Recent progress towards the gravity/CYBE correspondence

Kentaroh Yoshida (Dept. of Phys., Kyoto U.)

For supercoset construction, see [H. Kyono and KY, 1605.02519]

For the generalized SUGRA, see [Orlando, Reffert, Sakamoto and KY, 1607.00795]

The AdS/CFT correspondence

type IIB string on $AdS_5 \times S^5$ ⟷ 4D $\mathcal{N}=4$ SU(N) SYM $(N \to \infty)$

Recent progress: the discovery of integrability

[For a big review, Beisert et al., 1012.3982]

Integrability is so powerful!

The integrable structure provides a non-perturbative method to analyze.

➡ It enables us to check the conjectured relations even at finite coupling.

EX anomalous dimensions, amplitudes etc.

Indeed, there are many directions of study on the integrability.

Here, among them, we are concerned with

the classical integrability on the string-theory side.

↳ The existence of Lax pair (kinematical integrability)

The classical integrability of the $AdS_5 \times S^5$ superstring

The coset structure of $AdS_5 \times S^5$ is closely related to the integrability.

$$\mathrm{AdS}_5 \times \mathrm{S}^5 = \frac{SO(2,4)}{SO(1,4)} \times \frac{SO(6)}{SO(5)}$$

: symmetric coset

Z_2-grading → classical integrability

Including fermions

$$\frac{PSU(2,2|4)}{SO(1,4) \times SO(5)}$$

: super coset

[Metsaev-Tseytlin, 1998]

Z_4-grading → classical integrability

elucidated by
[Bena-Polchinski-Roiban, 2003]

This fact is the starting point of our later argument.

The next issue

Integrable deformations of the $AdS_5 \times S^5$ superstring

Integrable deformations → deformed $AdS_5 \times S^5$ geometries

(as exactly solvable models) $\xrightarrow{\text{expect}}$ Solutions of type IIB SUGRA

Integrability techniques $\overset{\text{correspond?}}{\longleftrightarrow}$ solution generation techniques

EX Integrable twists — EX TsT transformations

One may consider various kinds of integrable deformations.

→ There would be many associated solutions of type IIB SUGRA.

(This is our observation)

Motive

A classification of SUGRA solutions based on integrable deformations

A systematic deformation scheme

Integrable deformation!

Yang-Baxter deformations [Klimcik, 2002, 2008]

G-principal chiral model

Yang-Baxter sigma model

$$S = \int d^2x\, \eta^{\mu\nu} \mathrm{tr}(J_\mu J_\nu) \quad \longrightarrow \quad S^{(\eta)} = \int d^2x\, \eta^{\mu\nu} \mathrm{tr}\left(J_\mu \frac{1}{1-\eta R} J_\nu\right)$$

$$J_\mu = g^{-1}\partial_\mu g, \quad g \in G$$

η : a const. parameter

What is R?

$R \ : \ \mathfrak{g} \longrightarrow \mathfrak{g}$ ⟵ a classical r-matrix satisfying

a linear op.

the modified classical Yang-Baxter eq. (mCYBE)

An integrable deformation can be specified by a classical r-matrix.

Strong advantage

Given a classical r-matrix, a Lax pair follows automatically.

No need to construct it in an intuitive manner case by case

Relation between R-operator and classical r-matrix

A linear R-operator ⟷ A skew-symmetric classical r-matrix

$R \ : \quad \mathfrak{g} \longrightarrow \mathfrak{g}$ $\qquad\qquad$ $r \in \mathfrak{g} \otimes \mathfrak{g}$

$$R(X) \equiv \langle r_{12}, 1 \otimes X \rangle = \sum_i a_i \langle b_i, X \rangle \qquad \text{for} \quad X \in \mathfrak{g}$$

$$r_{12} = \sum_i a_i \otimes b_i \qquad \text{with} \qquad a_i, b_i \in \mathfrak{g}$$

Two sources of classical r-matrix

1) modified classical Yang-Baxter eq. (mCYBE) ⟵ the original work by Klimcik

$$[R(X), R(Y)] - R\left([R(X), Y] + [X, R(Y)]\right) = \underline{-c^2[X, Y]} \qquad (\, c \in \mathbb{C} \,)$$

2) classical Yang-Baxter eq. (CYBE) $\quad (c = 0)$ ⟵ a possible generalization

The list of generalizations of Yang-Baxter deformations (2 classes)

(i) modified classical Yang-Baxter eq. (trigonometric)

a) Principal chiral model [Klimcik, hep-th/0210095, 0802.3518]

b) Symmetric coset sigma model [Delduc-Magro-Vicedo, 1308.3581]

1) c) The $AdS_5 \times S^5$ superstring [Delduc-Magro-Vicedo, 1309.5850]

(ii) classical Yang-Baxter eq. (rational)

a) Principal chiral model [Matsumoto-KY, 1501.03665]

b) Symmetric coset sigma model [Matsumoto-KY, 1501.03665]

2) c) The $AdS_5 \times S^5$ superstring [Kawaguchi-Matsumoto-KY, 1401.4855]

NOTE bi-Yang-Baxter deformation [Klimcik, 0802.3518, 1402.2105]

(applicable only for principal chiral model)

Integrable deformations of the AdS_5 x S^5 superstring

Be careful! Some different names are utilized for the deformations.

1) η-deformation (standard q-deformation) $\leftarrow$ mCYBE [Delduc-Magro-Vicedo, 1309.5850]

λ-deformation [Sfetsos, Thompson, Siampos, Hollowood-Miramontes-Schmidt]

2) Jordanian deformations (non-standard q-deformations) $\leftarrow$ CYBE

[Kawaguchi-Matsumoto-KY, 1401.4855]

The gravity/CYBE correspondence

solutions of type IIB SUGRA $\xleftrightarrow{\text{expect}}$ classical r-matrices

Conjecture

The moduli space of a certain class of solutions of type IIB SUGRA would be able to be described by the classical Yang-Baxter equation.

The current status of two kinds of YB deformations

1) *η*-deformation ← mCYBE

Gleb's talk

- A *q*-deformed classical action with classical r-matrix of Drinfeld-Jimbo type [Delduc-Magro-Vicedo, 1309.5850]

  The metric and B-field [Arutyunov-Borsaro-Frolov, 1312.3542]

- Supercoset construction [Arutyunov-Borsaro-Frolov, 1507.04239]

  → NOT satisfy the e.o.m. of type IIB SUGRA c.f., [Hoare-Tseytlin, 1508.01150]

- Solution of a generalized SUGRA eqns.
- The world-sheet theory is not Weyl invariant, but scale invariant.

  [Arutyunov-Frolov-Hoare -Roiban-Tseytlin, 1511.05795]

- Kappa-invariance and the generalized SUGRA eqns. [Wulf-Tseytlin, 1605.04884]

+ many other works

minimal surfaces, quark-antiquark potential [Kameyama-KY, 1410.5544, 1602.06786]

The current status of two kinds of YB deformations

2) Jordanian deformations $\leftarrow$ CYBE

[Kawaguchi-Matsumoto-KY, 1401.4855]

A lot of classical r-matrices

i) Partial deformations are possible (i.e., only AdS_5 or only S^5)

This is an advantage of Jordanian deformations

ii) Candidates of gravity solutions have been argued for many classical r-matrices

[Matsumoto-KY, 1404.1838, 1404.3657, 1412.3658, 1502.00740] [S.J. van Tongeren, 1506.01023]

iii) Supercoset construction has been done for some examples. [Kyono-KY, 1605.02519]

[Orlando-Reffert-Sakamoto-KY, 1607.00795]

In the following, I will introduce

(a) The deformed classical action & the outline of supercoset construction

(b) Some classical r-matrices and the resulting backgrounds

YB deformation of the $AdS_5 \times S^5$ superstring

[Delduc-Magro-Vicedo, 1309.5850]

[Kawaguchi-Matsumoto-KY, 1401.4855]

$$S = -\frac{1}{2}\int_{-\infty}^{\infty} d\tau \int_0^{2\pi} d\sigma \; P_-^{\alpha\beta}\,\mathrm{Str}\left[A_\alpha d \circ \boxed{\frac{1}{1-\eta\,[R]_g \circ d}}(A_\beta)\right]$$

R satisfies (m)CYBE.

The undeformed limit: $\eta \to 0$ ➡ the Metsaev-Tseytlin action

[Metsaev-Tseytlin, hep-th/9805028]

- Lax pair is constructed : classical integrability
- Kappa invariance : a consistency as string theory at classical level

NOTE

The difference between mCYBE and CYBE is reflected in some coefficients of some quantities such as Lax pair and kappa-transformation.

A group element: $g = g_{\rm b}\, g_{\rm f} \quad \in SU(2,2|4)$

$$g_{\rm b} = {g_{\rm b}}^{\rm AdS_5}\, {g_{\rm b}}^{\rm S^5}\,,$$

[For a big review, Arutyunov-Frolov, 0901.4937]

$$g_{\rm f} = \exp(\mathbf{Q}^I \theta_I)\,, \quad \mathbf{Q}^I \theta_I \equiv (\mathbf{Q}^{\check{\alpha}\hat{\alpha}})^I\, (\theta_{\check{\alpha}\hat{\alpha}})_I \quad (I = 1,2;\ \check{\alpha},\hat{\alpha} = 1,\ldots,4)$$

When we take a parametrization like

$$\begin{aligned} {g_{\rm b}}^{\rm AdS_5} &= \exp\Big[x^0\, P_0 + x^1\, P_1 + x^2\, P_2 + x^3\, P_3\Big]\, \exp\Big[(\log\, z)\, D\Big]\,, \\ {g_{\rm b}}^{\rm S^5} &= \exp\Big[\frac{i}{2}(\phi_1\, h_1 + \phi_2\, h_2 + \phi_3\, h_3)\Big]\, \exp\Big[\xi\, \mathbf{J}_{68}\Big]\, \exp\Big[-i\, r\, \mathbf{P}_6\Big]\,, \end{aligned}$$

the metric of AdS_5 x S^5 is given by

$$\begin{aligned} ds^2 &= ds^2_{\rm AdS_5} + ds^2_{\rm S^5}\,, \\ ds^2_{\rm AdS_5} &= \frac{-(dx^0)^2 + (dx^1)^2 + (dx^2)^2 + (dx^3)^2}{z^2} + \frac{dz^2}{z^2}\,, \\ ds^2_{\rm S^5} &= dr^2 + \sin^2 r\, d\xi^2 + \cos^2\xi\, \sin^2 r\, d\phi_1^2 + \sin^2 r\, \sin^2\xi\, d\phi_2^2 + \cos^2 r\, d\phi_3^2 \end{aligned}$$

An outline of supercoset construction

[Arutyunov-Borsato-Frolov, 1507.04239]
[Kyono-KY, 1605.02519]

The deformed action can be rewritten into the canonical form:

$$\begin{aligned} S &= -\frac{\sqrt{\lambda_c}}{4}\int_{-\infty}^{\infty} d\tau \int_0^{2\pi} d\sigma \left[\gamma^{ab}\boxed{G_{MN}}\partial_a X^M \partial_b X^N - \epsilon^{ab}\boxed{B_{MN}}\partial_a X^M \partial_b X^N\right] \\ &\quad -\frac{\sqrt{\lambda_c}}{2}\, i\bar{\Theta}_I(\gamma^{ab}\delta^{IJ} - \epsilon^{ab}\sigma_3^{IJ})\, e_a^m \Gamma_m \boxed{D_b^{JK}}\Theta_K + \mathcal{O}(\theta^4) \end{aligned}$$

This action is expanded w.r.t the fermions.

In general, the covariant derivative D is given by

[Cvetic-Lu-Pope-Stelle, hep-th/9907202]

$$\begin{aligned} D_a^{IJ} &\equiv \delta^{IJ}\left(\partial_a - \frac{1}{4}\omega_a^{mn}\Gamma_{mn}\right) + \frac{1}{8}\sigma_3^{IJ}\, e_a^m H_{mnp}\Gamma^{np} \\ &\quad -\frac{1}{8}\mathrm{e}^{\Phi}\left[\epsilon^{IJ}\Gamma^p F_p + \frac{1}{3!}\sigma_1^{IJ}\Gamma^{pqr}F_{pqr} + \frac{1}{2\cdot 5!}\epsilon^{IJ}\Gamma^{pqrst}F_{pqrst}\right] e_a^m \Gamma_m \end{aligned}$$

From this expression, one can read off all of the fields of type IIB SUGRA.

3 Examples : supercoset construction works perfectly [Kyono-KY, 1605.02519]

i) gamma-deformations of S[5] c.f. Leigh-Strassler deformation

[Matsumoto-KY, 1404.1838]

Abelian classical r-matrix: $$r = \frac{1}{8}\left(\mu_3\, h_1 \wedge h_2 + \mu_1\, h_2 \wedge h_3 + \mu_2\, h_3 \wedge h_1\right)$$

where μ_i and h_i $(i = 1, 2, 3)$ are deformation parameters and the Cartan generators of $\mathfrak{su}(4)$.

Metric: $$ds^2 = ds^2_{\mathrm{AdS}_5} + \sum_{i=1}^{3}(d\rho_i^2 + G\rho_i^2 d\phi_i^2) + \eta^2 G \rho_1^2\rho_2^2\rho_3^2 \left(\sum_{i=1}^{3}\mu_i\, d\phi_i\right)^2 ,$$

B-field: $$B_2 = \eta\, G\left(\mu_3\, \rho_1^2\rho_2^2\, d\phi_1 \wedge d\phi_2 + \mu_1\, \rho_2^2\rho_3^2\, d\phi_2 \wedge d\phi_3 + \mu_2\, \rho_3^2\rho_1^2\, d\phi_3 \wedge d\phi_1\right) ,$$

dilaton: $$\Phi = \frac{1}{2}\log G\,, \quad G^{-1} \equiv 1 + \eta^2(1 + \mu_3^2\rho_1^2\rho_2^2 + \mu_1^2\rho_2^2\rho_3^2 + \mu_2^2\rho_3^2\rho_1^2)\,, \qquad \textstyle\sum_{i=1}^{3}\rho_i^2 = 1$$

R-R: $$F_3 = -4\eta \sin^3\alpha\, \cos\alpha\, \sin\theta\, \cos\theta \left(\sum_{i=1}^{3}\mu_i\, d\phi_i\right) \wedge d\alpha \wedge d\theta\,,$$

$$F_5 = 4\left[\omega_{\mathrm{AdS}_5} + G\,\omega_{\mathrm{S}^5}\right] .$$

$$\rho_1 = \sin\alpha\cos\theta\,, \quad \rho_2 = \sin\alpha\sin\theta\,, \quad \rho_3 = \cos\alpha\,.$$

[Lunin-Maldacena, Frolov, 2005]

ii) Gravity duals for SYM on non-commutative space

c.f. Seiberg-Witten, 1999

Abelian Jordanian r-matrix: $r = \frac{1}{2}\, p_2 \wedge p_3$

[Matsumoto-KY, 1404.3657]

where $p_\mu \equiv \frac{1}{2}\gamma_\mu - m_{\mu 5}\,, \quad m_{\mu 5} = \frac{1}{4}\left[\gamma_\mu, \gamma_5\right]\,, \quad \gamma_\mu\text{: a basis of } \mathfrak{su}(2,2)$

Metric: $$ds^2 = \frac{1}{z^2}(-dx_0^2 + dx_1^2) + \frac{z^2}{z^4+\eta^2}(dx_2^2 + dx_3^2) + \frac{dz^2}{z^2} + d\Omega_5^2$$

B-field: $$B_2 = \frac{\eta}{z^4+\eta^2}\, dx^2 \wedge dx^3\,,$$ dilaton: $$\Phi = \frac{1}{2}\log\left(\frac{z^4}{z^4+\eta^2}\right)$$

R-R : $$F_3 = \frac{4\eta}{z^5}\, dx^0 \wedge dx^1 \wedge dz\,, \qquad F_5 = 4\left[\mathrm{e}^{2\Phi}\omega_{AdS_5} + \omega_{\mathrm{S}^5}\right]\,.$$

[Hashimoto-Itzhaki, Maldacena-Russo, 1999]

Note: This background can be reproduced as a special limit of η-deformed AdS_5

[Arutyunov-Borsaro-Frolov, 1507.04239] [Kameyama-Kyono-Sakamoto-KY, 1509.00173]

iii) Schrödinger spacetimes

Mixed r-matrix: $r = -\frac{i}{4} p_- \wedge (h_4 + h_5 + h_6)$

[Matsumoto-KY, 1502.00740]

Metric: $$ds^2 = \frac{-2dx^+dx^- + (dx^1)^2 + (dx^2)^2 + dz^2}{z^2} - \eta^2 \frac{(dx^+)^2}{z^4} + ds^2_{\mathrm{S}^5}$$

B-field: $$B_2 = \frac{\eta}{z^2} dx^+ \wedge (d\chi + \omega)\,,$$

dilaton: $$\Phi = \mathrm{const.}$$

The R-R sector is the same as AdS_5 x S^5.

[Herzog-Rangamani-Ross, 0807.1099]
[Maldacena-Martelli-Tachikawa, 0807.1100]
[Adams-Balasubramanian-McGreevy, 0807.1111]

S^5 -coordinates: $$ds^2_{\mathrm{S}^5} = (d\chi + \omega)^2 + ds^2_{\mathbb{C}\mathrm{P}^2}\,,$$
$$ds^2_{\mathbb{C}\mathrm{P}^2} = d\mu^2 + \sin^2\mu \left(\Sigma_1^2 + \Sigma_2^2 + \cos^2\mu\, \Sigma_3^2\right)$$

NOTE: the dilaton and R-R sector have not been deformed.

In the middle of computation, the fermionic sector becomes really messy and quite complicated. So the cancellation of the deformation effect seems miraculous.

So far, we have discussed abelian classical r-matrices.

When the r-matrix is given by $r = a \wedge b$ and $a,\ b$ commutes with each other, the classical r-matrix is called ``abelian''. If not, it is ``non-abelian''.

It seems likely that the supercoset construction works perfectly and leads to complete solutions of type IIB SUGRA.

Here we have shown only 3 examples, but already checked many abelian examples.

[Kyono-Sakamoto-KY, 1607.NNNNN]

However, for non-abelian classical r-matrices, it is not the case.

We have tried some examples of non-abelian classical r-matrices, but none of them has led to a complete solution of type IIB SUGRA.

An example of non-abelian classical r-matrix

$$
\begin{aligned}
r &= E_{24} \wedge (c_1 E_{22} - c_2 E_{44}) \\
&= (p_0 - p_3) \wedge \left[a_1 \left(\frac{1}{2}\gamma_5 - n_{03} \right) - a_2 \left(n_{12} - \frac{i}{2} \mathbf{1}_4 \right) \right]
\end{aligned}
$$

$$
a_1 \equiv \frac{c_1 + c_2}{2} = \mathrm{Re}(c_1)\,, \qquad a_2 \equiv \frac{c_1 - c_2}{2\,i} = \mathrm{Im}(c_1)
$$

The resulting background: [Kyono-KY, 1605.02519]

$$
\begin{aligned}
ds^2 &= \frac{-2dx^+dx^- + d\rho^2 + \rho^2 d\phi^2 + dz^2}{z^2} - 4\eta^2 \left[(a_1^2 + a_2^2)\frac{\rho^2}{z^6} + \frac{a_1^2}{z^4} \right] (dx^+)^2 + ds^2_{\mathrm{S}^5}\,, \\
B_2 &= 8\eta \left[\frac{a_1 x^1 + a_2 x^2}{z^4} dx^+ \wedge dx^1 + \frac{a_1 x^2 - a_2 x^1}{z^4} dx^+ \wedge dx^2 + a_1 \frac{1}{z^3} dx^+ \wedge dz \right]\,, \\
F_3 &= 8\eta \left[\frac{a_2 x^1 - a_1 x^2}{z^5} dx^+ \wedge dx^1 \wedge dz + \frac{a_1 x^1 + a_2 x^2}{z^5} dx^+ \wedge dx^2 \wedge dz + \frac{a_1}{z^4} dx^+ \wedge dx^1 \wedge dx^2 \right]\,, \\
F_5 &= \text{undeformed}\,, \qquad \Phi = \text{const}
\end{aligned}
$$

$$
\boxed{dF_3 = 16\eta\, \frac{a_1}{z^5}\, dx^+ \wedge dx^1 \wedge dx^2 \wedge dz \neq 0}
$$

The Bianchi identity is broken.

The e.o.m. of B_2 is not satisfied as well.

Some comments

- Special case

The case with a1 = 0 is special. Then the classical r-matrix becomes abelian.

The background becomes a complete solution of type IIB SUGRA.

$\Rightarrow$ The Hubeney-Rangamani-Ross solution [hep-th/0504034]

- General case

The resulting background is not a solution of type IIB SUGRA,

but still satisfies the generalized equations.

Then one can perform ``T-dualities'' for this solution.

(a kind of generalized Buscher's rule)

Remarkably, this ``T-dualized'' background can be rewritten

into the undeformed $AdS_5 \times S^5$! [Orlando-Reffert-Sakamoto-KY, 1607.00795]

The generalized equations of type IIB SUGRA

[Arutyunov-Frolov-Hoare-Roiban-Tseytlin, 1511.05795]

$$
\begin{aligned}
& R_{MN} - \frac{1}{4} H_{MKL} H_N{}^{KL} - T_{MN} + D_M \boxed{X_N} + D_N \boxed{X_M} = 0 \,, \\
& \frac{1}{2} D^K H_{KMN} + \frac{1}{2} F^K F_{KMN} + \frac{1}{12} F_{MNKLP} F^{KLP} = \boxed{X^K} H_{KMN} + D_M \boxed{X_N} - D_N \boxed{X_M} \\
& R - \frac{1}{12} H^2 + 4 D_M \boxed{X^M} - \boxed{4 X_M X^M} = 0 \,, \\
& D^M \mathcal{F}_M - \boxed{Z^M} \mathcal{F}_M - \frac{1}{6} H^{MNK} \mathcal{F}_{MNK} = 0 \,, \qquad \boxed{I^M} \mathcal{F}_M = 0 \,, \\
& D^K \mathcal{F}_{KMN} - \boxed{Z^K} \mathcal{F}_{KMN} - \frac{1}{6} H^{KPQ} \mathcal{F}_{KPQMN} - (\boxed{I} \wedge \mathcal{F}_1)_{MN} = 0 \,, \\
& D^K \mathcal{F}_{KMNPQ} - \boxed{Z^K} \mathcal{F}_{KMNPQ} + \frac{1}{36} \epsilon_{MNPQRSTUVW} H^{RST} \mathcal{F}^{UVW} - (\boxed{I} \wedge \mathcal{F}_3)_{MNPQ} = 0
\end{aligned}
$$

$$\mathcal{F}_{n_1 n_2 \ldots} = \mathrm{e}^{\Phi} F_{n_1 n_2 \ldots}$$

$$T_{MN} \equiv \frac{1}{2} \mathcal{F}_M \mathcal{F}_N + \frac{1}{4} \mathcal{F}_{MKL} \mathcal{F}_N{}^{KL} + \frac{1}{4 \times 4!} \mathcal{F}_{MPQRS} \mathcal{F}_N{}^{PQRS} - \frac{1}{4} G_{MN} \left(\mathcal{F}_K \mathcal{F}^K + \frac{1}{6} \mathcal{F}_{PQR} \mathcal{F}^{PQR} \right)$$

Modified Bianchi identities

$$
\begin{aligned}
& (d\mathcal{F}_1 - \boxed{Z} \wedge \mathcal{F}_1)_{MN} - \boxed{I^K} \mathcal{F}_{MNK} = 0 \,, \\
& (d\mathcal{F}_3 - \boxed{Z} \wedge \mathcal{F}_3 + H_3 \wedge \mathcal{F}_1)_{MNPQ} - \boxed{I^K} \mathcal{F}_{MNPQK} = 0 \,, \\
& (d\mathcal{F}_5 - \boxed{Z} \wedge \mathcal{F}_5 + H_3 \wedge \mathcal{F}_3)_{MNPQRS} + \frac{1}{6} \epsilon_{MNPQRSTUVW} \boxed{I^T} \mathcal{F}^{UVW} = 0
\end{aligned}
$$

New ingredients: $X, \quad I, \quad Z$ 3 vector fields

But $X_M \equiv I_M + Z_M$, hence two of them are independent.

Then I & Z satisfy the following relations:

$$D_M I_N + D_N I_M = 0\,, \quad D_M I_N - D_N I_M + I^K H_{KMN} = 0\,, \quad I^M Z_M = 0$$

Assuming that I is chosen such that the Lie derivative

$$(\mathcal{L}_I B)_{MN} = I^K \partial_K B_{MN} + B_{KN} \partial_M I^K - B_{KM} \partial_N I^K$$

vanishes, the 2nd equation above can be solved by

$$Z_M = \partial_M \Phi - B_{MN} I^N \quad .$$

Thus only I is independent after all.

Note: When $I = 0$, the usual type IIB SUGRA is reproduced.

For the present case,

$$\boxed{I = -\frac{2\eta a_1}{z^2}\, dx^+ \,, \quad Z_M = 0}$$

[Orlando-Reffert-Sakamoto-KY, 1607.00795]

Performing ``T-dualities'' along x^+, x^-, ϕ_1, ϕ_2

$$\begin{aligned}
ds^2 &= -2z^2\, dx^+ dx^- + \frac{(d\rho - \eta a_1 \rho\, dx^-)^2 + \rho^2 (d\theta + \eta a_2\, dx^-)^2 + (dz - \eta a_1 z\, dx^-)^2}{z^2} \\
&\quad + dr^2 + \sin^2 r\, d\xi^2 + \frac{d\phi_1^2}{\cos^2\xi \sin^2 r} + \frac{d\phi_2^2}{\sin^2 r \sin^2 \xi} + \cos^2 r\, d\phi_3^2 \,, \\
\mathcal{F}_5 &= \frac{4i\rho}{z^3 \sin\xi \cos\xi \sin^2 r} (d\rho - \eta a_1 z\, dx^-) \wedge (d\theta + \eta a_2\, dx^-) \wedge (dz - \eta a_1 z\, dx^-) \wedge d\phi_1 \wedge d\phi_2 \\
&\quad + 4iz^2 \sin r \cos r\, dx^+ \wedge dx^- \wedge dr \wedge d\xi \wedge d\phi_3 \,, \\
\Phi &= -2\eta a_1 x^- + \log\left[\frac{z^2}{\sin^2 r \sin\xi \cos\xi}\right]
\end{aligned}$$

The other components are zero.

This is a solution of the usual type IIB SUGRA.

By performing the coordinate transformations,

$$\rho = \tilde{\rho}\,\mathrm{e}^{\eta\, x^-}\,, \quad z = \tilde{z}\,\mathrm{e}^{\eta\, x^-}\,, \quad \theta = \tilde{\theta} - \eta a_2 x^-\,, \quad \boxed{x^- = \frac{1}{2\eta}\log(2\eta\,\tilde{x}^-)\,,}$$

the ``T-dualized'' background can be rewritten as

$$\begin{aligned} ds^2 &= -2\tilde{z}^2\,dx^+ d\tilde{x}^- + \frac{d\tilde{\rho}^2 + \tilde{\rho}^2 d\tilde{\theta}^2 + d\tilde{z}^2}{\tilde{z}^2} \\ &\quad + dr^2 + \sin^2 r\,d\xi^2 + \frac{d\phi_1^2}{\cos^2\xi\,\sin^2 r} + \frac{d\phi_2^2}{\sin^2 r\,\sin^2\xi} + \cos^2 r\,d\phi_3^2\,, \\ \mathcal{F}_5 &= \frac{4i\tilde{\rho}}{\tilde{z}^3\sin\xi\cos\xi\sin^2 r}\,d\tilde{\rho}\wedge d\tilde{\theta}\wedge d\tilde{z}\wedge d\phi_1\wedge d\phi_2 \\ &\qquad + 4i\tilde{z}^2\sin r\cos r\,dx^+\wedge d\tilde{x}^-\wedge dr\wedge d\xi\wedge d\phi_3\,, \\ \Phi &= \log\left[\frac{\tilde{z}^2}{\sin^2 r\sin\xi\cos\xi}\right] \end{aligned}$$

After doing T-dualities along $x^0,\ x^3,\ \phi_1,\ \phi_3$, the undeformed AdS_5 x S^5 is reproduced.

[Orlando-Reffert-Sakamoto-KY, 1607.00795]

Can one interpret it as a twisted b.c. + alpha ? c.f., Frolov, Alday-Arutyunov-Frolov

Undoing a Drinfeld twist?

NOTES: We have studied other examples of non-abelian classical r-matrices.

[Orlando-Reffert-Sakamoto-KY, 1607.00795]

All of them satisfy the generalized equations,

but other properties are different depending on the cases.

- For some examples, ``T-dualized'' backgrounds are not equivalent to the undeformed $AdS_5 \times S^5$.
- For others, it seems that even ``T-dualized'' background would not exist.

⇒ It is very important to classify non-abelian classical r-matrices.

EX What kinds of classical r-matrix does lead to the undeformed $AdS_5 \times S^5$?

c.f., an argument based on scaling limits of the η-model

[Hoare-van Tongeren, 1605.03554]

Summary

Towards the gravity/CYBE correspondence

It works perfectly for abelian classical r-matrices.

EX Lunin-Maldacena-Frolov, Maldacena-Russo, Schrödinger spacetimes

Non-abelian classical r-matrices lead to solutions of the generalized SUGRA.

Further generalizations

1) YB deformations of non-integrable backgrounds:

EX $AdS_5 \times T^{1,1}$: non-integrability & chaos

[Basu-Pando Zayas, 1103.4107]
[Asano-Kawai-Kyono-KY, 1505.07583]

YB deformations can capture TsT trans. of $T^{1,1}$ as well. [Crichigno-Matsumoto-KY, 1406.2249]

2) YB deformations of 4D Minkowski space:

[Matsumoto-Orlando-Reffert-Sakamoto-KY, 1505.04553]
[Pachol, van Tongeren, 1510.02389]
[Borowiec-Kyono-Lukierski-Sakamoto-KY, 1510.03083]
[Kyono-Sakamoto-KY, 1512.00208]

Melvin, pp-wave, T-duals of (A)dS,
Hashimoto-Sethi, Spradlin-Takayanagi-Volovich, etc.

The limitation of the gravity/CYBE correspondence?

To what extent the correspondence can work?

What is the mathematical interpretation of YB deformations?

What are the associated deformations of N=4 SYM?

There are many open problems.

It is of great significance to reveal the fundamental aspects of Yang-Baxter deformations

Thank you!

Back up

Yang-Baxter deformations of 4D Minkowski spacetime

r-matrix	Type of Twist	Background
$p_i \wedge p_j \ (i,j=1,2,3)$	Melvin Shift Twist	Seiberg-Witten
$p_0 \wedge p_i$	Melvin Shift Twist	NCOS
$(p_0 + p_i) \wedge p_j \ (i \neq j)$	Null Melvin Shift Twist	light-like NC
$\frac{1}{2} p_3 \wedge n_{12}$	Melvin Twist	T–dual Melvin
$\frac{1}{2\sqrt{2}} p_2 \wedge (n_{01} + n_{13})$	Melvin Null Twist	Hashimoto–Sethi
$\frac{1}{2} n_{12} \wedge n_{03}$	R Melvin R Twist	Spradlin–Takayanagi–Volovich
$\frac{1}{2} p_1 \wedge n_{03}$	Melvin Boost Twist	T–dual of Grant space
$\frac{1}{2\sqrt{2}} (p_0 - p_3) \wedge n_{12}$	Null Melvin Twist	pp-wave
$\frac{1}{2\sqrt{2}} (\hat{d} - n_{03}) \wedge (p_0 - p_3)$	Non-Twist	pp-wave
$\frac{1}{2} \hat{d} \wedge p_0$	Non-Twist	T–dual of dS_4
$\frac{1}{2} \hat{d} \wedge p_1$	Non-Twist	T–dual of AdS_4
DJ-type (mCYBE)	Non-Twist	q-deformation (?)

Question

How prevalent is integrability in various kinds AdS/CFTs?

NOTE: Integrable subsectors are ubiquitous. EX integrable subsectors in large N QCD in 4 D

So we will concentrate on the full integrability below.

There are various kinds of AdS/CFT

Real (or complex) beta-deformations [Lunin-Maldacena, Frolov]

Gravity duals for NC gauge theories [Hashimoto-Itzhaki, Maldacena-Russo]

AdS_5 x $T^{1,1}$ [Klebanov-Witten] AdS_5 x $Y^{p,q}$ [Gauntlett-Martelli-Sparks-Waldram]

Klebanov-Strassler, Maldacena-Nunez

AdS BH [Horowitz-Strominger] AdS solitons [Witten, Horowitz-Myers]

AdS/NRCFT [Son,Balasubramanian-McGreevy, Kachru-Liu-Mulligan]

q-deformation of AdS_5xS^5 [Delduc-Magro-Vicedo, Arutyunov-Borsato-Frolov]

A classification list

(would not be complete)

Integrable backgrounds

Real beta-deformations [Frolov, hep-th/0503201]

Gravity duals for NC gauge theories [Matsumoto-KY, 1403.2703]

q-deformation of $AdS_5 \times S^5$ [Delduc-Magro-Vicedo, 1309.5850, Arutyunov-Borsato-Frolov, 1312.3542]

TsT transformatios of AdS_5 [Hubeny-Rangamani-Ross , hep-th/0504034] [Dhokarh-Haque-Hashimoto , 0801.3812] [Kawaguchi-Matsumoto-KY, 1401.4855]

Non-integrable backgrounds

Complex beta-deformations [Giataganas-Pando Zayas-Zoubos, 1311.3241]

$AdS_5 \times T^{1,1}$ [Basu-Pando Zayas, 1103.4107]

$AdS_5 \times Y^{p,q}$ [Basu-Pando Zayas, 1105.2540]

AdS BH [Pando Zayas-Terrero Escalante, 1007.0277]

AdS solitons [Basu-Das-Ghosh, 1103.4101]

Klebanov-Strassler, Maldacena-Nunez [Basu-Das-Ghosh-Pando Zayas, 1201.5634]

Schrödinger spacetime with $z = 4, 5, 6$ [Giataganas-Sfetsos, 1403.2703]

Lifshitz space (with hyper-scaling violation) [Giataganas-Sfetsos, 1403.2703] [Bai-Chen-Lee-Moon, 1406.5816]

p-brane backgrounds [Stepanchuk-Tseytlin, 1211.3727] [Chervonyi-Lunin, 1311.1521]

Question

How prevalent is integrability in various kinds AdS/CFTs?

NOTE: Integrable subsectors are ubiquitous. EX integrable subsectors in large N QCD in 4 D

So we will concentrate on the full integrability below.

There are various kinds of AdS/CFT

Real (or complex) beta-deformations [Lunin-Maldacena, Frolov]

Gravity duals for NC gauge theories [Hashimoto-Itzhaki, Maldacena-Russo]

AdS_5 x $T^{1,1}$ [Klebanov-Witten] AdS_5 x $Y^{p,q}$ [Gauntlett-Martelli-Sparks-Waldram]

Klebanov-Strassler, Maldacena-Nunez

AdS BH [Horowitz-Strominger] AdS solitons [Witten, Horowitz-Myers]

AdS/NRCFT [Son,Balasubramanian-McGreevy, Kachru-Liu-Mulligan]

q-deformation of AdS_5xS^5 [Delduc-Magro-Vicedo, Arutyunov-Borsato-Frolov]

Definitions of the quantities

Maurer-Cartan 1-form

$$A_\alpha \equiv g^{-1}\partial_\alpha g\,, \quad g \in SU(2,2|4) \quad ,$$

Projection on the group manifold

$$d \equiv P_1 + 2P_2 - P_3$$

Projection on the world-sheet

$$P_\pm^{\alpha\beta} \equiv \frac{1}{2}(\gamma^{\alpha\beta} \pm \epsilon^{\alpha\beta})$$

$$\left[\begin{array}{l} \gamma^{\alpha\beta} = \mathrm{diag}(-1,1) \\ \epsilon^{\alpha\beta}:\ \text{anti-symm. tensor} \end{array}\right.$$

A chain of operations

$$R_g(X) \equiv g^{-1}R(gXg^{-1})g\,, \qquad \forall X \in \mathfrak{su}(2,2|4)$$