumGravity30,235002 (2013)

dual redshift on Planck-scale-curved momentum spaces (but with flat spacetime) **produces time-of-arrival effects which at leading order are of the form ($n \in \{1,2\}$)**

$$\Delta t \simeq \eta \left(\frac{E}{M_P} \right)^n t$$

and could be described in terms of an energy-dependent "physical velocity" of ultrarelativistic particles

$$v \simeq c + \eta \left(\frac{E}{M_P} \right)^n c$$

these are very small effects but (at least for the case n=1) they could cumulate to an observably large ΔT if the distances travelled T are cosmological and the energies E are reasonably high (GeV and higher)!!!

GRBs are ideally suited for testing this:
cosmological distances (established in 1997)
photons (and neutrinos) emitted nearly simultaneously
with rather high energies (GeV…..TeV…100 TeV…)

GAC+Ellis+Mavromatos+Nanopoulos+Sarkar, Nature393,763(1998)
GAC, NaturePhysics10,254(2014)

solid theory is for (curved momentum space and) flat spacetime

phenomenological opportunities are for propagation over cosmological distances, whose analysis requires curved spacetime

study of theories with both curved momentum space and curved spacetime still in its infancy

GAC+Rosati, PhysRevD86,124035(2012)
KowalskiGlikman+Rosati,ModPhysLettA28,135101(2013)
Heckman+Verlinde,arXiv:1401.1810(2014)

Jacob and Piran [JCAP0801,031(2008)] used a compelling heuristic argument for producing a formula of energy-dependent time delay applicable to FRW spacetimes, which has been the only candidate so far tested

$$\Delta t = \eta \frac{E}{M_P} D(z) \qquad \text{with} \qquad D(z) = \int_0^z d\zeta \frac{(1+\zeta)}{H_0\sqrt{\Omega_\Lambda + (1+\zeta)^3\Omega_m}}$$

where as usual H_0 is the Hubble parameter, Ω_Λ is the cosmological constant and Ω_m is the matter fraction.
However, it is now understood that Jacob-Piran formula implicitly makes restrictive assumptions about the nature of space-and-time translation transformations…next goal is to include nontrivial effects in the translation sector because explicit models suggest that the same effects affecting Lorentz sector also affect translation sector

Rosati + GAC +Marcianò +Matassa,
arXiv:1507.02056, PysRevD(2015)

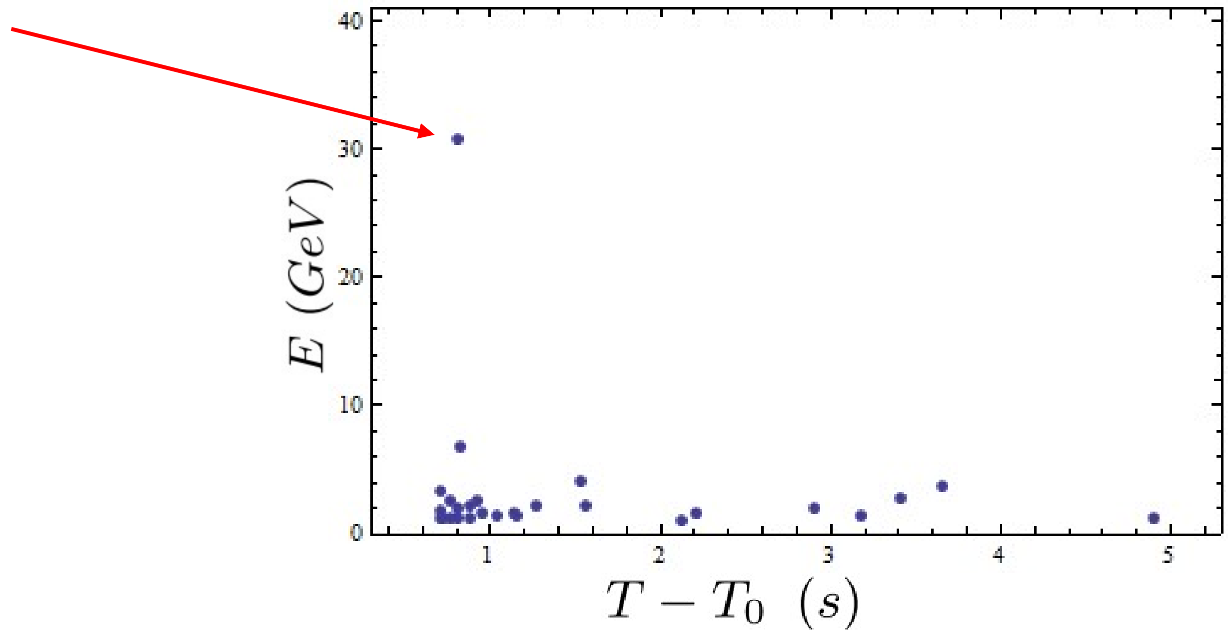
testing Jacob-Piran formula:

focus on n=1 case (sensitivity to the n=2 case still far beyond our reach presently but potentially within reach of neutrino astrophysics)

first came GRB080916C data providing a limit of $M_{QG}>10^{-1}M_{planck}$ for hard spectral lags and $M_{QG}>10^{-2}M_{planck}$ for soft spectral lags

analogous studies of blazars lead to comparable limits

then came GRB090510 (magnificent short burst) allowing to establish a limit at M_{planck} level on both signs of dispersion (soft and hard spectral lags)



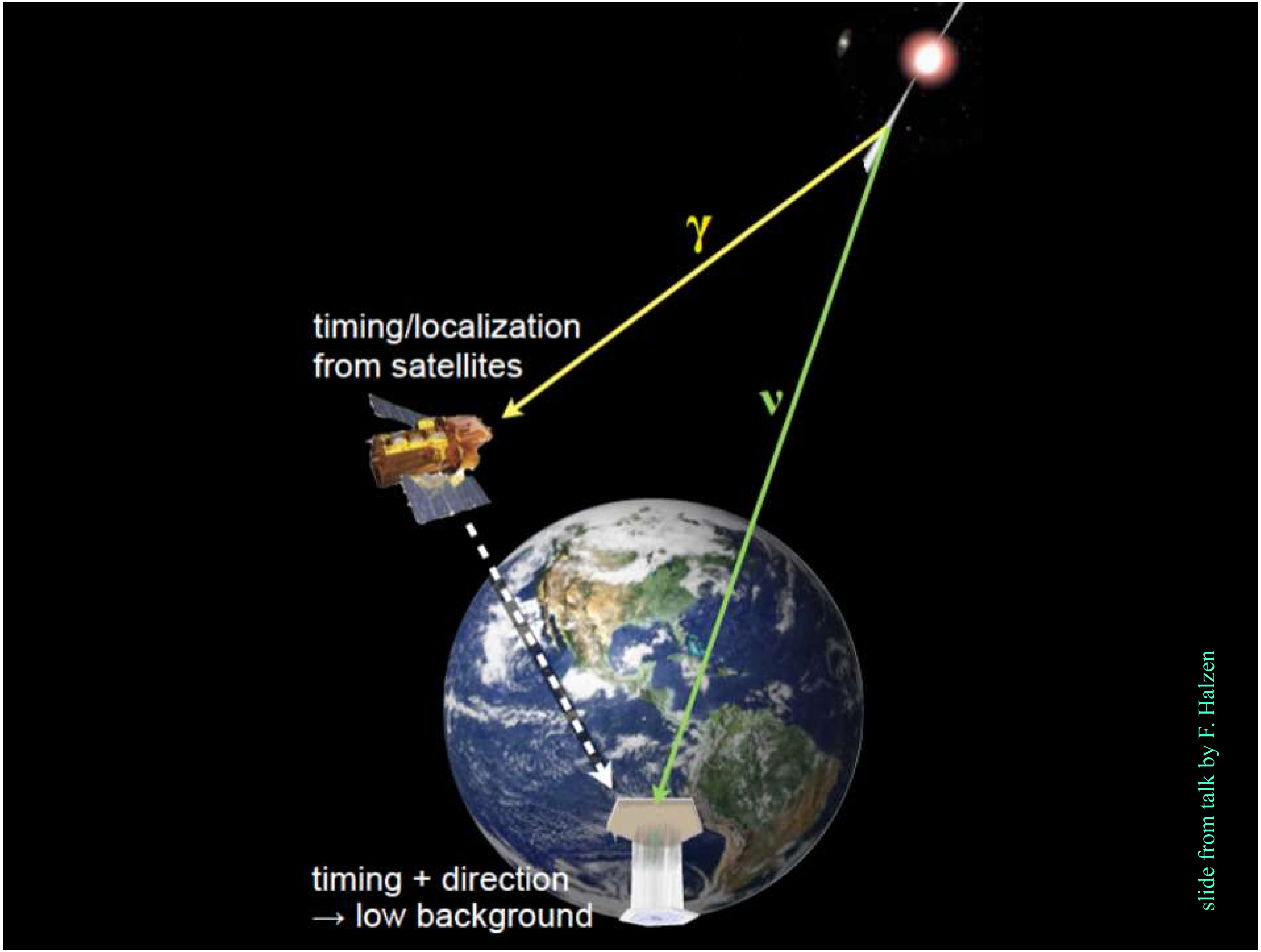
a test with accuracy of about one part in 10^{20}!!!

the whole field of quantum-gravity phenomenology is now covered by a "living review":

GAC, Living Reviews in Relativity 16 (2013) 5

my next update of the "living" review is due next year…any suggestions for additions of course welcome….

the field has grown from non-existence 15 years ago to being now a pretty large field, an effort which is articulated over several research lines, each covering a potential opportunity for the first ever discovery of a quantum-gravity effect


γ
ν
timing/localization
from satellites
timing + direction
→ low background
slide from talk by F. Halzen

GAC+Barcaroli+D'Amico+Loret+Rosati, arXiv1605.00496

IceCube has reported so far 21 shower neutrinos with energy between 60 and 500 TeV

We consider a IceCube neutrino as a "GRB-neutrino candidate" only
IF it was observed within 3 days of a GRB
AND its direction is compatible with the direction of the relevant GRB within 2 sigmas

[main novelty is wide temporal window…most other analyses assume a window of only 500 seconds…in rare cases a 1-day window…]

Assuming the validity of

$$\Delta t = \eta \frac{E}{M_P} D(z) \qquad \textbf{with} \qquad D(z) = \int_0^z d\zeta \frac{(1+\zeta)}{H_0\sqrt{\Omega_\Lambda + (1+\zeta)^3\Omega_m}}$$

we should find that at least some of our GRB-neutrino candidates have difference of time of arrival with respect to the relevant GRB which grows linearly with energy, modulo the uncertainties in redshift

in a simple-minded data analysis the values of Δt would not scale linearly with energy because of the redshift dependence, and redshift is often the least known physical quantity in the analysis

we can however absorbe the redshift dependence into an accordingly rescaled Δt which we call Δt*

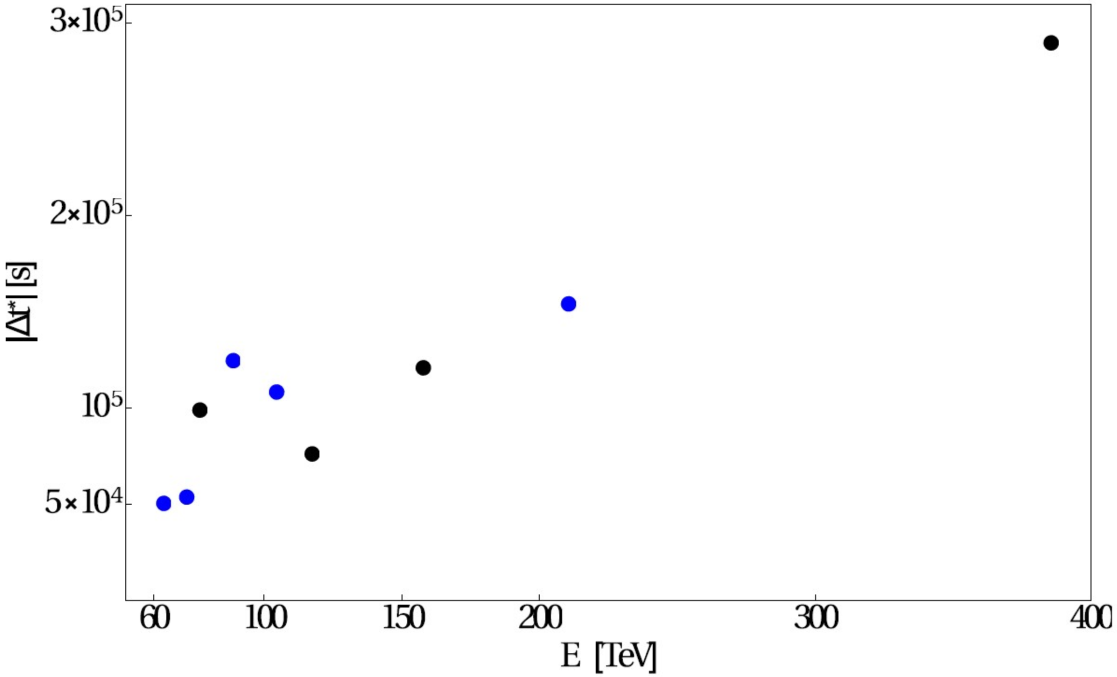
the large uncertainties in redshift will still be present, disguised as corresponding uncertainties for the determinations of Δt*, but at least we will be working with a linear relationship:

$$\Delta t^* = \eta \frac{E}{M_P} D(1)$$

$$\Delta t^* \equiv \Delta t \frac{D(1)}{D(z)} \qquad\qquad D(z) = \int_0^z d\zeta \frac{(1+\zeta)}{H_0\sqrt{\Omega_\Lambda + (1+\zeta)^3\Omega_m}}$$

Among the mentioned 21 IceCube neutrinos only 9 turned out to be "GRB-neutrino candidates" with our angular and temporal selection criteria.
So let's see if they provided some support for the linear dependence between Δt* and energy

GAC+Barcaroli+D'Amico+Loret+Rosati, arXiv1605.00496



the correlation found in data is 0.95, which is amazingly high

particularly amazing considering that our Δt^* should carry a strong uncertainty due to the limited knowledge about redshift of GRBs….we find that it makes sense if the redshift distribution of neutrino-producing GRBs is peaked at about z=1.5 with uncertainty of 0.5 (whereas the distribution of GRBs observed only in photons is found to be peaked at about z=2.1 with uncertainty of 1.3)

even more amazing considering that the expected number of background neutrinos that should sneak in our list of GRB-neutrino candidates is 4

the false alarm probability is 0.03% (probability of finding such a high correlation if all neutrinos are background neutrinos that happened to fit by accident our GRB-neutrino selection criteria)

the false alarm probability is amazingly low!!!!
but chances are the curse of this season of fundamental physics will strike again….new data should be reported by IceCube within a few months…

still interesting to devote a few thoughts to the hypothesis that this preliminary evidence actually holds up:

1. **The scale of the effect is at about 0.1 the Planck scale while bounds on the same effect for photons, even the most conservative bounds, exclude the effect up to about 1.2 Planck scale….could we have different effects for photons and neutrinos in a Hopf-algebra setup? [The difference is actually expected in a setup with effective field theory and a preferred frame]**
2. **Our 9 GRB-neutrino candidates are composed of 4 "early neutrinos" and 5 "late neutrinos". Could the sign of the effect depend on helicity in a Hopf-algebra setup? [The helicity dependence is actually expected in a setup with effective field theory and a preferred frame]**

	E[TeV]	GRB	z	Δt^* [s]	
IC9	63.2	110503A	1.613	50227	*
IC19	71.5	111229A	1.3805	53512	*
IC42	76.3	131117A	4.042	5620	
		131118A	1.497 *	-98694	*
		131119A	?	-146475	
IC11	88.4	110531A	1.497 *	124338	*
IC12	104.1	110625B	1.497 *	108061	*
IC2	117.0	100604A	?	10372	
		100605A	1.497 *	-75921	*
		100606A	?	-135456	
IC40	157.3	130730A	1.497 *	-120641	*
IC26	210.0	120219A	1.497 *	153815	*
		120224B	?	-117619	
IC33	384.7	121023A	0.6 *	-289371	*

Relative locality limit

$$\hbar \to 0\,, \quad G_N \to 0\,, \quad \text{but with fixed } \sqrt{\frac{\hbar}{G_N}} = M_p$$

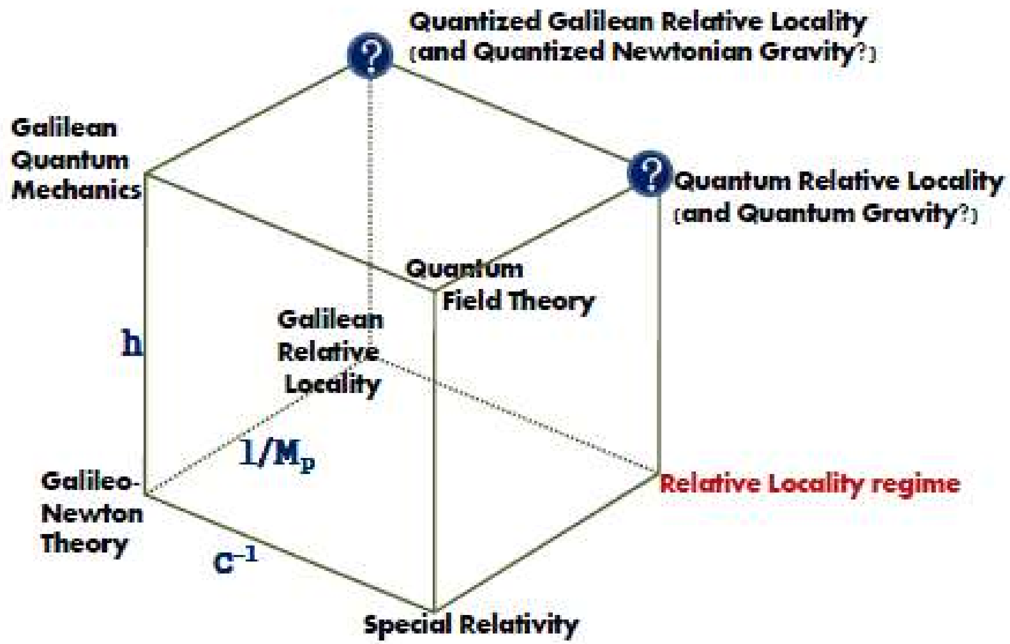
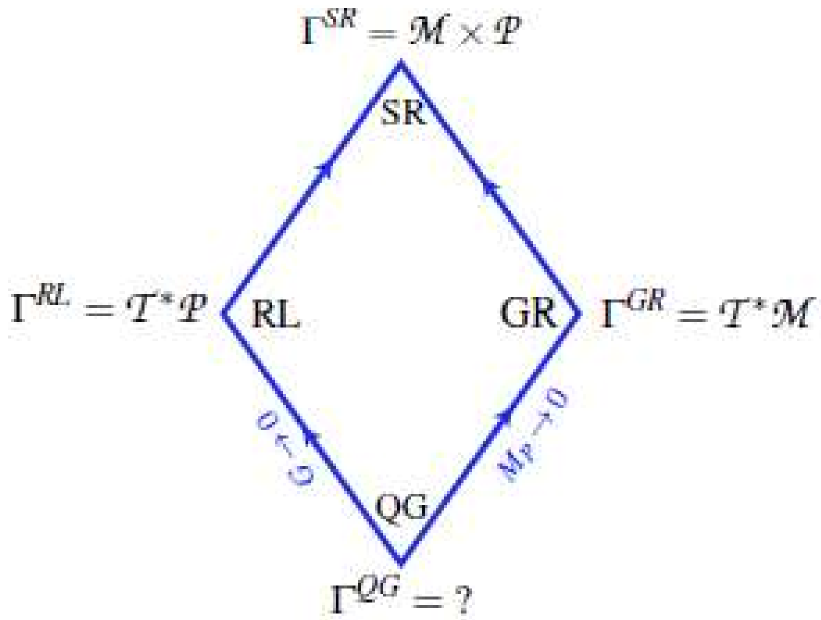


Figure 4: If the Planck scale is primitive (together with $\hbar$ and c) while Newton constant is derived, the Bronstein cube would have to be redrawn as here shown. I am assuming that the relative-locality regime is described by the relative-locality framework or something similar to it. This may also suggest that that the theory encompassing all these regimes ("quantum gravity") could be obtained as a quantum theory on the relative-locality momentum space.



These novel phenomena have a consistent mathematical description in which the notion of spacetime gives way to an invariant geometry formulated in a phase space. In special relativity, the phase space associated with each particle is a product of spacetime and momentum space, *i.e.* $\Gamma^{SR} = \mathcal{M} \times \mathcal{P}$.

In general relativity, the spacetime manifold $\mathcal{M}$ has a curved geometry. The particle phase space is no longer a product. Instead, there is a separate momentum space, $\mathcal{P}_x$ associated to each spacetime point $x \in \mathcal{M}$. This is identified with the cotangent space of $\mathcal{M}$ at x, so that $\mathcal{P}_x = \mathcal{T}_x^*(\mathcal{M})$. The whole phase space is the cotangent bundle of $\mathcal{M}$, *i.e.* $\Gamma^{GR} = \mathcal{T}^*(\mathcal{M})$

Within the framework of relative locality, it is the momentum space $\mathcal{P}$ that is curved. There then must be a separate spacetime, $\mathcal{M}_p$ for each value of momentum, $\mathcal{M}_p = \mathcal{T}_p^*(\mathcal{P})$. The whole phase space is then the cotangent bundle over momentum space, *i.e.* $\Gamma^{RL} = \mathcal{T}^*(\mathcal{P})$.